

**EFFECT OF CHEMICAL REACTION ON THE UNSTEAD CONVECTIVE
HEAT AND MASS TRANSFER FLOW IN A VERTICAL WAVY CHANNEL
WITH OSCILLATORY FLUX AND HEAT SOURCES**

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ABSTRACT

The unsteadiness in the flow is due to the oscillatory flux in the flow region. Here the effect of chemical reaction on unsteady free convective heat and mass transfer flow through a porous medium with oscillatory flux and heat sources is considered in a vertical wavy channel with the slope δ as the perturbation parameter the perturbation technique is used to solve the coupled equations governing the flow, heat and mass transfer. The expression for the velocity, the temperature, the concentration, the rate of heat and mass transfer are derived and are analyzed for different variations of the governing parameters.

Keywords: Unsteady convective flow, chemical reaction, porous medium, oscillatory flux, heat sources, vertical wavy channel and

1. INTRODUCTION

Underground spreading chemical wastes and other pollutants, grain storage, evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed. Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair[2], Lai and Kulacki [5] and Murthy and Singh[9]. Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous media has been analyzed by Lai [6]. The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al [1]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably Nelson and Wood [10,11], Lee et al [7] and others [15, 16, 17].

Unsteady convection flows play an important role in aerospace technology, turbo machinery and chemical engineering. Such flows arise due to either unsteady motion of a boundary temperature. Unsteadiness may also be due to oscillatory free stream velocity or temperature. These oscillatory free convective flows are important from technological point of view. Effects of chemical reaction on MHD unsteady free convective Walter's memory flow with constant suction and heat sink is studied by Nityananda Senapati et al [12]. Sreekantha reddy *et al.* [14] studied the effect of chemical reaction and thermo-diffusion on non-Darcy convective heat and mass transfer flow in a vertical channel with heat sources.

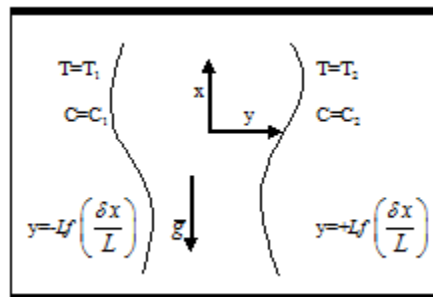
In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing. Das *et al.* [3] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumaraswamy [9] has studied the effects of reaction on a long surface with suction. Gnaneswar [4] has studied radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation. Raptis and Perdikis [13] have explained clearly the Radiation and free convection flow past a moving plate.

Recently Madhusudan Reddy [8] has analyzed the effect of chemical reaction on double diffusive heat transfer flow of a viscous fluid in a wavy channel. Here in this paper we discuss the effect of chemical reaction on unsteady free convective heat and mass transfer flow through a porous medium in a vertical wavy channel the unsteadiness in the flow is due to the oscillatory flux in the flow region. The coupled equations governing the flow, heat and mass transfer

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have been solved by using a perturbation technique with the slope δ as the perturbation parameter. The expression for the velocity, the temperature, the concentration, the rate of heat and mass transfer are derived and are analyzed for different variations of the governing parameters.



Configuration of the Problem

2. FORMULATION OF THE PROBLEM

We consider the effect of chemical reaction on the unsteady motion of viscous, incompressible electrically conducting fluid in a vertical channel bounded by wavy walls. The thermal buoyancy in the flow field is created by an oscillatory flux in the fluid region. The walls are maintained at constant temperature and concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are neglected in comparison with heat by conduction and convection in the energy equation. Also the Kinematic viscosity ν , the thermal conducting k are treated as constants. We choose a rectangular Cartesian system $O(x, y)$ with x -axis in the vertical direction and y -axis normal to the walls. The walls of the channel are at $y = \pm Lf\left(\frac{\delta x}{L}\right)$

The equations governing the unsteady flow, heat and mass transfer are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\rho_e \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho g - (\sigma \mu_e^2 H_o^2) u \quad (2)$$

$$\rho_e \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (3)$$

$$\rho_e C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - Q(T - T_e) \quad (4)$$

$$\left(\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D_1 \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - k_1(C - C_e) \quad (5)$$

$$\rho - \rho_e = -\beta \rho_e (T - T_e) - \beta^* \rho_e (C - C_e) \quad (6)$$

where ρ_e is the density of the fluid in the equilibrium state, T_e , C_e are the temperature and concentration in the equilibrium state, (u, v) are the velocity components along $O(x, y)$ directions, p is the pressure, U vector velocity, T , C are the temperature and Concentration in the flow region, ρ is the density of the fluid, μ is the constant coefficient of viscosity, C_p is the specific heat at constant pressure, λ is the coefficient of thermal conductivity, k is the permeability of the porous medium, β is the coefficient of thermal expansion, Q is the strength of the constant internal heat source, σ is the electrical conductivity, μ_e is the magnetic permeability, β^* is the volumetric expansion with mass fraction coefficient, D_1 is the molecular diffusivity and k_1 is the chemical reaction coefficient.

In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_e g \quad (7)$$

where $p = p_e + p_D$, p_D being the hydrodynamic pressure.

The flow is maintained by an oscillatory volume flux for which a characteristic velocity is defined as

$$q(1 + k e^{i\omega t}) = \frac{1}{L} \int_{-L}^{L} u dy \quad (8)$$

The boundary conditions for the velocity and temperature fields are

$$u = 0, v = 0, T = T_1, C = C_1 \text{ on } y = -Lf \left(\frac{\delta x}{L} \right)$$

$$u = 0, v = 0, T = T_2, C = C_2 \text{ on } y = +Lf \left(\frac{\delta x}{L} \right) \quad (9)$$

In view of the continuity equation we define the stream function ψ as

$$u = -\psi_y, v = \psi_x \quad (10)$$

Eliminating pressure p from equation (2) - (5) using the equations governing the flow in terms of ψ are

$$[(\nabla^2 \psi)_t + \psi_x (\nabla^2 \psi)_y - \psi_y (\nabla^2 \psi)_x] = \nu \nabla^4 \psi - \beta g (T - T_0)_y - \beta^* g (C - C_0)_y - (\sigma \mu_e^2 H_o^2) \frac{\partial^2 \psi}{\partial y^2} \quad (11)$$

$$\rho_e C_p \left(\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = \lambda \nabla^2 T - Q(T - T_0) \quad (12)$$

$$\left(\frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = D \nabla^2 C - k_1 (C - C_0) \quad (13)$$

Introducing the non-dimensional variables in (11) - (13) as

$$x' = x/L, y' = y/L, t' = t\omega, \Psi' = \Psi/\nu, \theta = \frac{T - T_2}{T_1 - T_2}, C' = \frac{C - C_2}{C_1 - C_2} \quad (14)$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$R \left(\gamma^2 (\nabla^2 \psi)_t + \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, y)} \right) = \nabla^4 \psi + \left(\frac{G}{R} \right) (\theta_y + N C_y) - M^2 \frac{\partial^2 \psi}{\partial y^2} \quad (15)$$

$$P \left(\gamma^2 \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla^2 \theta - \alpha \theta \quad (16)$$

$$Sc \left(\gamma^2 \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \nabla^2 C - k C \quad (17)$$

where

$$R = \frac{UL}{\nu} \quad (\text{Reynolds number})$$

$$P = \frac{\mu C_p}{k_1} \quad (\text{Prandtl number})$$

$$Sc = \frac{\nu}{D_1} \quad (\text{Schmidt Number})$$

$$G = \frac{\beta g \Delta T_e L^3}{\nu^2} \quad (\text{Grashof number})$$

$$M^2 = \frac{\sigma \mu_e^2 H_o^2 L^2}{\nu^2} \quad (\text{Hartman Number})$$

$$\alpha = \frac{QL^2}{\lambda} \quad (\text{Heat source parameter})$$

$$k_1 = \frac{K_1 L^2}{D_1} \quad (\text{Chemical reaction parameter}) \quad \gamma^2 = \frac{\omega L^2}{\nu} \quad (\text{Wormsely Number})$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The corresponding boundary conditions are

$$\psi(+f) - \psi(-f) = 1$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = \pm f \quad (18)$$

$$\theta(x, y) = 1, C = 1 \quad \text{on } y = -f$$

$$\theta(x, y) = 0, C = 0 \quad \text{on } y = f$$

$$\frac{\partial \theta}{\partial y} = 0, \frac{\partial C}{\partial y} = 0 \quad \text{at } y = 0 \quad (19)$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis (8). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t .

3. METHOD OF SOLUTION

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to traveling thermal wave imposed on the boundaries. The perturbation analysis is carried out by assuming that the aspect ratio δ to be small.

