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STAR COLOURING OF CENTRAL GRAPHS

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ABSTRACT

In this paper, we discuss about the star colouring and its chromatic number of a Central graph of Complete bipartite graph, Complete graph, Cycle, Path denoted by $C(K_{m,n}),C(K_n),C(C_n),C(P_n)$ *respectively.*

Keywords: *Chromatic number, Central graph, Star colouring, Star chromatic number.*

Subject Classification: *05C15, 05C75.*

1. INTRODUCTION

Let G be a finite undirected graph with no loops and multiple edges. The Central graph of a graph G [11], $C(G)$ is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G.

The notion of star chromatic number was introduced by Grunbaum in 1973. A Star colouring of a graph G [1, 4, 5] is a proper vertex colouring (no two adjacent vertices of G have the same colour) such that every path of G on four vertices is not bicoloured. The minimum number of colours needed to star colour G is called as Star chromatic number and is denoted by $Xs\ (G)$.

A number of results exist for star colourings of graphs formed by certain graph operations. Guillaume Fertin et al. [4] gave the exact value of the star chromatic number of different families of graphs such as Trees, Cycles, Complete bipartite graphs, Outerplanar graphs and 2-dimensional grids. They also investigated and gave bounds for the star chromatic number of other families of graphs, such as planargraphs, hypercubes, d-dimensional grids ($d \geq 3$), d-dimensional tori ($d \geq 2$), graphs with bounded tree width and cubic graphs.

Albertson *et al.* [1] showed that it is NP-complete to determine whether $Xs\ (G) \leq 3$, even when G is a graph that is both planar and bipartite. The problem of finding star colouring is NP-hard and remains so even for bipartite graphs [9].

2. THE STAR COLOURING OF $C\ (K_{m,n})$

2.1 Definition: A graph whose vertices can be partitioned into two sets such that every edge joins a vertex in one set with a vertex in the other and each vertex in one set is joined to each vertex in the other by exactly one edge.

2.2 Theorem: Let $K_{m,n}$ be a Complete bipartite graph on m and n vertices. Then $X_s\ [C\ (K_{m,n})] = \text{Max}\ \{m, n\}$.

Proof: Consider the Complete bipartite graph $K_{m,n}$ with bipartitions (X,Y) where $X=\{v_1,v_2,\ldots\ldots\ldots v_m\}$ and $Y=\{u_1,u_2,\ldots\ldots\ldots u_n\}$ in $C(K_{m,n})$. Let v_i and u_j be the vertices of $K_{m,n}$ where $\{v_i\!: 1 \leq i \leq m\}$ and $\{u_j\!: 1 \leq j \leq n\}$. By the definition of Complete bipartite graph, every vertex from $v_{i:}\ 1 \leq i \leq m$ is adjacent to every vertex in $u_j\!: 1 \leq j \leq n$. Let $e_{ij}\!: 1 \leq i \leq m, 1 \leq j \leq n\}$ be the set of edges of $K_{m,n}$. By the definition of Central graph, the edges $\{e_{ij}\!: 1 \leq i \leq m, 1 \leq j \leq n\}$ be subdivided by the vertex $\{w_{ij}\!: 1 \leq i \leq m, 1 \leq j \leq n\}$ in $C(K_{m,n})$. Let w_{ij} represents the newly added vertex in the edge joining v_i and u_j in $C\ (K_{m,n})$. So $(v_{i:}\ 1 \leq i \leq m),(u_j : 1 \leq j \leq n)$ are complete graphs in $C\ (K_{m,n})$.

Now assign a colouring to the vertices of $C\ (K_{m,n})$ as follows.

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Case 1: When $m \geq n$
Assign the colour c_i to the vertex v_i for $i=1,2,\ldots.m$ and assign the colour c_j to the vertex u_j for $j=1,2,\ldots\ldots.n$. So for a proper colouring we require minimum m colours to colour the vertices of v_i and u_j, which produces a star colouring. Next we assign a colouring to the vertices $\{w_{ij}\colon 1 \leq i\leq m,\ 1 \leq j\leq n\}$. Suppose if we assign any new colour c_{m+1} to the vertices w_{ij}, $\{w_{ij}\colon 1 \leq i\leq m,\ 1 \leq j\leq n\}$, it will not produce a star colouring because none of the vertices w_{ij} does not realize its own colour. Therefore, the only possibility is to assign an existing colour to the vertices w_{ij}. Hence by colouring procedure the above said colouring is minimal and star colouring.

