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EFFECT OF RADIATION AND DISSIPATION ON CONVECTIVE HEAT TRANSFER FLOW OF A VISCOUS ELECTRICALLY CONDUCTING FLUID IN A NON-UNIFORMLY HEATED VERTICAL CHANNEL

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ABSTRACT

The effect of radiation on mixed convective heat transfer flow of a viscous fluid, incompressible electrically conducting fluid in a vertical channel bounded by flat walls. A non-uniform temperature is imposed on the walls on the walls. The viscous dissipation is taken in to account in the energy equation. Assuming the slope of the boundary temperature to be small. We solve the governing momentum, energy and diffusion equations by a perturbation technique. The velocity, the temperature, the shear stress and the rate of heat transfer have been analyzed for different variations of the governing parameters. The dissipative effects and radiation effects on the flow, heat and mass transfer are clearly brought out

Keywords: *Radiation Dissipation, Heat Transfer, Viscous clued, Vertical Channel.*

1. INTRODUCTION

The process of free convection as a mode of heat transfer has wide applications in the fields of Chemical Engineering, Aeronautical and Nuclear power generation. It was shown by Gill and Casal (1962) that the buoyancy significantly affects the flow of low Prandtl number fluids which is highly sensitive to gravitational force and the extent to which the buoyancy force influences a forced flow is a topic of interest. Free convection flows between two long vertical plates have been studied for many years because of their engineering applications in the fields of nuclear reactors, heat exchangers, cooling appliances in electronic instruments. These flows were studied by assuming the plates at two different constant temperatures or temperature of the plates varying linearly along the plates etc. The study of fully developed free convection flow between two parallel plates at constant temperature was initiated by Ostrach (1952). Combined natural and forced convection laminar flow with linear wall temperature profile was also studied by Ostrach (1954). The first exact solution for free convection in a vertical parallel plate channel with asymmetric heating for a fluid of constant properties was presented by Anug (1972). Many of the early works on free convection flows in open channels have been reviewed by Manca *et al.* (2000). Recently, Campo *et al.* (2006) considered natural convection for heated iso-flux boundaries of the channel containing a low-Prandtl number fluid. Pantokratoras (2006) studied the fully developed free convection flow between two asymmetrically heated vertical parallel plates for a fluid of varying thermophysical properties. However, all the above studies are restricted to fully developed steady state flows. Very few papers deal with unsteady flow situations in vertical parallel plate channels. Transient free convection flow between two long vertical parallel plates maintained at constant but unequal temperatures was studied by Singh *et al.* (2006). Jha *et al.* (2003) extended the problem to consider symmetric heating of the channel walls.

In the risk assessment of nuclear power plants, the possibility and the consequences of a melt down of the reactor core are usually considered. During the course of such an accident molten fuel and coolant may interact. Violent thermal reaction can dispose the molten fuel into fine particles. These small particles quickly solidify in the coolant and settle on internal structures of the reactor pressure vessel forming a saturated porous bed. The question arises, under what conditions the nuclear decay heat can be removed from the particle bed to the ambient coolant by natural convection. Thus the problem of natural convection in saturated porous layers. This analysis of heat transfer in a viscous heat generating fluid also important in engineering processes pertain to flow in which a fluid supports an exothermal chemical or nuclear reaction or problems concerned with dissociating fluids(Less(1950), Lighthil(1958)). The Volumetric heat generation has been assumed to be constant (Ajay(2003), Bejamin(1988), Bejan(1985), Bejan (1984), Bird(1955), Cheng (1979, Ostrach(1954), Palm(1975)) or a function of space variable (Chambre(1957), Costa(1980), Gill (1962), Greosh(1958), Helman(1950), Krishna(2002)). For example a hypothetical core-disruptive accident in a liquid metal fast breeder reactor (LMFBR) could result in the setting of fragmented fuel debris as horizontal surfaces below the core. The porous debris could be saturated sodium coolant and heat generation will result from the radioactive decay of the fuel particulate(1974) The heat losses from the geothermal system in some cases can be treated as if the heat comes from the heat generating sources (1974). Keeping this in view, porous medium with internal heat source have been discussed by several authors (Buretta (1972), Gabor (1974), Hardec (1974), Hardec (1979), Palm (1954)).