Introduce the transformation such that

$$\bar{x} = \delta x, \quad \frac{\partial}{\partial x} = \delta \frac{\partial}{\partial \bar{x}}$$

$$\text{Then, } \frac{\partial}{\partial x} \approx O(\delta) \rightarrow \frac{\partial}{\partial \bar{x}} \approx O(1)$$

For small values of $\delta \ll 1$, the flow develops slowly with axial gradient of order δ

$$\text{And hence we take } \frac{\partial}{\partial \bar{x}} \approx O(1)$$

Using the above transformation the equations (15-17) reduces to

$$\delta R \left(\gamma^2 (\nabla_1^2 \psi)_t + \frac{\partial(\psi, \nabla_1^2 \psi)}{\partial(x, y)} \right) = \nabla_1^4 \psi + \left(\frac{G}{R} \right) (\theta_y + N C_y) - M^2 \frac{\partial^2 \psi}{\partial y^2} \quad (20)$$

$$\delta P \left(\gamma^2 \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla_1^2 \theta - \alpha \theta \quad (21)$$

$$\delta Sc \left(\gamma^2 \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \nabla_1^2 C - k C \quad (22)$$

$$\text{where, } \nabla_1^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\text{Introducing the transformation, } \eta = \frac{y}{f(\bar{x})}$$

the equations(20-22)reduces to

$$\delta R f \left(\gamma^2 (F^2 \psi)_t + \frac{\partial(\psi, F^2 \psi)}{\partial(\bar{x}, \eta)} \right) = F^4 \psi + \left(\frac{G f^3}{R} \right) (\theta_\eta + N C_\eta) - (M^2 f^2) \frac{\partial^2 \psi}{\partial \eta^2} \quad (23)$$

$$\delta P \left(\gamma^2 \frac{\partial \theta}{\partial t} + f \left(\frac{\partial \psi}{\partial \eta} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial \eta} \right) \right) = F^2 \theta - \alpha f^2 \theta \quad (24)$$

$$\delta Sc \left(\gamma^2 \frac{\partial C}{\partial t} + f \left(\frac{\partial \psi}{\partial \eta} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial \eta} \right) \right) = F^2 C - KC \quad (25)$$

$$\text{where, } F^2 = \delta^2 \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \eta^2}$$

We adopt the perturbation scheme and write

$$\begin{aligned} \psi(x, \eta, t) &= \psi_0(x, \eta, t) + ke^{it} \bar{\psi}_0(x, \eta, t) + \delta(\psi_1(x, \eta, t) + ke^{it} \bar{\psi}_1(x, \eta, t)) + \dots \\ \theta(x, \eta, t) &= \theta_0(x, \eta, t) + ke^{it} \bar{\theta}_0(x, \eta, t) + \delta(\theta_1(x, \eta, t) + ke^{it} \bar{\theta}_1(x, \eta, t)) + \dots \\ C(x, \eta, t) &= C_0(x, \eta, t) + ke^{it} \bar{C}_0(x, \eta, t) + \delta(C_1(x, \eta, t) + ke^{it} \bar{C}_1(x, \eta, t)) + \dots \end{aligned} \quad (26)$$

On substituting (20) in (23) - (25) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,\eta\eta\eta\eta} - (M_1^2 f^2) \psi_{0,\eta\eta} = - \left(\frac{Gf^3}{R} \right) (\theta_{0,\eta} + NC_{0,\eta}) \quad (27)$$

$$\theta_{0,\eta\eta} - (\alpha f^2) \theta_0 = 0 \quad (28)$$

$$C_{0,\eta\eta} - (KScf^2) C_0 = 0 \quad (29)$$

with

$$\psi_{0(+1)} - \psi_{0(-1)} = 1,$$

$$\psi_{0,\eta} = 0, \psi_{0,x} = 0 \text{ at } \eta = \pm 1 \quad (30)$$

$$\begin{aligned} \theta_0 &= 1, \quad C_0 = 1 \quad \text{on } \eta = -1 \\ \theta_0 &= 0, \quad C_0 = 0 \quad \text{on } \eta = 1 \end{aligned} \quad (31)$$

$$\bar{\theta}_{0,\mu\eta} - (iP\gamma^2 f^2) \bar{\theta}_0 = 0 \quad (32)$$

$$\bar{C}_{0,\eta\eta} - (KSc\gamma^2 f^2) \bar{C}_0 = 0 \quad (33)$$

$$\bar{\psi}_{0,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2) f^2) \bar{\psi}_{0,\eta\eta} = - \left(\frac{Gf^3}{R} \right) (\bar{\theta}_{0,\eta} + N\bar{C}_{0,\eta}) \quad (34)$$

$$\bar{\theta}_0(\pm 1) = 0, \quad \bar{C}_0(\pm 1) = 0$$

$$\bar{\psi}_0(+1) - \bar{\psi}_0(-1) = 1 \quad \bar{\psi}_{0,\eta}(\pm 1) = 0, \bar{\psi}_{0,x}(\pm 1) = 0 \quad (35)$$

The first order equations are

$$\psi_{1,\eta\eta\eta\eta} - (M_1^2 f^2) \psi_{1,\eta\eta} = - \left(\frac{Gf^3}{R} \right) (\theta_{1,\eta} + NC_{1,\eta}) + (Rf)(\psi_{0,\eta} \psi_{0,x\eta\eta} - \psi_{0,x} \psi_{0,\eta\eta\eta}) \quad (36)$$

$$\theta_{1,\eta\eta} - (\alpha f^2) \theta_1 = (PRf)(\psi_{0,x} \theta_{0,\eta} - \psi_{0,\eta} \theta_{0,x}) \quad (37)$$

$$C_{1,\eta\eta} - (KScf^2) C_1 = (Scf)(\psi_{0,x} C_{0,\eta} - \psi_{0,\eta} C_{0,x}) \quad (38)$$

$$\bar{\psi}_{1,\eta\eta\eta\eta} - ((M_1^2 + i\gamma^2) f^2) \bar{\psi}_{1,\eta\eta} = - \left(\frac{Gf^3}{R} \right) (\bar{\theta}_{1,\eta} + N\bar{C}_{1,\eta}) + (Rf)(\bar{\psi}_{0,\eta} \bar{\psi}_{0,x\eta\eta} + \bar{\psi}_{0,\eta} \bar{\psi}_{0,x\eta\eta} - \bar{\psi}_{0,x} \bar{\psi}_{0,\eta\eta\eta} - \bar{\psi}_{0,x} \bar{\psi}_{0,\eta\eta\eta}) \quad (39)$$

$$\bar{\theta}_{1,\eta\eta} - ((iP\gamma^2 + \alpha)f^2)\bar{\theta}_1 = (PRf)(\psi_{0,\eta}\bar{\theta}_{o,x} + \bar{\psi}_{0,\eta}\theta_o - \bar{\psi}_{0,x}\theta_{o,\eta} - \psi_{0,x}\bar{\theta}_{o\eta}) \quad (40)$$

$$\bar{C}_{1,\eta\eta} - ((K + i\gamma^2)Scf^2)\bar{C}_1 = (Scf)(\psi_{0,\eta}\bar{C}_{o,x} + \bar{\psi}_{0,\eta}C_o - \bar{\psi}_{0,x}C_{o,\eta} - \psi_{0,x}\bar{C}_{o\eta}) \quad (41)$$

with $\psi_{1(+1)} - \psi_{1(-1)} = 0$

$$\psi_{1,\eta} = 0, \psi_{1,x} = 0 \text{ at } \eta = \pm 1 \quad (42)$$

$$\theta_1(\pm 1) = 0, C_1(\pm 1) = 0$$

$$\bar{\theta}_1(\pm 1) = 0, \bar{C}_1(\pm 1) = 0$$

$$\bar{\psi}_1(+1) - \bar{\psi}_1(-1) = 1, \bar{\psi}_{1,\eta}(\pm 1) = 0, \bar{\psi}_{1,x}(\pm 1) = 0 \quad (43)$$

