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EFFECT OF CONCEPTUAL ACHIEVEMENT OF CALCULUS ON ABILITY OF DOING PROBLEMS OF CALCULUS OF STUDENTS AT HIGHER SECONDARY LEVEL IN BARPETA DISTRICT OF ASSAM

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ABSTRACT

Students' ability of grasping calculus has been found to be of great concern. Students are often locked into a process-oriented style of thinking that appears as an obstacle for proper understanding the important concepts of the subject. Here, our study would be based on this area for identifying the role of conceptual achievements on various aspects of the subject. Some of these, in particular, that have been taken into consideration are functions and variables, limit and continuity, differentiation, indeterminate forms and integration to explore some concrete mathematics or statistics biased information on ability of working out problems of these topics correctly by the students at higher secondary level. Only keeping this in note, data were collected by means of two questionnaires for 106 higher secondary students and by Karl Pearson correlation coefficient as the tool. This has been analyzed and found to be fairly positive (0.818). Thus, the findings reveal a significant correlation between two factors in discussion. Moreover, the expertise of ability of working out problems of calculus has been found to be same as their conceptual achievement.

Keywords: *calculus, function, limit, continuity, indeterminate form, differentiation, integration, data, Karl Pearson correlation coefficient, random sample.*

1. INTRODUCTION

Calculus occupies an important place in higher secondary level mathematics syllabus. It is the entry-point to undergraduate mathematics, science, engineering and many other courses. Yet the students find difficult to understand various concepts of it and they fail to work out problems of calculus in examinations. There have been many research studies in the students' understanding of calculus.

Romeburg and Tufte [1] argue that students view mathematics as a static collection of concepts and skills. Because of the character of skills mathematics- the subject itself, sometime, appear as annoying one. To be specific,calculus reform programmes are now well underway in some countries in an attempt to explore the difficulties faced by the students in calculus study and to take appropriate remedial measures. Many researchers like D.O.Tall [3], [4], L.Steen [2]**,** M.Bernes [5] and P.W. Thompson [6] uncovered students' conceptual difficulties with the topics limits, differentiation and integration.

Here, the conceptual achievement and ability of doing problems of students of higher secondary level were tested on the following topics:

- Functions and variables
- Limits and continuity
- Derivatives
- Indeterminate forms
- Integrals and graphical representation of areas

2. METHODOLOGY

The research adopted the descriptive survey method. Data were collected through questionnaires. Each questionnaire contains 50 questions having four options each. Students are asked to put tick mark ($\surd$) against its correct option.

To find the correlation between the conceptual achievement of calculus and the ability of doing problems of calculus, 106 students are drawn from two leading colleges and seven higher secondary schools of Barpeta district of Assam. The researcher conducted two Mathematics Achievement Test (MAT) – one for conceptual achievement of calculus and other for ability of doing problems of calculus

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In addition to these, we tried to make a comparative study of the conceptual achievement and ability of doing problems on limit and continuity, derivative and integrals of functions defined as a (i) single formula (ii) breaking formula.

Also, we have made a comparative study on the conceptual achievement and ability of doing problems on finding the limiting value of a function appeared as indeterminate forms ($\frac{0}{0}$ and $\frac{\infty}{\infty}$)

3. Hypothesis
The investigator has made the following hypotheses:
(i) There is no difference between the conceptual achievement and ability of doing problems of calculus.
(ii) There is no correlation between the two variables of our discussion

4. Analysis of data
The original data of the scores of 106 students on the conceptual achievement of calculus are taken as *x* and on ability of doing problems of calculus are taken as *y*.