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In the above mentioned investigations the bounding walls are maintained at constant temperature. However, there are a few physical situations which warrant the boundary temperature to be maintained non-uniform. It is evident that in forced or free convection flow in a channel (pipe) a secondary flow can be created either by corrugating the boundaries or by maintaining non-uniform wall temperature such a secondary flow may be of interest in a few technological process. For example in drawing optical glass fibres of extremely low loss and wide bandwidth, the process of modified chemical vapour deposition (MCVD) (Krishna (2002), Simpikins (1979)) has been suggested in recent times. Performs from which these fibres are drawn are made by passing a gaseous mixture into a fused – silica tube which is heated locally by an oxy-hydrogen flame. Particulates of Sio_2- Geo_2 composition are formed from the mixture and collect on the interior of the tube. Subsequently these are fined to form a victorious deposit as the flame traversed along the tube. The deposition is carried out in the radial direction as the flame traversed along the tube. The deposition is carried out in the radial direction through the secondary flow created due to non-uniform.

All the above mentioned studies are based on the hypothesis that the effect of dissipation is neglected. This is possible in case of ordinary fluid flow like air and water under gravitational force. But this effect is expected to be relevant for fluids with high values of the dynamic viscous flows. Moreover Gehart(1962), Gebhart and Mollen dorf(1969) have shown that that viscous dissipation heat in the natural convective flow is important when the flow field is of extreme size or at extremely low temperature or in high gravitational filed. On the other hand Barletta (1997) has pointed out that relevant effect of viscous dissipation on the temperature profiles and on the Nusselt numbers may occur in the fully developed forced convection in tubes. In view of this several authors notably, Soudalgekar and Pop (1974) Raptis etc al (1995), Barletta (Barletta (1997, 1998)), Sreevani (1992). El-hakeing (2000), Bulent Ypsilanti (2002). Rossidi schio (2001) and Israel *et al* (2003) have studied the effect of viscous dissipation on the convective flows past on infinite vertical plates and through vertical channels and Ducts. The effect of viscous dissipation on natural convection has been studied for some different cases including the natural convection from horizontal cylinder. The natural convection from horizontal cylinder embedded in a porous media has been studied by Fand and Brucker (1996). They reported that the viscous dissipation may not be neglected in all cases of natural convection from horizontal cylinders and further, that the inclusion of a viscous dissipation term in porous medium may lead to more accurate correlation equations. The effect of viscous dissipation has been studied by Nakayama and Pop (1989) for steady free convection boundary layer over a non-isothermal bodies of arbitrary shape embedded in porous media. They used integral method to show that the viscous dissipation results in lowering the level of the heat transfer rate from the body. Costa (2005) has analyzed a natural convection in enclosures with viscous dissipation. Recently Jambal et al have discussed the effects of viscous dissipation and fluid axial heat conduction heat transfer for non-Newtonian fluids in ducts with uniform wall temperature. Recently Prasad (2006)has discussed the effect of dissipation on the mixed convective heat and mass transfer flow of a viscous fluid through a porous medium in a vertical channel bounded by flat walls.

In this chapter, we discuss the effect of radiation on mixed convective heat transfer flow of a viscous fluid, incompressible electrically conducting fluid in a vertical channel bounded by flat walls. A non-uniform temperature is imposed on the walls on the walls. The viscous dissipation is taken in to account in the energy equation. Assuming the slope of the boundary temperature to be small. We solve the governing momentum, energy and diffusion equations by a perturbation technique. The velocity, the temperature, the shear stress and the rate of heat transfer have been analyzed for different variations of the governing parameters. The dissipative effects and radiation effects on the flow, heat and mass transfer are clearly broughtout.

2. FORMULATION OF THE PROBLEM

We analyze the steady motion of viscous, incompressible fluid in a vertical channel bounded by flat walls which are maintained at a non-uniform wall temperature in the presence of a constant heat source. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous dissipations and the joule heating are taken into account in the energy equation. Also the kinematic viscosityν, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0(x, y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at y = ± L. The equations governing the steady flow, heat and mass transfer are Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

Equation of linear momentum**:**

$$\rho_e\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho g-(\sigma\mu_e^2H_o^2)u \tag{2.2}$$

$$\rho_e\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2.3}$$

Equation of Energy:

$$\rho_e C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial v}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial v^2}\right)+Q+\mu\left(\left(\frac{\partial u}{\partial v}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right)-\frac{\partial(q_R)}{\partial v} \tag{2.4}$$

Equation of State:

$$\rho-\rho_e=-\beta\rho_e(T-T_e) \tag{2.5}$$

where ρ_e is the density of the fluid in the equilibrium state, Te is the temperature in the equilibrium state, (u, v) are the velocity components along O(x, y) directions, p is the pressure, T is the temperature in the flow region, ρ is the density of the fluid,μ is the constant coefficient of viscosity, Cp is the specific heat at constant pressure, λ is the coefficient of thermal conductivity, μ_e is the *the magnetic* permeability, σ is the electrical conductivity, β is the coefficient of thermal expansion, Q is the strength of the constant internal heat source , qr is the radiative heat flux.