4. SOLUTION OF THE PROBLEM

Solving the equations (3.8) - (3.22) subject to the relevant boundary conditions we obtain

$$C_0 = 0.5 \left(\frac{Ch(\beta_1\eta)}{Ch(\beta_1)} - \frac{Sh(\beta_1\eta)}{Sh(\beta_1)} \right)$$

$$\psi_o = a_{13}Ch(M_1\eta) + a_{14}Sh(M_1\eta) + a_{15}\eta + a_{16} + f_1(\eta)$$

$$f_1(\eta) = a_9Sh(\beta_1\eta) + a_{10}Ch(\beta_1\eta) + a_{11}Sh(\beta_2\eta) + a_{12}Ch(\beta_2\eta)$$

$$\bar{C}_0 = 0$$

$$\bar{\theta}_0 = 0$$

$$\bar{\psi}_0 = a_{16}Sh(M_1\eta) + a_{17}\eta + a_{18}$$

$$C_1 = a_{92}Ch(\beta_2\eta) + a_{93}Sh(\beta_2\eta) + f_2(\eta)$$

$$f_2(\eta) = (a_{67} + \eta a_{79})Ch(\beta_6\eta) + (a_{68} + \eta a_{80})Ch(\beta_7\eta) + a_{69}Sh(\beta_6\eta) + a_{70}Ch(\beta_7\eta) +$$

$$+ (a_{71} + \eta a_{83})Sh(\beta_8\eta) + a_{72}Sh(\beta_9\eta) + (a_{73} + \eta a_{81})Ch(\beta_8\eta) + (a_{74} + \eta a_{82})Ch(\beta_9\eta) +$$

$$+ (a_{75} + \eta(a_{78} + a_{88}))Sh(2\beta_1\eta) + (a_{76} + \eta(a_{77} + a_{87}))Ch(2\beta_1\eta) + a_{84}\eta Sh(\beta_5\eta) + a_{85}\eta Sh(\beta_1\eta) +$$

$$+ a_{86}\eta Ch(\beta_1\eta) + a_{89}\eta^2 Ch(\beta_1\eta) + a_{90}\eta^2 Sh(\beta_1\eta) - a_{91}\eta^2$$

$$\theta_1 = b_{92}Ch(\beta_1\eta) + b_{93}Sh(\beta_1\eta) + f_3(\eta)$$

$$\begin{aligned} f_3(\eta) = & d_1Ch(\beta_{10}\eta) + d_2Ch(\beta_{11}\eta) + d_3Sh(\beta_{10}\eta) + d_4Sh(\beta_{11}\eta) + d_5Ch(\beta_9\eta) + d_6Sh(\beta_9\eta) \\ & + d_7Ch(\beta_8\eta) + d_8Sh(\beta_8\eta) + d_9Ch(\beta_7\eta) + d_{10}Sh(\beta_7\eta) + d_{11}Ch(\beta_6\eta) + d_{12}Sh(\beta_6\eta) \\ & + d_{13}Sh(2\beta_1\eta) + d_{14}Ch(2\beta_1\eta) + d_{15}Ch(\beta_2\eta) + d_{16}Sh(\beta_2\eta) + d_{17}Sh(2\beta_2\eta) + d_{18}Ch(2\beta_2\eta) \\ & + d_{19}\eta Ch(\beta_1\eta) + d_{20}\eta S(\beta_1\eta) + d_{21}\eta^2 Ch(\beta_1\eta) + d_{22}\eta^2 S(\beta_1\eta) + d_{23}\eta^3 Ch(\beta_1\eta) \\ & + d_{24}\eta^3 Sh(\beta_1\eta) + d_{25}\eta Sh(2\beta_1\eta) + d_{26}\eta Sh(2\beta_2\eta) + d_{27}\eta Ch(2\beta_1\eta) + d_{28}\eta Ch(\beta_2\eta) \\ & + d_{29}\eta Sh(\beta_2\eta) + d_{30}\eta Ch(\beta_2\eta) + d_{31}\eta Ch(\beta_6\eta) + d_{32}\eta Ch(\beta_7\eta) + d_{34}\eta Ch(\beta_7\eta) \\ & + d_{35}\eta Sh(\beta_8\eta) + d_{36}\eta Sh(\beta_9\eta) + d_{37}\eta Sh(\beta_6\eta) + d_{38}\eta Sh(\beta_7\eta) + d_{39}\eta Sh(\beta_{10}\eta) \\ & + d_{40}\eta Sh(\beta_{11}\eta) + d_{41}\eta Sh(\beta_{11}\eta) + d_{42}\eta Ch(\beta_{10}\eta) + d_{43}\eta Ch(\beta_{11}\eta) - d_{43}\eta^2 - d_{44} \end{aligned}$$

$$\psi_1 = p_1 + p_2\eta + p_3Ch(M_1\eta) + p_4Sh(M_1\eta) + f_4(\eta)$$

$$\begin{aligned}
 f_4(\eta) = & g_1 Sh(\beta_1 \eta) + g_2 Ch(\beta_1 \eta) + g_3 Sh(\beta_2 \eta) + g_4 Ch(\beta_2 \eta) + g_5 Sh(\beta_6 \eta) \\
 & + g_6 Sh(\beta_7 \eta) + g_7 Sh(\beta_8 \eta) + g_8 Sh(\beta_9 \eta) + g_9 Sh(\beta_{10} \eta) + g_{10} Sh(\beta_{11} \eta) \\
 & + g_{11} Ch(\beta_6 \eta) + g_{12} Ch(\beta_7 \eta) + g_{13} Ch(\beta_8 \eta) + g_{14} Ch(\beta_9 \eta) + g_{15} Ch(\beta_{10} \eta) \\
 & + g_{16} Ch(\beta_{11} \eta) + g_{17} Ch(2\beta_1 \eta) + E_{97} \eta^2 Sh(M_1 \eta) + E_{98} \eta^2 Ch(M_1 \eta) \\
 & + E_{99} \eta Sh(M_1 \eta) + E_{104} \eta Sh(2M_1 \eta) + E_{105} \eta^2 Sh(2M_1 \eta) + E_{106} Ch(2M_1 \eta) \\
 & + a_{21} Sh(\beta_{12} \eta) + a_{22} Sh(\beta_{13} \eta) + a_{23} Sh(\beta_{14} \eta) + a_{24} Sh(\beta_{15} \eta) + a_{25} Ch(\beta_{12} \eta) \\
 & + a_{26} Ch(\beta_{13} \eta) + a_{27} Ch(\beta_{14} \eta) + a_{28} Ch(\beta_{15} \eta) + E_{57} \eta^3 Sh(\beta_1 \eta) + g_{29} \eta^2 Sh(\beta_1 \eta) + \\
 & + g_{30} \eta Sh(\beta_1 \eta) + E_{72} \eta^3 Ch(\beta_1 \eta) + g_{31} \eta^2 Ch(\beta_1 \eta) + g_{32} \eta Ch(\beta_1 \eta)
 \end{aligned}$$

5. NUSSELT NUMBER and SHERWOOD NUMBER

$$(\tau)_{y=-1} = d_6 + Ecd_7 + \delta d_8 + O(\delta^2)$$

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w} \left(\frac{\partial \theta}{\partial y} \right)_{\eta=\pm 1}$$

where

$$\theta_m = 0.5 \int_{-1}^1 \theta d\eta$$

and the corresponding expressions are

$$\begin{aligned}
 (Nu)_{\eta=+1} &= \frac{(d_9 + \delta d_{11})}{(\theta_m - \sin(x + \gamma t))} \\
 (Nu)_{\eta=-1} &= \frac{(d_8 + \delta d_{10})}{(\theta_m - 1)}
 \end{aligned}$$

$$\text{Where } \theta_m = d_{14} + \delta d_{15}$$

The local rate of mass transfer coefficient (Sherwood Number Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w} \left(\frac{\partial C}{\partial y} \right)_{y=\pm 1}$$

$$\text{where } C_m = 0.5 \int_{-1}^1 C dy$$

and the corresponding expressions are,

$$\begin{aligned}
 (Sh)_{\eta=+1} &= \frac{(d_4 + \delta d_6)}{(C_m)} \\
 (Sh)_{\eta=-1} &= \frac{(d_5 + \delta d_7)}{(C_m - 1)}
 \end{aligned}$$

$$\text{where } C_m = d_{12} + \delta d_{13}$$

where d_1, d_2, \dots, d_{14} are constants given in the appendix.

6. DISCUSSION OF THE RESULTS

In this analysis, we investigate the effect of chemical reaction and radiation absorption on unsteady convective heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel in the presence of heat

generating sources. The axial velocity is shown in figure 1 – 7 for different values of G , M , α , Sc , γ and β . The actual axial flow is in the vertically downward direction i.e. $u < 0$ is actual flow and $u > 0$ is a reversal flow. Fig 1 represents the variation of u with Grashof number G . It is found that u exhibits a reversal flow in entire fluid region and the region of reversal flow enlarges with increase in $|G|$ with maximum attained at $\eta = -0.4$. The magnitude of u enhances with increase in $|G|$. Higher the Lorentz force lesser $|u|$ in the flow region (fig.3). An increase in the strength of the heat source enhances $|u|$ in fluid region. Also for $\alpha \geq 4$, u exhibits a reversal flow in the entire flow region (fig.4). The variation of u with Schmidt number (Sc) shows that lesser the molecular diffusivity larger $|u|$ in the flow region. Also the reversal flow zone enhances with increase in Sc (fig.5). The variation of u with chemical reaction parameter k shows that the axial velocity enhances with increase in k (fig.7). The influence of the surface geometry on u is shown in (fig.6). It is found that higher the dilation of the channel walls, larger $|u|$ for $\beta \leq 0.5$ and larger for $|\beta| \geq 0.7$. An increase in the Wormesely number γ reduces with increase in $\gamma \leq 4$ and enhances marginally with higher $\gamma \geq 6$ (fig.5). Moving along the axial direction of the channel $|u|$ depreciates with x in the entire flow region (fig.5).