TABLE

X	Y	$\bar{x}$	$x_i - \bar{x}$	$\bar{y}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})2$	$(x_i - \bar{x})(y_i - \bar{y})$
38	31	21.26	16.74	21.75	9.25	280.2276	85.5625	154.845
39	33	21.26	17.74	21.75	11.25	314.7076	126.5625	199.575
39	42	21.26	17.74	21.75	20.25	314.7076	410.0625	359.235
39	42	21.26	17.74	21.75	20.25	314.7076	410.0625	359.235
39	33	21.26	17.74	21.75	11.25	314.7076	126.5625	199.575
40	33	21.26	18.74	21.75	11.25	351.1876	126.5625	210.825
40	33	21.26	18.74	21.75	11.25	351.1876	126.5625	210.825
40	33	21.26	18.74	21.75	11.25	351.1876	126.5625	210.825
38	42	21.26	16.74	21.75	20.25	280.2276	410.0625	338.985
40	39	21.26	18.74	21.75	17.25	351.1876	297.5625	323.265
46	45	21.26	24.74	21.75	23.25	612.0676	540.5625	575.205
45	44	21.26	23.74	21.75	22.25	563.5876	495.0625	528.215
44	40	21.26	22.74	21.75	18.25	517.1076	333.0625	415.005
38	35	21.26	16.74	21.75	13.25	280.2276	175.5625	221.805
37	42	21.26	15.74	21.75	20.25	247.7476	410.0625	318.735
39	37	21.26	17.74	21.75	15.25	314.7076	232.5625	270.535
39	40	21.26	17.74	21.75	18.25	314.7076	333.0625	323.755
26	20	21.26	4.74	21.75	-1.75	22.4676	3.0625	-8.295
18	14	21.26	-3.26	21.75	-7.75	10.6276	60.0625	25.265
27	26	21.26	5.74	21.75	4.25	32.9476	18.0625	24.395
13	13	21.26	-8.26	21.75	-8.75	68.2276	76.5625	72.275
18	19	21.26	-3.26	21.75	-2.75	10.6276	7.5625	8.965
31	28	21.26	9.74	21.75	6.25	94.8676	39.0625	60.875
19	25	21.26	-2.26	21.75	3.25	5.1076	10.5625	-7.345
10	10	21.26	-11.26	21.75	-11.75	126.7876	138.0625	132.305
11	15	21.26	-10.26	21.75	-6.75	105.2676	45.5625	69.255
23	34	21.26	1.74	21.75	12.25	3.0276	150.0625	21.315
36	34	21.26	14.74	21.75	12.25	217.2676	150.0625	180.565
30	30	21.26	8.74	21.75	8.25	76.3876	68.0625	72.105
22	21	21.26	0.74	21.75	-0.75	0.5476	0.5625	-0.555
16	15	21.26	-5.26	21.75	-6.75	27.6676	45.5625	35.505
15	15	21.26	-6.26	21.75	-6.75	39.1876	45.5625	42.255