Case 2: When $n \geq m$
Assign the colour c_i to the vertex v_i for $i=1,2,\ldots..m$ and assign the colour c_j to the vertex u_j for $j=1,2,\ldots\ldots\ldots..n$. So for a proper colouring we require minimum n colours to colour the vertices of v_i and u_j, which produces a star colouring. Next we assign a colouring to the vertices $\{w_{ij}\colon 1 \leq i \leq m,\ 1 \leq j \leq n\}$. Suppose if we assign any new colour c_{n+1} to the vertices w_{ij}, $\{w_{ij}\colon 1 \leq i \leq m,\ 1 \leq j \leq n\}$, it will not produce a star colouring because none of the vertices of w_{ij} does not realize its own colours. Therefore, the only possibility is to assign an existing colour to the vertices w_{ij}. Hence by colouring procedure the above said colouring is minimal and star colouring.

Case 3: When $m = n$
In this case m will become n. Thus we get $n+ 1$ complete graph for n vertices. Proceeding similarly as in case (1), we get $Xs\ [C\ (K_{m,n})] = n$

$\therefore\ X_s\ [C\ (K_{m,n})] = \text{Max}\ \{m, n\}$.

Example:

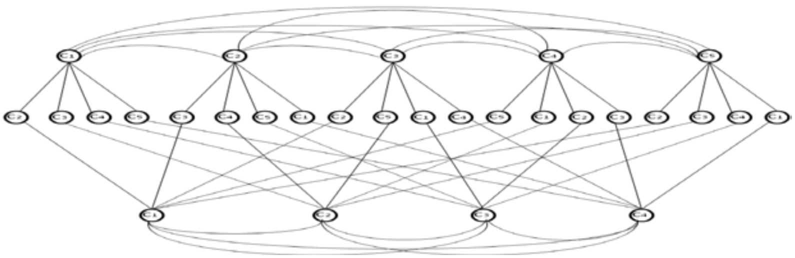

Figure: 1 $X_s\ [C\ (K_{5,\,4})] = 5$

3. STAR COLOURING OF $C\ (K_n)$

3.1 Definition: A complete graph is a simple graph G with n vertices in which every vertex is adjacent to each other. Each vertex is of degree $n\text{-}1$ and is denoted by K_n

.

3.2 Theorem: Let K_n be a complete graph on n-vertices .Then $X_s\ [C\ (K_n)] = n,\ n>3$

Proof: Consider a complete graph K_n on n-vertices, $n>3$. Let v_{ij} represents the newly added vertex in the edge joining v_i and v_j.

Now assign a colouring to the vertices of $C(K_n)$ as follows. Consider the colour class $C = \{c_1, c_2\ , c_3\ldots\ c_n\}$. Assign the colour c_i to the vertex v_i for $1 \leq i \leq n$. So for a proper colouring, we need minimum n colours to colour the vertices of v_i which results in star colouring.

Next we assign a colouring to the vertices v_{ij} for $i =1,2,\ldots\ldots\ldots n$. Suppose if we assign a new colour c_{n+1} to vertex v_{ij} for $i=1,2,\ldots\ldots n$ then it will not be a star colouring because none of the vertices v_{ij}'s realizes its own colours. Thus to make a proper star colouring, we use only the existing colours to the vertex to realizes its own colour. Therefore, we have to assign only the existing colours to the vertices of v_{ij}'s. Thus colouring procedure is minimal and thus results in star colouring.

$\therefore X_s\ [C\ (K_n)] = n,\ n>3$

Example:

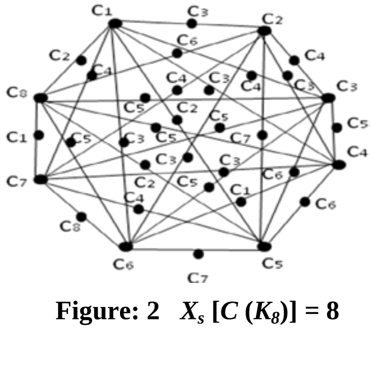

Figure: 2 $X_s\ [C\ (K_8)] = 8$

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4. STAR COLOURING OF CYCLE (C_n)

4.1 Definition: A Cycle is a circuit in which no vertex except the first (which also the last) appears more than once. A Cycle with n vertices is denoted as C_n.

4.2Theorem: Let C_n be the Cycle of length n, $n>3$ then $X_s\ [C\ (C_n)] = n\text{-}2$.

Proof: Let C_n be the Cycle of length n with vertices $v_1, v_2, v_3 \ldots\ldots v_n$. Let v_{ij} represents the newly added vertex in the edge joining v_i and v_j. Now in $C\ (C_n)$, we can note that the vertex v_i is adjacent with all vertices except v_{i+1} and v_{i-1} for $i=1,2,3,\ldots..n\text{-}1$. v_1 is adjacent with all the vertices except v_2 and v_n . v_n is adjacent with all the vertices except v_{n-1} and v_1.

Now assign a star colouring to the vertices of $C\ (C_n)$ as follows. Consider the colour class $C = \{c_1, c_2, c_3\ldots.c_n\}$. Suppose if we assign the colour c_i to the vertices v_i for $i=1, 2\ldots n$, it require n distinct colours which will not result in star colouring.