Invoking Rosseland approximation for radiation

$$q_r=-\frac{4\sigma^{\bullet}}{3\beta_R}\frac{\partial(T'^4)}{\partial y}$$

Expanding T'^4 in Taylor's series about Te neglecting higher order terms (2.5)

$$T'^4\cong 4T_e^3T'-3T_e^4$$

where $\sigma^{\bullet}$ is the Stefan-Boltzmann constant β_R is the Extinction coefficient.

In the equilibrium state

$$0=-\frac{\partial p_e}{\partial x}-\rho_e g \tag{2.6}$$

where $p=p_e+p_D$, p_D being the hydrodynamic pressure.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$Q=\frac{1}{2L}\int_{-L}^{L}u\,dy. \tag{2.7}$$

The boundary conditions for the velocity and temperature fields are

u = 0, v = 0 on y = ±L

$$T-T_e=\gamma(\delta x/L) \quad \text{on } y=\pm L \tag{2.8}$$

γ is chosen to be twice differentiable function,δ is a small parameter characterizing the slope of the temperature variation on the boundary.

In view of the continuity equation we define the stream function ψ as

$$u=-\psi_y,\ v=\psi_x \tag{2.9}$$

The equation governing the flow in terms of ψ are

$$\left[\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial(\nabla^2\psi)}{\partial x}\right]=\nu\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)^2-\beta g\frac{\partial T}{\partial y}-\left(\frac{\sigma\mu_e^2H_o^2}{\rho}\right)\frac{\partial^2\psi}{\partial y^2} \tag{2.10}$$

$$\rho_e C_p\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)=\lambda\nabla^2\theta+Q+\mu\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\left(\frac{\partial^2\psi}{\partial x^2}\right)\right)+\frac{4}{3N_1}\frac{\partial^2\theta}{\partial y^2} \tag{2.11}$$

Introducing the non-dimensional variables in (2 .10) - (2.11) as

$$(x',\ y') = (x, y)/L,\ (u',v') = (u,v)/U,\ \theta = \frac{T - T_e}{\Delta T_e}$$

$$p' = \frac{p_D}{\rho_e U^2}, \gamma' = \frac{\gamma}{\Delta T_e} \tag{2.12}$$

(Under the equilibrium state $\Delta T_e = T_e(L) - T_e(-L) = \left(\frac{QL^2}{\lambda}\right)$

The governing equations in the non-dimensional form (after dropping the dashes) are

$$R\frac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)} = \nabla^4\psi + \frac{G}{R}\theta_y - M^2\frac{\partial^2\psi}{\partial y^2} \tag{2.13}$$

and the energy diffusion equations in the non-dimensional form are

$$P_1R\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \nabla^2\theta + N_2 + \left(\frac{P_1R^2E_c}{G}\right)\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 + \left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right) \tag{2.14}$$

where $R = \frac{UL}{\nu}$ (Reynolds number) $G = \frac{\beta g \Delta T_e L^3}{\nu^2}$ (Grashof number)

$M^2 = \frac{\sigma\mu_e^2 H_o^2 L^2}{\upsilon^2}$ (Hartmann Number) $\mathrm{P} = \frac{\mu c_p}{k_1}$ (Prandtl number),

$E_c = \frac{\beta g L^3}{C_p}$ (Eckert number) $N_1 = \frac{\beta_R \lambda}{4\sigma^r\ T_e^3}$ (Radiation parameter)

$N_2 = \frac{3N_1}{3N_1 + 4}, P_1 = PN_2$

The corresponding boundary conditions are

$$\psi(+1) - \psi(-1) = 1$$

$$\frac{\partial\psi}{\partial x} = 0,\ \frac{\partial\psi}{\partial y} = 0 \quad at\ y = \pm 1 \tag{2.15a}$$

$$\theta(x, y) = f(\delta x) \qquad on\ \ y = \pm 1 \tag{2.15b}$$

$$\frac{\partial\theta}{\partial y} = 0\ \ at \quad y = 0 \tag{2.16}$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis (2.7) .Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function γ(x).

3. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to non-uniform slowly varying temperature imposed on the boundaries. We introduce the transformation

$$\bar{x} = \delta\, x$$

With this transformation the equations (2.13), (2.14) reduce to

$$R\delta\frac{\partial(\psi, F^2\psi)}{\partial(x, y)} = F^4\psi + \frac{G}{R}\theta_y - M^2\frac{\partial^2\psi}{\partial y^2} \tag{3.1}$$

and the energy &diffusion equations in the non-dimensional form are

$$P_1 R\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)=F^2\theta+N_2+\left(\frac{P_1R^2E_c}{G}\right)\left(\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\delta^2\left(\frac{\partial^2\psi}{\partial x^2}\right)^2\right)+M^2\left(\frac{\partial\psi}{\partial y}\right)^2 \qquad (3.2)$$

where $F^2=\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

for small values of the slope δ,the flow develops slowly with axial gradient of order δ and hence we take

$$\frac{\partial}{\partial\bar{x}}\approx O(1)$$

We follow the perturbation scheme and analyze through first order as a regular perturbation problem at finite values of R, G, P, Sc and M

Introducing the asymptotic expansions

$\psi(x, y) = \psi_0(x, y) + \delta\psi_1(x, y) + \delta^2\psi_2(x, y) + \ldots\ldots$

$\theta(x, y) = \theta_0(x, y) + \delta\theta_1(x, y) + \delta^2\theta_2(x, y) +$ (3.3)

On substituting (3.3) in (3.1) & (3.2) and separating the like powers of δ the equations and respective conditions to the zeroth order are

$$\psi_{0,yyyyy}-M_1^2\psi_{0,yy}=-\frac{G}{R}(\theta_{0,y}) \qquad (3.4)$$

$$\theta_{0,yy}=-\frac{P_1R^2Ec}{G}\psi_{0,yy}^2-\frac{P_1M_1^2Ec}{G}\psi_{0,y}^2 \qquad (3.5)$$

with

$\psi_0(+1)-\Psi(-1) = 1$,

$\psi_{0,y} = 0$, $\psi_{0,x} = 0$ at $y = \pm1$ (3.6a)

$\theta o(\pm1) = \gamma(x)$ at $y = \pm1$ (3.6b)

and to the first order are

$$\psi_{1,yyyyy}-M_1^2\psi_{1,yy}=-\frac{G}{R}(\theta_{1,y})+R(\psi_{0,y}\psi_{0,xyy}-\psi_{0,x}\psi_{0,yyy}) \qquad (3.7)$$

$$\theta_{1,yy}=P_1R(\psi_{0,x}\theta_{0,y}-\psi_{0,y}\theta_{0,x})-\frac{P_1Ec}{G}(R^2\psi_{1,yy}^2+M_1^2\psi_{1,y}^2) \qquad (3.8)$$

$\psi_{1(+1)} - \psi_{1(-1)} = 0$

$\psi_{1,y} = 0$, $\psi_{1,x} = 0$ at $y = \pm1$ (3.9)

$\theta_1(\pm1) = 0$ at $y = \pm 1$ (3.10)

Assuming Ec<<1 to be small we take the asymptotic expansions as

$$\psi_0(x,y)=\psi_{00}(x,y)+Ec\psi_{01}(x,y)+\ldots\ldots\ldots\ldots$$
$$\psi_1(x,y)=\psi_{10}(x,y)+Ec\psi_{11}(x,y)+\ \ \ldots\ldots\ldots\ldots$$
$$\theta_0(x,y)=\theta_{00}(x,y)+Ec\theta_{01}(x,y)+\ldots\ldots\ldots\ldots\ldots$$
$$\theta_1(x,y)=\theta_{10}(x,y)+Ec\theta_{11}(x,y)+\ldots\ldots\ldots\ldots\ldots \qquad (3.11)$$