25	25	21.26	3.74	21.75	3.25	13.9876	10.5625	12.155
28	13	21.26	6.74	21.75	-8.75	45.4276	76.5625	-58.975
18	21	21.26	-3.26	21.75	-0.75	10.6276	0.5625	2.445
20	18	21.26	-1.26	21.75	-3.75	1.5876	14.0625	4.725
21	13	21.26	-0.26	21.75	-8.75	0.0676	76.5625	2.275
20	20	21.26	-1.26	21.75	-1.75	1.5876	3.0625	2.205
21	18	21.26	-0.26	21.75	-3.75	0.0676	14.0625	0.975
16	13	21.26	-5.26	21.75	-8.75	27.6676	76.5625	46.025
15	14	21.26	-6.26	21.75	-7.75	39.1876	60.0625	48.515
21	16	21.26	-0.26	21.75	-5.75	0.0676	33.0625	1.495
14	18	21.26	-7.26	21.75	-3.75	52.7076	14.0625	27.225
29	29	21.26	7.74	21.75	7.25	59.9076	52.5625	56.115
16	21	21.26	-5.26	21.75	-0.75	27.6676	0.5625	3.945
27	13	21.26	5.74	21.75	-8.75	32.9476	76.5625	-50.225
22	18	21.26	0.74	21.75	-3.75	0.5476	14.0625	-2.775
21	18	21.26	-0.26	21.75	-3.75	0.0676	14.0625	0.975
22	30	21.26	0.74	21.75	8.25	0.5476	68.0625	6.105
23	15	21.26	1.74	21.75	-6.75	3.0276	45.5625	-11.745
13	13	21.26	-8.26	21.75	-8.75	68.2276	76.5625	72.275
14	4	21.26	-7.26	21.75	-17.75	52.7076	315.0625	128.865
12	20	21.26	-9.26	21.75	-1.75	85.7476	3.0625	16.205
17	9	21.26	-4.26	21.75	-12.75	18.1476	162.5625	54.315
15	9	21.26	-6.26	21.75	-12.75	39.1876	162.5625	79.815
7	10	21.26	-14.26	21.75	-11.75	203.3476	138.0625	167.555
7	21	21.26	-14.26	21.75	-0.75	203.3476	0.5625	10.695
3	5	21.26	-18.26	21.75	-16.75	333.4276	280.5625	305.855
6	9	21.26	-15.26	21.75	-12.75	232.8676	162.5625	194.565
16	16	21.26	-5.26	21.75	-5.75	27.6676	33.0625	30.245
14	13	21.26	-7.26	21.75	-8.75	52.7076	76.5625	63.525
13	14	21.26	-8.26	21.75	-7.75	68.2276	60.0625	64.015
8	10	21.26	-13.26	21.75	-11.75	175.8276	138.0625	155.805
10	13	21.26	-11.26	21.75	-8.75	126.7876	76.5625	98.525
16	14	21.26	-5.26	21.75	-7.75	27.6676	60.0625	40.765
14	18	21.26	-7.26	21.75	-3.75	52.7076	14.0625	27.225
18	24	21.26	-3.26	21.75	2.25	10.6276	5.0625	-7.335
14	16	21.26	-7.26	21.75	-5.75	52.7076	33.0625	41.745
15	21	21.26	-6.26	21.75	-0.75	39.1876	0.5625	4.695
11	19	21.26	-10.26	21.75	-2.75	105.2676	7.5625	28.215
13	25	21.26	-8.26	21.75	3.25	68.2276	10.5625	-26.845
10	19	21.26	-11.26	21.75	-2.75	126.7876	7.5625	30.965
14	16	21.26	-7.26	21.75	-5.75	52.7076	33.0625	41.745
12	11	21.26	-9.26	21.75	-10.75	85.7476	115.5625	99.545
3	14	21.26	-18.26	21.75	-7.75	333.4276	60.0625	141.515
13	26	21.26	-8.26	21.75	4.25	68.2276	18.0625	-35.105
12	18	21.26	-9.26	21.75	-3.75	85.7476	14.0625	34.725
39	19	21.26	17.74	21.75	-2.75	314.7076	7.5625	-48.785
14	19	21.26	-7.26	21.75	-2.75	52.7076	7.5625	19.965