The secondary velocity (v) which is due to the wall waviness of the channel is shown in figs 8-14. It is found from fig.8 that for $G > 0$, v is towards the mid region and is towards the boundary for $G < 0$. $|v|$ enhances with increase in $|G|$ with maximum attained at $\eta = 0.4$. Higher the Lorentz force smaller $|u|$ in the flow region. The variation of v with heat

source parameter α shows that the magnitude of v enhances remarkably with increase in $\alpha \leq 4$ and depreciates with higher $\alpha \geq 6$. (fig.10). With respect to Sc we find that lesser the molecular diffusivity larger $|v|$ in the region (fig. 11). The variation of v with chemical reaction parameter k shows that $|v|$ enhances with increase in $k \leq 3.5$ and depreciates with higher $k \geq 4.5$ (fig.14). we find that higher the dilation of the channel walls larger $|v|$ in the flow region. Also an increase in γ reduces $|v|$ (fig.12).

The non-dimensional temperature (θ) is shown in figures (15-21) for different parametric values. We follow the convention that the non-dimensional temperature is positive or negative according as the actual temperature is greater (or) lesser than T_2 . (Fig. 15) represents θ with Grashof number G . It is found the actual temperature enhances with $G > 0$ and depreciates with $G < 0$. The variation of θ with heat source parameter α exhibits an increasing tendency with α (fig.17). With respect to Sc we find that lesser the molecular diffusivity larger the actual temperature in the flow region. (fig 18) An increase in the chemical reaction parameter $k \leq 2.5$ leads to a depreciation in the actual temperature, enhances at $k = 3.5$ and again depreciates with higher $k \geq 4.5$ (fig.21). The influence of the wall waviness is shown in (fig.20). It is found that higher the dilation of the channel walls larger the actual temperature. An increase in γ results in a depreciation in the actual temperature (fig.19).

The non-dimensional concentration (C) is shown in figs 22-28 for different parametric values. (Fig.22) represents the concentration C with Grashof number G . It is found that the actual concentration enhances with increase in $|G|$ with maximum at $\eta = 0.6$. The variation of C with Hartmann number M shows that higher the Lorentz force smaller the actual temperature and for further higher Lorentz force larger in the left half and smaller in the right half (fig.23). An increase in the heat source parameter α reduces the actual concentration. It is found that for higher $\alpha = 6$ we notice a remarkable depreciation in the actual concentration (fig.24). With respect to Sc we find that lesser the molecular diffusivity higher the actual concentration (fig 25). For $k \leq 2.5$, we find that the actual concentration enhances in the region, except in the region $(-0.2, 0.2)$ where it depreciates and for higher $k \geq 3.5$, it depreciates in the regions adjacent to $\eta = \pm 1$ and enhances in the central region (fig.28). From fig.24 we find that the actual temperature depreciates irrespective of the directions of the buoyancy forces.

The rate of heat transfer (Nu) at $\eta = \pm 1$ is shown tables 1-4 for different values of Sc , α and N . It is found that the rate of heat transfer enhances at $\eta = 1$ and reduces at $\eta = -1$ with $G > 0$ and for $G < 0$, we notice an enhancement at both the walls. With respect to Sc , we find that at $\eta = +1$ $|Nu|$ depreciates with $Sc \leq 1.3$ and enhances with higher $Sc \geq 2.01$. At $\eta = -1$, it reduces with Sc for all G . When the molecular buoyancy force dominates over the thermal buoyancy force the rate of heat transfer reduces at $\eta = +1$ and enhances at $\eta = -1$ when the buoyancy forces act in the same direction and for the forces acting in opposite directions a reversed effect is observed in the behavior of $|Nu|$. The variation of Nu with heat source parameter α shows that $|Nu|$ enhances with $\alpha \leq 4$ and reduces with $\alpha \geq 6$ at $\eta = +1$ while at $\eta = -1$, it enhances with α for all G (tables 1 & 2). The variation of Nu with chemical reaction parameter k shows that $|Nu|$ enhances with k at $\eta = \pm 1$. Moving along the axial direction the Nusselt number reduces at $\eta = +1$ and enhances at $\eta = -1$ for all G (tables 1 & 2).

The rate of mass transfer (Sh) at $\eta = \pm 1$ is shown in tables 3& 4 for different parametric values. An increase in $G > 0$ enhances $|Sh|$ at $\eta = \pm 1$ while it enhances at $\eta = +1$ and reduces at $\eta = -1$ for $G < 0$. The rate of mass transfer enhances at $\eta = \pm 1$ with $\alpha \leq 4$ and for higher $\alpha \geq 6$, it enhances at $\eta = +1$ and reduces at $\eta = -1$ (tables 3 & 4). With references to Sc , we find that lesser the molecular diffusivity lesser $|Sh|$ and for further lowering of the diffusivity larger $|Sh|$ and for still lowering of the diffusivity lesser $|Sh|$ while at $\eta = -1$, smaller $|Sh|$ for all G . An increase in Wormesely number γ results

in a depreciation in $|Sh|$ at $\eta = \pm 1$. The variation of Nu with N shows that the rate of mass transfer reduces at $\eta = +1$ and enhances at $\eta = -1$ for $G > 0$ and for $G < 0$, it reduces at $\eta = 1$ when the buoyancy forces act in the same direction and for the forces acting in opposite directions, $|Sh|$ enhances at $\eta = +1$, reduces at $\eta = -1$, for all G . The variation of Sh with k shows that higher the chemical reaction parameter $k < 1.5$ larger Sh and for further higher $k > 2.5$, smaller Sh in the heating case and in the cooling case a reversed effect is observed in the behavior of Sh at $\eta = 1$. At $\eta = -1$, Sh depreciates with increase in k for all

CONCLUSIONS

In this article a mathematical model has been presented for the effect of chemical reaction on convective heat and mass transfer flow of a viscous fluid through a porous medium in a vertical wavy channel. The non dimensional governing equations are solved with the help of regular perturbation method.

1. The variation of u with chemical reaction parameter k shows that the axial velocity enhances with increase in k and an increase in the strength of the heat source enhances $|u|$ in fluid region.
2. The secondary velocity (v) which is due to the wall waviness of the channel. The variation of v with chemical reaction parameter k shows that $|v|$ enhances with increase in $k \leq 3.5$ and depreciates with higher $k \geq 4.5$. And the variation of v with heat source parameter α shows that the magnitude of v enhances remarkably with increase in $\alpha \leq 4$ and depreciates with higher $\alpha \geq 6$.
3. An increase in the chemical reaction parameter $k \leq 2.5$ leads to a depreciation in the actual temperature, enhances at $k = 3.5$ and again depreciates with higher $k \geq 4.5$ and the variation of θ with heat source parameter α exhibits an increasing tendency with α .
4. It is found that the actual concentration enhances with increase in $|G|$ with maximum at $\eta = 0.6$. For $k \leq 2.5$, we find that the actual concentration enhances in the region, except in the region $(-0.2, 0.2)$ where it depreciates and for higher $k \geq 3.5$, it depreciates in the regions adjacent to $\eta = \pm 1$ and enhances in the central region. And the variation of v with heat source parameter α shows that the magnitude of v enhances remarkably with increase in $\alpha \leq 4$ and depreciates with higher $\alpha \geq 6$.
5. We observe that at $\eta = +1$ Nu depreciates with some starting & lesser value of Sc and then enhance with the higher values we have considered within the preferable range. When the molecular buoyancy force dominates over the thermal buoyancy force the rate of heat transfer reduces at $\eta = +1$ and enhances at $\eta = -1$ when the buoyancy forces acting in opposite directions a reversed effect is observed in the behavior of Nu .
6. Considering Sc to rate of mass transfer, we find that lesser the molecular diffusivity lesser $|Sh|$ and for further lowering of the diffusivity larger $|Sh|$ and for still lowering of the diffusivity $r |Sh|$ while at $\eta = -1$, smaller $|Sh|$ for all G . An increase in Wormsely number γ results in a depreciation in $|Sh|$ at $\eta = \pm 1$.