Substituting the expansions (3.11) in equations (3.4) - (3.10) and separating the like powers of Ec we get the following equations

$$\theta_{00,yy}=-1,\ \ \theta_{00}(\pm1)=f(\bar{x}) \qquad (3.12)$$

$$\psi_{00,yyyy} - M_1^2\psi_{00,yy} = -\frac{G}{R}(\theta_{00,y}),$$
$$\psi_{00}(+1) - \psi_{00}(-1) = 1, \psi_{00,y} = 0, \psi_{00,x} = 0 \quad at \quad y = \pm 1 \tag{3.13}$$

$$\theta_{01,yy} = -\frac{P_1 M_1^2}{G}\psi^2{}_{00,y} - \frac{P_1 R^2}{G}\psi^2{}_{00,yy}, \qquad \theta_{01}(\pm 1) = 0 \tag{3.14}$$

$$\psi_{01,yyyy} - M_1^2\psi_{01,yy} = -\frac{G}{R}(\theta_{01,y}),$$
$$\psi_{01}(+1) - \psi_{01}(-1) = 0, \tag{3.15}$$
$$\psi_{01,y} = 0, \psi_{01,x} = 0 \quad at \quad y = \pm 1$$

$$\theta_{10,yy} = RP_1(\psi_{00,y}\theta_{00,x} - \psi_{00,x}\theta_{00,y}) \qquad \theta_{10}(\pm 1) = 0 \tag{3.16}$$

$$\psi_{10,yyyy} - M_1^2\psi_{10,yy} = -\frac{G}{R}(\theta_{10,y}) + R(\psi_{00,y}\psi_{00,xyy} - \psi_{00,x}\psi_{00,yyy}),$$
$$\psi_{10}(+1) - \psi_{10}(-1) = 0, \psi_{10,y} = 0, \psi_{10,x} = 0 \quad at \quad y = \pm 1 \tag{3.17}$$

$$\theta_{11,yy} = RP_1(\psi_{00,y}\theta_{1,x} - \psi_{1,x}\theta_{00,y}), \quad \theta_1(\pm 1) = 0 \tag{3.18}$$

$$\psi_{11,yyyy} - M_1^2\psi_{1,yy} = -\frac{G}{R}(\theta_{11,y}) + R(\psi_{00,y}\psi_{11,xyy} - \psi_{00,x}\psi_{01,yyy} + \psi_{01,y}\psi_{00,xyy} - \psi_{01.x}\psi_{00,yyy}),$$
$$\psi_{11}(+1) - \psi_{11}(-1) = 0, \psi_{11,y} = 0, \psi_{11,x} = 0 \quad at \quad y = \pm 1 \tag{3.19}$$

4. SOLUTION OF THE PROBLEM

Solving the equations (3.12)- (3.19) subject to the relevant boundary conditions we obtain

$$\theta_{00} = 0.5(1 - y^2) + \gamma(\bar{x})$$
$$\psi_{00} = a_4 Ch(M_1 y) + a_5 Sh(M_1 y) + a_6 y + a_7 + a_3 y^3 + d_2 y^2$$
$$\theta_{01} = 0.5PM_1^2(y^2 - 1)$$
$$\psi_{01} = a_{10} Sh(M_1 y) + a_{11} y + a_{12} + a_8 y^3$$
$$\theta_{10} = a_{24}y^2 + a_{25}y^3 + a_{26}y^4 + a_{27}y^5 + a_{28}y^6 + (a_{20} + ya_{22})Ch(M_1 y) + (a_{21} + ya_{23})Sh(M_1 y)$$

$$\psi_{10} = b_8 Ch(M_1 y) + b_9 Sh(M_1 y) + b_{10} y + b_{11} + \phi(y)$$
$$\begin{aligned}\varphi(y) = {} & a_{70}y^2 + a_{71}y^3 + a_{72}y^4 + a_{73}y^5 + a_{74}y^6 \\ & + a_{75}y^7 + (b_1 y + b_3 y^2 + b_5 y^3)Ch(M_1 y) \\ & + (b_2 y + b_4 y^2 + b_6 y^4)Sh(M_1 y) + b_7 y^4 Sh(M_1 y)\end{aligned}$$