19	18	21.26	-2.26	21.75	-3.75	5.1076	14.0625	8.475
24	24	21.26	2.74	21.75	2.25	7.5076	5.0625	6.165
25	30	21.26	3.74	21.75	8.25	13.9876	68.0625	30.855
22	31	21.26	0.74	21.75	9.25	0.5476	85.5625	6.845
25	26	21.26	3.74	21.75	4.25	13.9876	18.0625	15.895
22	29	21.26	0.74	21.75	7.25	0.5476	52.5625	5.365
17	21	21.26	-4.26	21.75	-0.75	18.1476	0.5625	3.195
13	15	21.26	-8.26	21.75	-6.75	68.2276	45.5625	55.755
16	14	21.26	-5.26	21.75	-7.75	27.6676	60.0625	40.765
16	18	21.26	-5.26	21.75	-3.75	27.6676	14.0625	19.725
7	21	21.26	-14.26	21.75	-0.75	203.3476	0.5625	10.695
16	13	21.26	-5.26	21.75	-8.75	27.6676	76.5625	46.025
15	14	21.26	-6.26	21.75	-7.75	39.1876	60.0625	48.515
9	21	21.26	-12.26	21.75	-0.75	150.3076	0.5625	9.195
14	10	21.26	-7.26	21.75	-11.75	52.7076	138.0625	85.305
11	10	21.26	-10.26	21.75	-11.75	105.2676	138.0625	120.555
20	21	21.26	-1.26	21.75	-0.75	1.5876	0.5625	0.945
19	29	21.26	-2.26	21.75	7.25	5.1076	52.5625	-16.385
25	20	21.26	3.74	21.75	-1.75	13.9876	3.0625	-6.545
28	25	21.26	6.74	21.75	3.25	45.4276	10.5625	21.905
27	27	21.26	5.74	21.75	5.25	32.9476	27.5625	30.135
26	19	21.26	4.74	21.75	-2.75	22.4676	7.5625	-13.035
25	23	21.26	3.74	21.75	1.25	13.9876	1.5625	4.675
18	17	21.26	-3.26	21.75	-4.75	10.6276	22.5625	15.485
12	17	21.26	-9.26	21.75	-4.75	85.7476	22.5625	43.985
17	27	21.26	-4.26	21.75	5.25	18.1476	27.5625	-22.365
19	25	21.26	-2.26	21.75	3.25	5.1076	10.5625	-7.345
2254	**2306**					**11414.61**	**9319.625**	**8440.87**

Mean of x = $\bar{x}$ = 2254/106 =21.26

Mean of y = $\bar{y}$ =2306/106 = 21.75

Correlation coefficient given by Karl Pearson is

$$r_{xy} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} \qquad (1)$$

where

$$\text{cov}(x, y) = \frac{1}{n}\Sigma(x_i - \bar{x})(y_i - \bar{y}), \quad \sigma_x^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2, \quad \sigma_y^2 = \frac{1}{n}\Sigma(y_i - \bar{y})^2$$

From the above original data (Table), we obtain

$\text{cov}(x, y)$ = 79.63

σ_x = 10.377

σ_y = 9.376

Putting these values in (1)

$$r_{xy} = \frac{79.63}{10.3 \times 9.32} = 0.818$$

5. TEST OF SIGNIFICANCE

(1) For mean:

Applying z-test $|z| = \dfrac{|\bar{x}-\bar{y}|}{\sqrt{\dfrac{\sigma_x^2}{n_1}+\dfrac{\sigma_y^2}{n_2}}} = \dfrac{|\bar{x}-\bar{y}|}{\sqrt{\dfrac{\sigma_x^2+\sigma_y^2}{n}}} = \dfrac{0.49}{\sqrt{\dfrac{(10.377)^2+(9.376)^2}{106}}}. = 0.361 < 1.96$

where $n_1=n_2=n=106$

This is not significant. Thus, the hypothesis (i) is not rejected at 5 % level of significance. Thus one can conclude that conceptual achievement of calculus and ability of working skill of doing problems of calculus are same.

(2) For correlation coefficient:

Applying t-test

$$|t| = \frac{|r|}{\sqrt{1-r^2}}\sqrt{n-2} = \frac{.818\times\sqrt{104}}{\sqrt{1-.818^2}} = 14.5 \rangle 1.96$$

This is highly significant.

Thus the hypothesis (ii) is rejected.

Thus one can conclude that the two variables are highly correlated at 5% level of significance.

The investigation shows that many students have primitive concept of a function when it is represented by a single formula. They view the function as a static quantity thinking of only one point at a time and can find limits, derivatives etc. easily. But when a function is defined in two or three parts and not by a single formula, some students failed to do such problems correctly or reject it to attempt.