7. REFERENCES

- [1] Angirasaa, D, Peterson ,G.P and Pop, I :Combined heat and mass transfer by natural convection with opposing buoyancy effects in a fluid saturated porous medium Int. J. Heat Mass Transfer (1997), V.40, pp. 2755-2773
- [2] Bejan, A and Khair, K. R: Heat and Mass transfer by natural convection in a porous medium, Int. J. Heat Mass transfer, (1985) V.28,pp.908-918
- [3] Das, U. N, Deka, R.K and Soundalgekar, V. M: Forsching in Inge, (1994), V. 80, p.284
- [4] Gnaneswar Reddy: Acta Ciencia Indica, (2008),V.34M,No,21,p.639
- [5] Lai, F.C and Kulacki, F. A : Coupled heat and mass transfer by natural convection from vertical surfaces in porous medium., Int. J. Heat Mass Transfer, (1991), V.34, pp.1189-1194
- [6] Lee, T. S, Parikh, P.G, Archivos, A and Bershader, D: Int. J. Heat Mass Transfer, (1982) V, 25,pp. 499-522
- [7] Madhusadana Reddy, Y: Mixed convective heat and mass transfer through a porous medium in channels with radiation effect, Ph.D. thesis, S.K. University, Anantapur, India (2010).
- [8] Murthy, P.V.S. N and Singh, P: Heat and Mass transfer by natural convection in a Non-Darcy porous medium, Acta Mech, (1990),V.26,pp.567
- [9] Muthukumaraswamy, R:Acta Mechnica, (2002),V.155,p.65
- [10] Nelson, D. J and Wood, B. D:Combined heat and mass transfer by natural convection between vertical plates ,Int. J., Heat Mass transfer, (1989), V.82,pp.1789-1792
- [11] Nelson,D.J and Wood, B.D: Combined heat and mass transfer by natural convection between vertical plates with uniform flux boundary conditions, Heat transfer, (1986),V.4,pp.1587-1952
- [12] Nityananda Senapati1, Rajendra Kumar Dhal and Tusharkanta Das: Effects Of Chemical Reaction On MHD Unsteady Free Convective Walter's Memory Flow With Constant Suction And Heat Sink, International Journal of Mathematical Archive (2012), 3(10), 3633-3641
- [13] Raptis, A. A and Perdikis, C :Radiation and free convection flow past a moving plate, Appl. Mech. Eng, (1999), Vol.4, pp 817-821
- [14] Sreekantha Reddy M. V, Srinivasa Kumar C and Prasada Rao D. R.V: Effect of Chemical Reaction and Thermo-Diffusion on Non-Darcy Convective Heat and Mass International Journal of Mathematical Archive (2012), 3(9), 3380-339

- [15] Trevison ,D.V and Bejan,A :Combined heat and mass transfer by natural convection in vertical enclosure, Trans. ASME, (1987),V.109, pp.104-11
- [16] Vajravelu K and Ali Neyfeh: Influence of wall waviness on friction and pressure drop in channels, Int. J. Maths and Math. Sc., (1981), V.4, pp.805-818
- [17] Vajravelu, K and Sastry, K. S : Heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall, J. Fluid Mech. (1978), V.86, No.2, p.

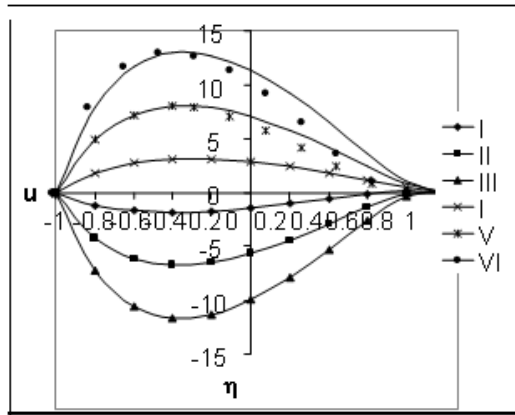


Fig. 1: Variation of u with G

| | | | | | | | | | | |
|---|--------|-----------------|-----------------|---------|------------------|------------------|---|---|----|-----|
| | I | II | III | IV | V | VI | | I | II | III |
| G | 10^2 | 3×10^2 | 5×10^2 | -10^2 | -3×10^2 | -5×10^2 | M | 2 | 5 | 10 |

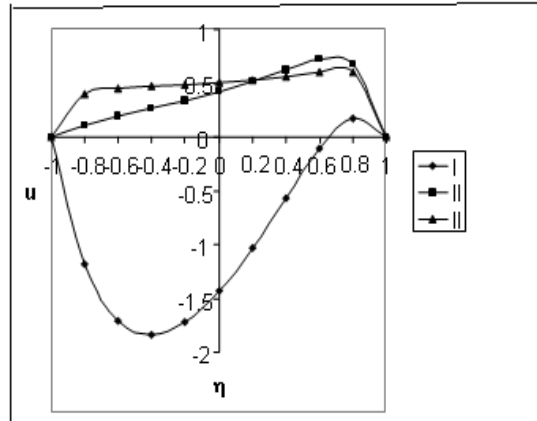


Fig. 2: Variation of u with M

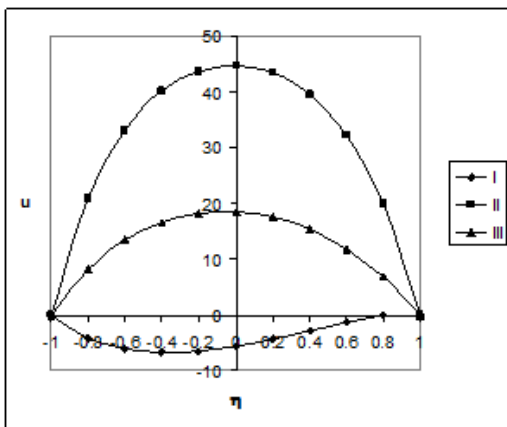


Fig. 3: Variation of u with α

| | | | | | | | | |
|---|---|----|-----|----|------|------|------|------|
| | I | II | III | | I | II | III | IV |
| α | 2 | 4 | 6 | Sc | 0.24 | 0.60 | 1.30 | 2.01 |

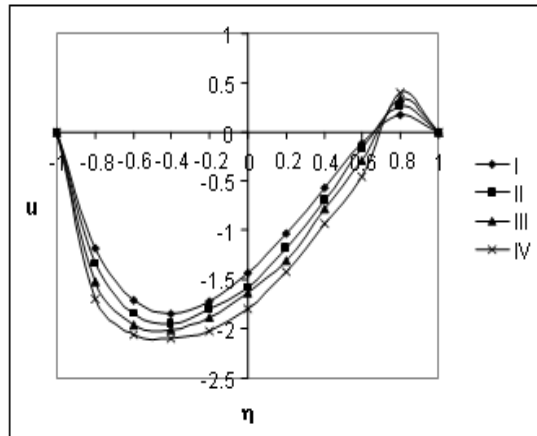


Fig. 4: Variation of u with Sc

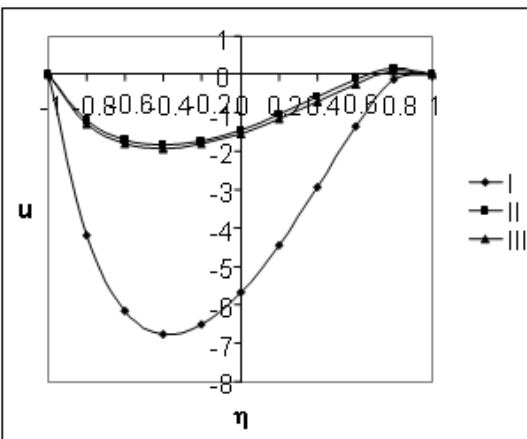


Fig. 5: Variation of u with γ

| | | | | | | | | | |
|---|---|----|-----|---|-----|-----|-----|-----|-----|
| | I | II | III | | I | II | III | IV | V |
| γ | 2 | 4 | 6 | β | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |

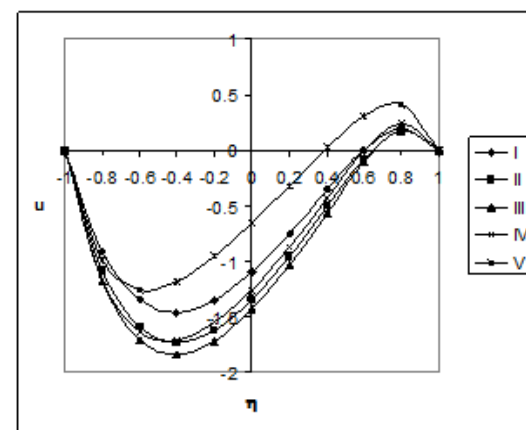


Fig.6: Variation of u with β

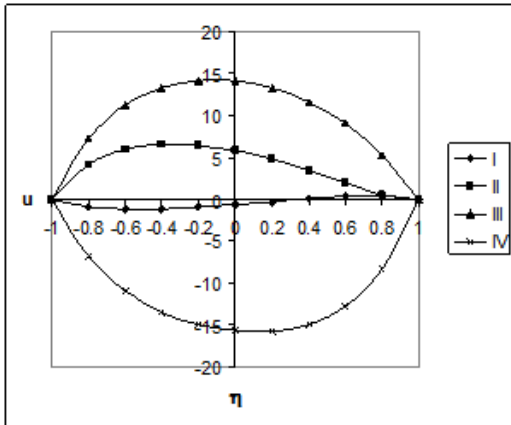


Fig. 7: Variation of u with k

| | I | II | III | IV |
|-----|-----|-----|-----|-----|
| k | 1.5 | 2.5 | 3.5 | 4.5 |

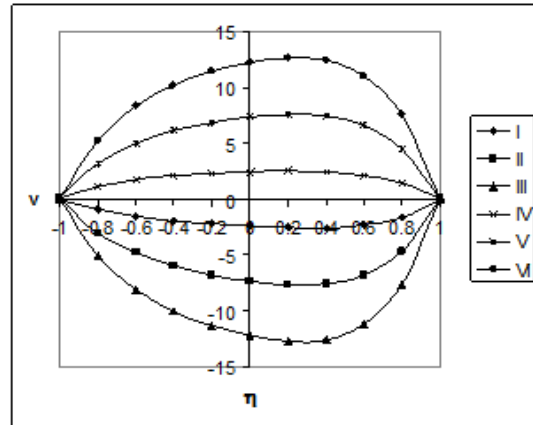


Fig. 8: Variation of v with G

| | I | II | III | IV | V | VI |
|-----|--------|-----------------|-----------------|---------|------------------|------------------|
| G | 10^2 | 3×10^2 | 5×10^2 | -10^2 | -3×10^2 | -5×10^2 |

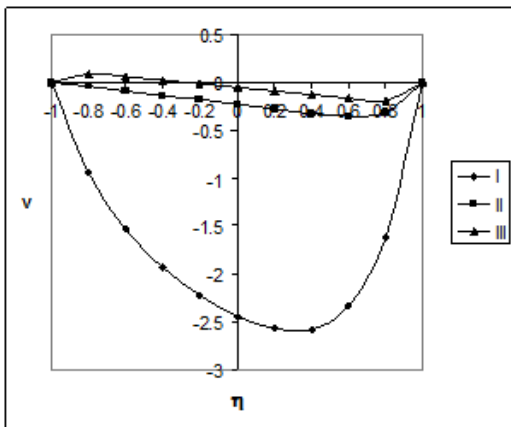


Fig. 9: Variation of v with M

| | I | II | III |
|-----|---|----|-----|
| M | 2 | 5 | 10 |

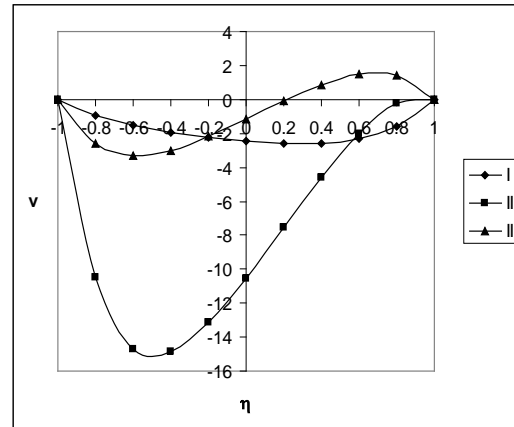


Fig. 10: Variation of v with α

| | I | II | III |
|----------|---|----|-----|
| α | 2 | 4 | 6 |

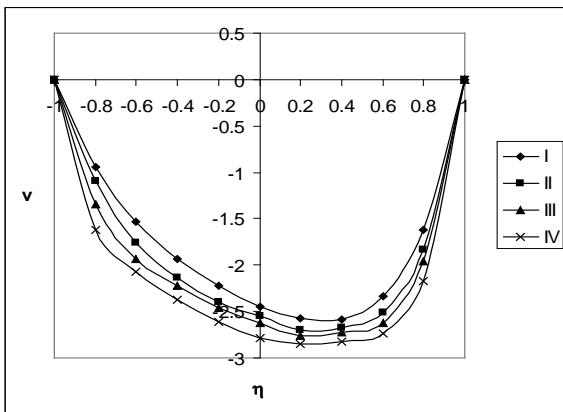


Fig. 11: Variation of v with Sc

| | I | II | III | IV |
|------|------|------|------|------|
| Sc | 0.24 | 0.60 | 1.30 | 2.01 |

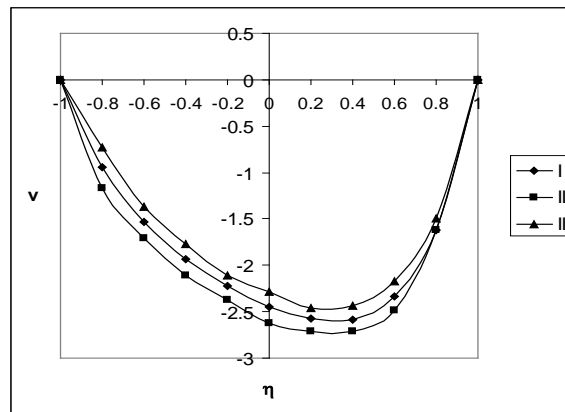


Fig. 12: Variation of v with γ

| | I | II | III |
|----------|---|----|-----|
| γ | 2 | 4 | 6 |

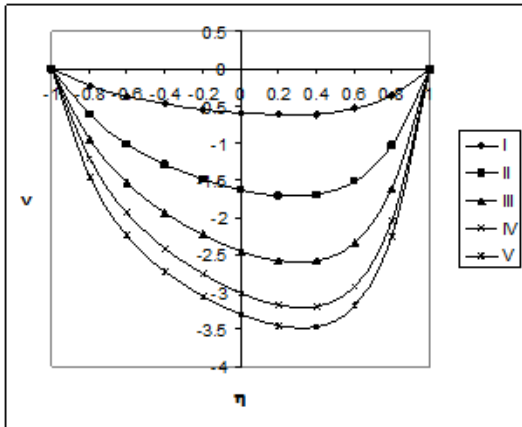


Fig. 13: Variation of v with β

| | I | II | III | IV | V |
|---------|-----|-----|-----|-----|-----|
| β | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |

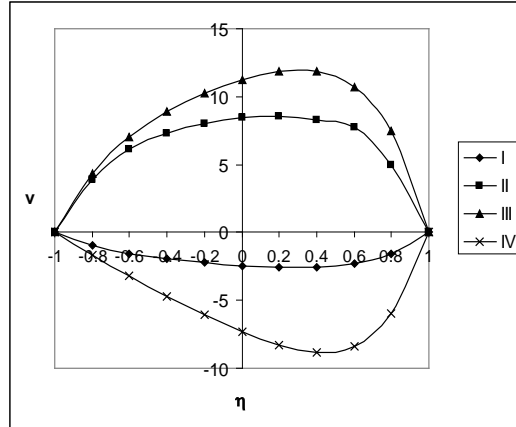


Fig. 14: Variation of v with k

| | I | II | III | IV |
|-----|-----|-----|-----|-----|
| k | 1.5 | 2.5 | 3.5 | 4.5 |

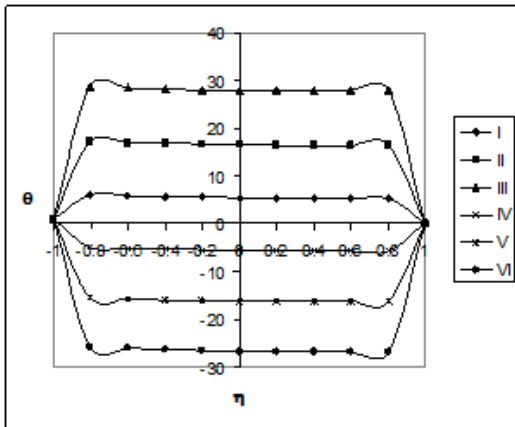


Fig. 15: Variation of θ with G

| | I | II | III | IV | V | VI |
|-----|--------|-----------------|-----------------|---------|------------------|------------------|
| G | 10^2 | 3×10^2 | 5×10^2 | -10^2 | -3×10^2 | -5×10^2 |