Thus we assign the same colour to the non-adjacent vertices. Assign c_i to v_i and $v_{i+1.}$ Suppose if we assign c_1 to v_1 and v_2 as they are non adjacent in $C\ (C_n)$. Assign c_2 to v_3 and v_4 as they are non adjacent in $C\ (C_n)$. Assign c_3 to v_5 and v_6 as they are non adjacent in $C\ (C_n)$. As the same, we cannot assign the same colour to all the non–adjacent vertices v_i and v_{i+1}.

Due to the above mentioned non adjacency of v_i's this colouring will not be a star colouring. Thus to make a star colouring, we should assign a proper colouring to v_{ij}'s. Suppose if we assign the new colour c_{i+1} to the vertex v_{ij}, for all $1 \le i \le n,\ 1 \le j \le n,$ which does not produces a star colouring. So we cannot assign new colours to the vertex v_{ij}. Thus assign only the existing colours to the vertices of v_{ij}'s. So the colouring procedure is minimal and thus results in star colouring.

$\therefore\ X_s\ [C\ (C_n)] = n\text{-}2$

Example:

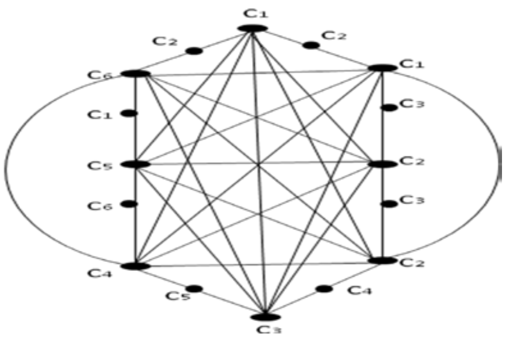

Figure: 3 $X_s[C\ (C_8)] = 6$

5. STAR COLOURING OF Path (P_n)

5.1 Definition: A Path is a sequence of consecutive edges in a graph and the length of the Path is the number of edges traversed. A Path with n vertices is denoted as P_n.

5.2 Theorem: For any Path P_n *of* $n>3$, then $X_s\ [C\ (P_n)] = n\text{-}2$.

Proof: Let P_n be the Path of length $n\text{-}1$ with vertices $v_1, v_2, v_3\ldots..v_n$. Let v_{ij} represents the newly added vertex in the edge joining v_i and v_j. Now in $C\ (P_n)$, we can note that the vertex v_i is adjacent with all vertices except v_{i+1} and v_{i-1} for $i=1, 2, 3,\ldots\ldots\ldots\ldots..n\text{-}1$. v_1 is adjacent with all the vertices except v_2 and v_n . v_n is adjacent with all the vertices except v_{n-1} and v_1 .

Now assign a star colouring to the vertices of $C\ (P_n)$ as follows. Consider the colour class $C = \{c_1, c_2, c_3\ldots.c_n\}$.Suppose assign the colour c_i to the vertices v_i for $i=1, 2,3,\ldots\ldots n$ it require n distinct colours which will not result in star colouring.

So we assign the same colour to the non-adjacent vertices. Assign c_i to v_i and v_{i+1}. Suppose if we assign c_1 to v_1 and v_2 as they are non adjacent in $C(P_n)$. Assign c_2 to v_3 and v_4 as they are non adjacent in $C(P_n)$. Assign c_3 to v_5 and v_6 as they are non adjacent in $C(P_n)$. As the same, we cannot assign the same colour to all the non–adjacent vertices $v_{i\ \&}\ v_{i+1}$.

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Due to the above mentioned non adjacency of v_i's this colouring will not be a star colouring. Thus to make a star colouring, we should assign a proper colouring to v_{ij}'s. Suppose if we assign the colour c_{i+1} to the vertex v_{ij}, for all $1 \le i \le n$, $1 \le j \le n$, which does not produces star colouring. So we cannot assign new colours to the vertex v_{ij}. Thus assign only the existing colours to the vertices of v_{ij}'s. So the colouring procedure is minimal and thus results in star colouring.

$\therefore X_s [C (P_n)] = n\text{-}2.$

Example:

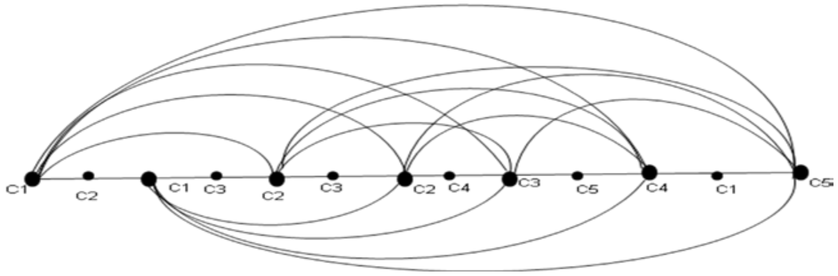

Figure: 4 $X_s [C (P_7)] = 5$

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