Percentage of students doing totally correct answers And partially correct, wrong & rejected answers

Functions defined as	**limit**		**continuity**		**derivative**		**integration**	
	Totally correct	Partially correct,, wrong & rejected	Totally correct	Partially correct, wrong & rejected	Totally correct	Partially correct, wrong & rejected	totally correct	Partially correct, wrong & rejected
single formula	35%	65%	28%	72%	27%	73%	22%	78%
breaking formula	18%	82%	19%	81%	12%	88%	04%	96%

In case of indeterminateness, 5 questions in each questionnaire are given to 106 students from the topic indeterminate forms ($\frac{0}{0}$ and $\frac{\infty}{\infty}$) . The performance of the students is shown below:

Percentage of students doing totally correct answers and partially correct, wrong & rejected answers

Indeterminate forms	**Totally correct answer**	**Partially correct, wrong & rejected answer**
conceptual achievement	11%	89%
ability of doing problems	14%	86%

6. FINDINGS AND CONCLUSIONS

The investigation reveals the exact correlation between the conceptual achievement of calculus and ability of the working skill of doing problems of calculus to be 0.818 which is a fairly strong positive relationship. Using this correlation coefficient t –test is done. Based on the study, the following conclusions are made:

1) t-test shows that the correlation coefficient is highly significant at 5% level of significance. Since achievement of students is a focal objective of teaching mathematics, stress should be given on making a sound conception of calculus thereby enhancing ability of working skill of doing problems of calculus.

2) z-test shows that difference of two means is insignificant and hence the ability of earning concept of calculus and ability of doing problems of calculus are homogeneous.

The following table reveals the comparative observations of students' response to different types of problems of above mentioned topics of calculus.

Topics	Students are found
Functions and Variables	• to view a function as a single formula • unable to connect algebraic and graphical representation of functions
Limits and Continuity	• able to find the limit of f(x) as x→a by putting x=a • not able to find one-sided limit • not able to interpret the result geometrically
Derivative	• able to find the derivative by the process of differentiation • not able to find the derivatives when the functions are given as breaking formula • not able to understand the rate of change from straight line graph
Integration	• to overlook the integration as the limit of a sum • able to integrate by applying the basic techniques of integration
Indeterminate forms	• not able to find limit properly • to reject such problems

The study showed that some students reject certain graphs of functions because of a correspondence being discontinuous at some point or points. Vinner and Drefus are of the view that "students usually pay less attention to the conceptual aspects of a given notion and move attention to computational or operational aspects". From the investigation, it has been found that students have a tendency to be process-oriented rather than concept-oriented in their approach. In the words of Skemp, it has been instrumental not relational. This may be explained from the higher percentage of students either worked out wrongly or rejected to do when the functions are given in breaking formulas.

4) The students difficulty in the understanding of limit problems lies in the concept of 'getting close to' or 'as near as we please' or 'for sufficiently large *n*' For them limit problems are the results to be evaluated by simply putting a for x when $x \rightarrow a$.

5) This difficulty in understanding of limit problems is the basic cause of difficulties in doing problems of continuity and derivatives. The reform of calculus teaching has suggested the teaching of both symbolic manipulation and dynamic graphics for illustration and computation. The perceptual notion of continuity is based on drawing a curve with a pencil without taking the pencil off the paper. To make sense the concepts of limit, continuity, tangent, derivative and so on, we need to consider how we, as individuals, think about these ideas.

6) Orton suggests that activities in which the students can explore the idea of a limit in an intuitive way have to be developed. When learners begin to study the calculus, their success or failure depend on their previous experience and current knowledge. .

7) Using information technology (software) it becomes possible now a days to plot the numerical value of the slope of a curve as a point. As these happen dynamically, students can see the graph of the slope function and to realise.

8) The investigation could be carried out to a wide range of area covering a large number of educational institutions where students come from various backgrounds such as caste, creed, religion, economic condition, urban, rural etc.