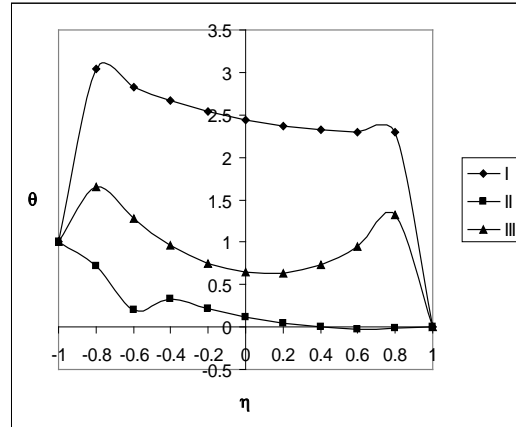


Fig. 16: Variation of θ with M

| | I | II | III |
|-----|---|----|-----|
| M | 2 | 5 | 10 |

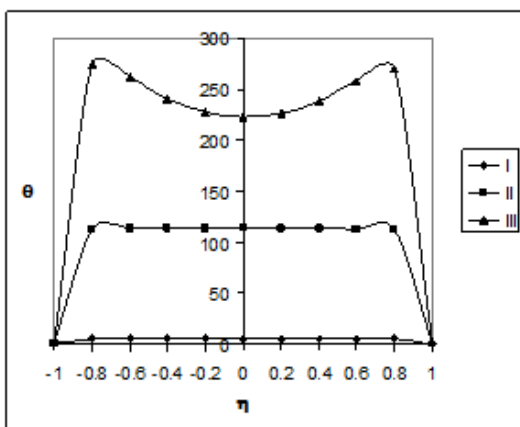


Fig. 17: Variation of θ with α

| | I | II | III |
|----------|---|----|-----|
| α | 2 | 4 | 6 |

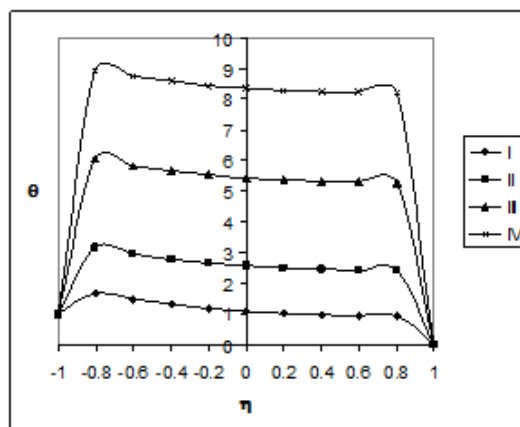


Fig. 18: Variation of θ with Sc

| | I | II | III | IV |
|------|------|------|------|------|
| Sc | 0.24 | 0.60 | 1.30 | 2.01 |

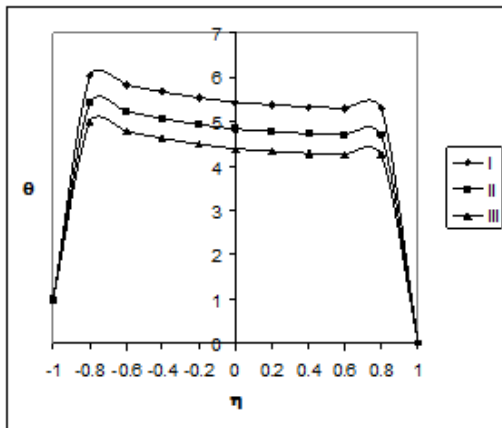


Fig.19: Variation of θ with γ

| | I | II | III |
|----------|---|----|-----|
| γ | 2 | 4 | 6 |

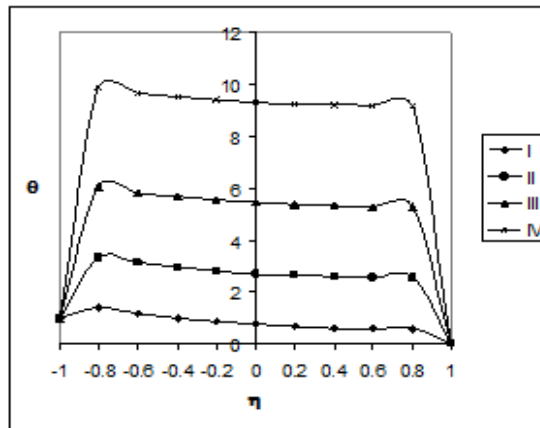


Fig. 20: Variation of θ with β

| | I | II | III | IV | V |
|---------|-----|-----|-----|-----|-----|
| β | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |

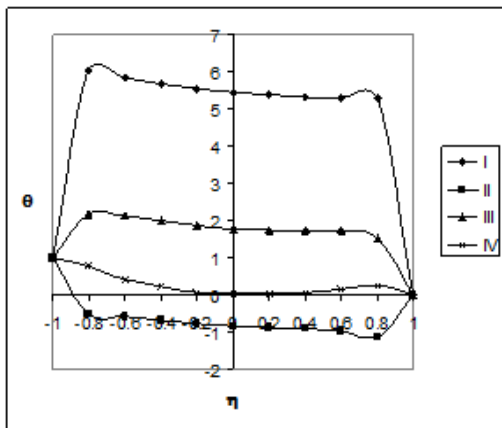


Fig. 21: Variation of θ with k

| | I | II | III | IV |
|-----|-----|-----|-----|-----|
| k | 1.5 | 2.5 | 3.5 | 4.5 |

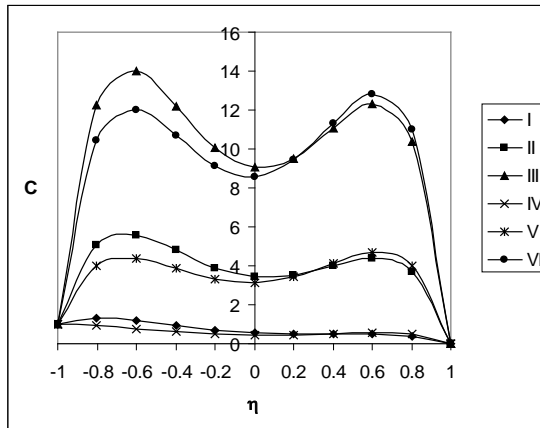


Fig. 22: Variation of C with G

| | I | II | III | IV | V | VI |
|-----|--------|-----------------|-----------------|---------|------------------|------------------|
| G | 10^2 | 3×10^2 | 5×10^2 | -10^2 | -3×10^2 | -5×10^2 |

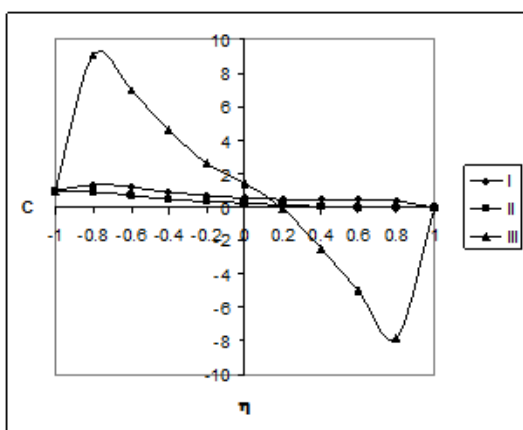


Fig. 23: Variation of C with M

| | I | II | III |
|-----|---|----|-----|
| M | 2 | 5 | 10 |

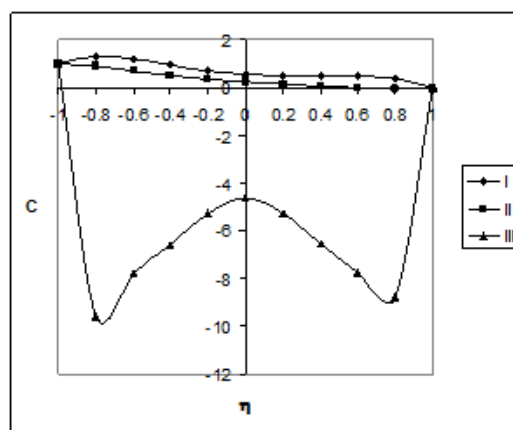


Fig. 24: Variation of C with α

| | I | II | III |
|----------|---|----|-----|
| α | 2 | 4 | 6 |

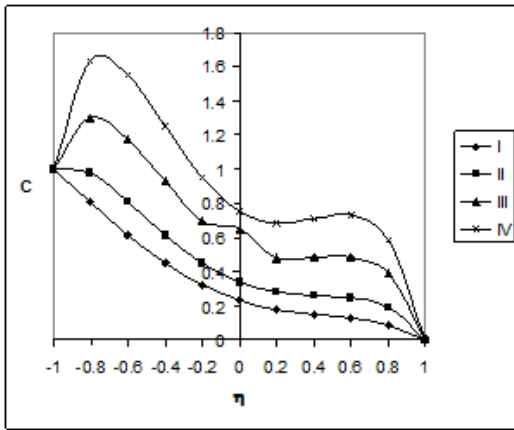


Fig. 25: Variation of C with Sc

| | | | | |
|----|------|------|------|------|
| | I | II | III | IV |
| Sc | 0.24 | 0.60 | 1.30 | 2.01 |

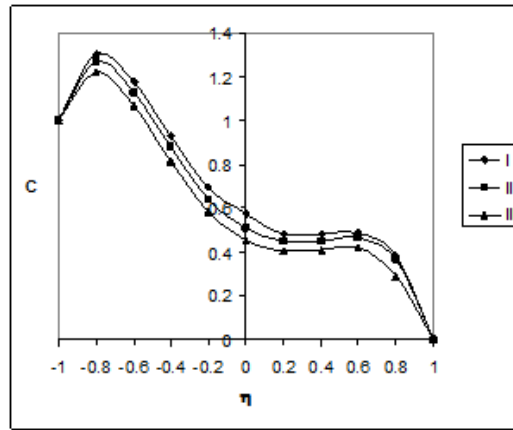


Fig. 26: Variation of C with γ

| | | | |
|----------|---|----|-----|
| | I | II | III |
| γ | 2 | 4 | 6 |

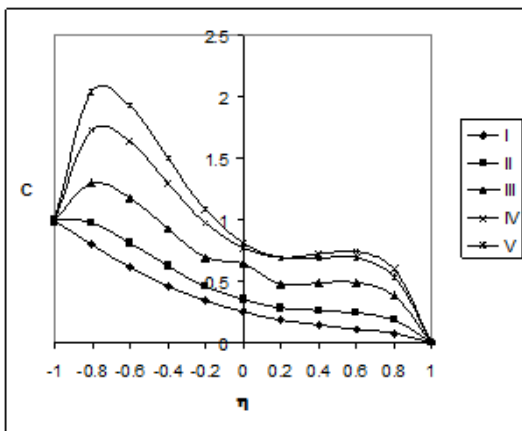


Fig. 27: Variation of C with β

| | | | | | |
|---------|-----|-----|-----|-----|-----|
| | I | II | III | IV | V |
| β | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |

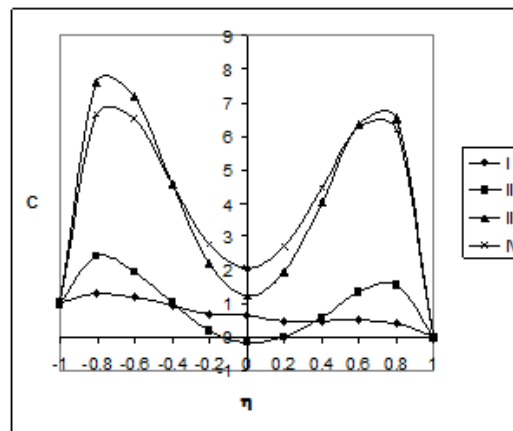


Fig. 28: Variation of C with k

| | | | | |
|---|-----|-----|-----|-----|
| | I | II | III | IV |
| k | 1.5 | 2.5 | 3.5 | 4.5 |

Table-1
Nusselt number (Nu) at $\eta = +1$

| G | I | II | III | IV | V | VI | VII | VIII | IX |
|------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10^3 | -0.16633 | -0.03163 | -0.02137 | 0.12502 | -0.00479 | -0.00584 | 0.00492 | -0.02435 | -0.02998 |
| 3×10^3 | 4.58437 | -0.02745 | -0.01943 | 0.00264 | -0.01013 | -0.01246 | -0.00731 | -0.05699 | -0.07721 |
| 5×10^3 | 1.76228 | -0.02535 | -0.02013 | -0.00731 | -0.01686 | -0.01988 | -0.03585 | -0.08160 | -0.11895 |
| -10^3 | 0.28334 | -0.03943 | -0.02869 | 0.02312 | -0.00294 | -0.00130 | -0.03532 | 0.02319 | 0.02691 |
| -3×10^3 | 0.31308 | -0.05442 | -0.04988 | 0.04036 | -0.01779 | -0.00664 | -0.74550 | 0.10352 | 0.10300 |
| -5×10^3 | 0.25329 | -0.08722 | -0.13184 | 0.03795 | 0.18091 | 0.11405 | 0.54701 | 0.28235 | 0.22598 |
| Sc | 1.3 | 0.24 | 0.6 | 2.01 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 |
| γ | 2 | 2 | 2 | 2 | 4 | 6 | 2 | 2 | 2 |
| N | 1 | 1 | 1 | 1 | 1 | 1 | 2 | -0.5 | -0.8 |

Table-2
Nusselt number (Nu) at $\eta = -1$

| G | I | II | III | IV | V | VI | VII | VIII | IX |
|------------------|----------|---------|-----------|----------|----------|----------|----------|---------|---------|
| 10^3 | -0.08687 | 0.30744 | 0.30517 | 0.16605 | 0.29956 | 0.29816 | 0.29373 | 0.30644 | 0.30638 |
| 3×10^3 | 2.65625 | 0.24242 | 0.22590 | 0.18371 | 0.19781 | 0.19503 | 0.17833 | 0.19095 | 0.17911 |
| 5×10^3 | 1.53743 | 0.19872 | 0.17614 | 0.12486 | 0.14075 | 0.13723 | 0.09900 | 0.11563 | 0.08566 |
| -10^3 | 0.35264 | 0.41655 | 0.45877 | 0.68536 | 0.56007 | 0.56256 | 0.58726 | 0.51182 | 0.50289 |
| -3×10^3 | 0.35835 | 0.64295 | 0.91506 | -2.15037 | 3.72464 | 3.77411 | -1.48120 | 0.99651 | 0.87942 |
| -5×10^3 | 0.28428 | 1.42239 | -37.24785 | -0.37579 | -0.75000 | -0.76766 | -0.06686 | 3.73909 | 2.04993 |
| Sc | 1.3 | 0.24 | 0.6 | 2.01 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 |
| γ | 2 | 2 | 2 | 2 | 4 | 6 | 2 | 2 | 2 |
| N | 1 | 1 | 1 | 1 | 1 | 1 | 2 | -0.5 | -0.8 |

Table-3
Sherwood number (Sh) at $\eta = +1$

| G | I | II | III | IV | V | VI | VII | VIII | IX |
|------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10^3 | -6.28048 | 0.15517 | -1.65257 | -0.00433 | -0.00441 | -0.00452 | 0.08152 | -0.20638 | -0.25734 |
| 3×10^3 | 59.15698 | 0.73996 | 0.10114 | 0.22910 | 0.22849 | 0.22774 | 0.17241 | -0.35132 | -0.56428 |
| 5×10^3 | 16.67761 | 1.31453 | 0.56839 | 0.39619 | 0.39473 | 0.39294 | -0.20886 | -0.52582 | -0.98312 |
| -10^3 | -3.09650 | -0.43906 | 1.00843 | -0.29950 | -0.29956 | -0.29964 | -0.48704 | -0.08942 | -0.05770 |
| -3×10^3 | 6.73608 | -1.04191 | -0.39133 | -0.65134 | -0.65202 | -0.65286 | -1.49391 | 0.00128 | 0.04189 |
| -5×10^3 | 6.45649 | -1.65257 | -0.90692 | -1.05440 | -1.05645 | -1.05900 | -2.86270 | 0.06758 | 0.05055 |
| Sc | 1.3 | 0.24 | 0.6 | 2.01 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 |
| γ | 2 | 2 | 2 | 2 | 4 | 6 | 2 | 2 | 2 |
| N | 1 | 1 | 1 | 1 | 1 | 1 | 2 | -0.5 | -0.8 |

Table-4
Sherwood number (Sh) at $\eta = -1$

| G | I | II | III | IV | V | VI | VII | VIII | IX |
|----------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 10^3 | - 18.25392 | -9.28312 | -3.97834 | -9.13616 | -9.15039 | -9.16807 | - 11.32324 | -6.60309 | -6.19487 |
| 3×10^3 | - 11.81682 | - 11.21569 | -9.23283 | - 11.00023 | - 11.04884 | - 11.10962 | - 20.76539 | -3.57309 | -2.86901 |
| 5×10^3 | - 13.57864 | - 13.23784 | - 11.14021 | - 13.31674 | - 13.41038 | - 13.52812 | -36.6240 | 1- 1.08234 | -0.59741 |
| -10^3 | - 30.11087 | -7.43545 | - 13.26602 | -7.75220 | -7.73931 | -7.72340 | -6.48857 | - 10.38062 | - 11.02130 |
| - 3×10^3 | - 22.59347 | -5.66846 | -7.54073 | -6.90270 | -6.86572 | -6.82040 | -5.90355 | - 15.23539 | - 18.16160 |
| - 5×10^3 | - 20.45791 | -3.97834 | -6.06446 | -6.68133 | -6.61701 | -6.53886 | - 10.69353 | - 21.72232 | - 29.23937 |
| Sc | 1.3 | 0.24 | 0.6 | 2.01 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 |
| γ | 2 | 2 | 2 | 2 | 4 | 6 | 2 | 2 | 2 |
| N | 1 | 1 | 1 | 1 | 1 | 1 | 2 | -0.5 | -0.8 |

Source of support: Nil, Conflict of interest: None Declared