7. REFORM OF CALCULUS CONTENT AND PRESENTATION

Above investigation shows that there is a need to review the teaching –learning process of calculus education in higher secondary level of Assam (the place of investigation). To make calculus sensible to the learners, we must depend upon the evidence of our human senses and use these as a meaningful basis for later developments. "Its major advantage is that it need not be based initially on concepts known to cause student difficulty, but allows fundamental ideas of the calculus to develop naturally from sensible origins, in such a way as to make sense in its own origin right for general purposes, support the intuitions necessary for applications, provide a meaning for the limit concept to be used later in standard analysis and further, to provide a sensible basis for infinitesimal concepts in non-standard analysis [8]".

Many researchers found that students show little intuition about the concepts and processes of calculus [9].

Calculus reform movement, started in1980s decade in USA, advocates the fundamental changes in the content and presentation of the course. A reformed calculus course differs from a traditional course in methods of instruction. Here a teacher is no longer the central focus of the class. In reference to the changes in the content of the calculus course, real –world application whether it is in physics, economics, or medicine and context of mathematical principles emphasizing active learning where students participation is a must, should be included. The participation of students in the learning process in traditional teaching method has not been focused. Many focused on application of calculus and conceptual understanding as important to computational skills. Thompson has rightly suggests that calculus curriculum should

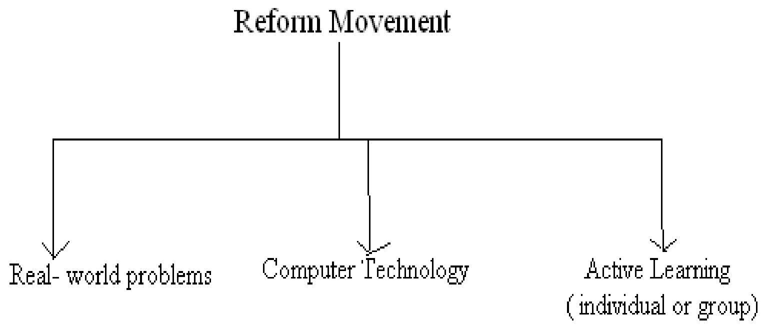
- be problem based
- promote reflective abstraction
- contain questions that focused on relationship
- have as its objective a cognitive structure that allows one to think with the structure of the subject matter

For the improvement of conceptual understanding in calculus reform programme includes –

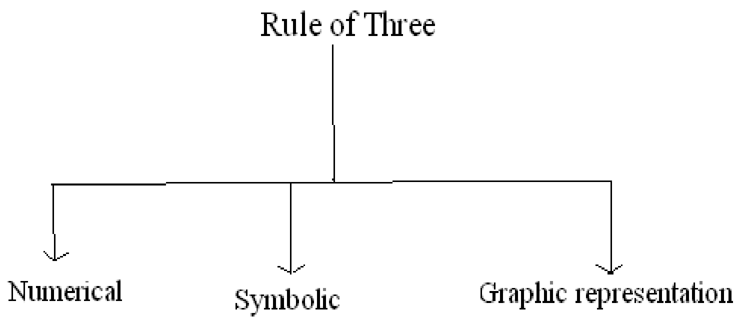
- more students involvement
- taking advantage of technology
- emphasis on problem solving and modeling

The National Science Foundation (NSF) of USA defend application-based approach, argue that students often fail to understand how, why or when to apply their knowledge after studying mathematical theorems and proofs.

The emphasis on reform focused on students understanding with the help of computer technology but not on computer technology. In this regard Mathematica and Matlab can help students' understanding in graphs, limiting behavior, continuity, derivative and integral of functions.



Integral to the new approach to teaching calculus is the 'Rule of Three'



It is clear that computers help students in enhancing their understanding and make them free from the meaningless plug and chug approach to solving problems. It will help students for understanding of interaction among numerical, symbolic and graphic representation.

A study by Sarah Rebecca Kueffer in "Reform and Traditional Undergraduate Calculus 1: A Meta-analysis" shows that reform teaching techniques increased students' conceptual understanding.

The reform-text book will ask students not to be concerned with the answers, but to be concerned with the explanation arrived at and its correctness.

In the long run, the impact of calculus reform will produce a community of mathematicians and hence the calculus reform movement is the need of the hour in India also.

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