$$\begin{aligned}\theta_{11} = {} & b_{15}y^2 + b_{16}y^3 + b_{17}Ch(2M_1 y) + b_{18}Sh\left(2M_1 y\right) + b_{19}y^4 + b_{20}y^6 \\ & + b_{21}y^8 + b_{22}y^{10} + b_{23}y^{11} + b_{23}y^{11} + b_{24}y^{12} + b_{25}y^8 Sh\left(2M_1 y\right) \\ & + b_{26}y^7 Ch(2M_1 y) + b_{27}y^6 Sh\left(2M_1 y\right) + b_{28}y^5 Ch\left(2M_1 y\right) \\ & + b_{29}y^4 Sh(2M_1 y) \; b_{30}y^3 Ch\left(2M_1 y\right) + b_{31}y^2 Sh\left(2M_1 y\right) \\ & + b_{32}yCh(2M_1 y) + b_{33}Sh\left(2M_1 y\right) + b_{34}y + b_{35}\end{aligned}$$

$$\psi_{11} = b_{54} + b_{55}y + b_{56}Ch(M_1 y) + b_{57}Sh(M_1 y) + \phi_1(y)$$

$$\begin{aligned}\varphi(y) &= b_{34}y^{14} + b_{35}y^{13} + b_{36}y^{12} + b_{37}y^{11} + b_{38}y^{10} + b_{39}y^{9} + b_{40}y^{6} + b_{41}y^{5} \\ &+ b_{42}y^{4} + b_{43}y^{3} + b_{44}Ch(2M_1y) + b_{45}Sh(2M_1y) + b_{46}y^{8}Ch(2M_1y) \\ &+ b_{47}y^{7}Sh(2M_1y) + b_{48}y^{6}Ch(2M_1y) + b_{49}y^{5}Sh(2M_1y) \\ &+ b_{50}y^{4}Ch(2M_1y) + b_{51}y^{3}Sh(2M_1y) + b_{52}y^{2}Ch(2M_1y) \\ &+ b_{53}ySh(2M_1y)\end{aligned}$$

where $a_1, a_2, ...a_{75}, b_1, b_2, ...b_{53}$ are constants.

5. SHEAR STRESS AND NUSSELT NUMBER

The shear stress on the channel walls is given by

$\tau = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=\pm L}$ which in the non- dimensional form reduces to

$$\tau = \left(\frac{\frac{\tau}{\mu U}}{a}\right) = (\psi_{yy} - \delta^2\psi_{xx}) = [\psi_{00,yy} + Ec\psi_{01,yy} + \delta(\psi_{10,yy} + Ec\psi_{11,yy} + O(\delta^2)]_{y=\pm 1}$$

and the corresponding expressions are

$$(\tau)_{y=+1} = d_3 + Ecd_4 + \delta d_5 + O(\delta^2)$$

$$(\tau)_{y=-1} = d_6 + Ecd_7 + \delta d_8 + O(\delta^2)$$

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$Nu = \frac{1}{\theta_m - \theta_w}\left(\frac{\partial\theta}{\partial y}\right)_{y=\pm 1}$ and the corresponding expressions are

$$(Nu)_{y=+1} = \frac{(d_{10} + \delta(d_{11} + d_{12})}{(d_8 - \gamma(x) + \delta d_9)} \qquad (Nu)_{y=-1} = \frac{(-d_{10} + \delta(d_{12} - d_{11})}{(d_8 - \gamma(x) + \delta d_9)},$$

where $d_3, d_4, ..., d_{10}$ are constants.

6. DISCUSSION OF THE NUMERIC RESULTS

In this analysis we analyze the effect of radiation and dissipation on convective heat transfer flow of a viscous electrically conducting fluid in a non – uniformly heated vertical channel. The axial velocity u is shown in figs.1-6 for different values of G, M, α_1, N_1, Ec and x. The fig.1 represents the axial velocity u with Grashof number G. It is found that the actual axial velocity is the vertically upward direction and hence u<0 represents the reversal flow. We notice a reversal flow in the left half of the channel at G = 10^3 and in the right half at G=- 10^3. The region of the reversal flow enlarges with increase in $|G|$ with maximum attained at y = 0.6. The variation of u with Hartmann number M shows that the reversal flow which appears in the left half at M = 2, disappears at higher values of M . Higher the Lorentz force larger $|u|$ in the flow region. The variation in u is comparably large at higher M = 6. The point of maximum shifts towards the mid- region with increase in M(fig.2). The variation of u with the amplitude α_1 of the boundary temperature is shown in fig.3. It is found that the reversal flow appears in the left half for all values of α_1. $|u|$ depreciates with increase in α_1 and enhances with higher $\alpha_1 \geq 0.5$ (fig.3). Fig.4 represents the variation of u with radiation parameter N_1.

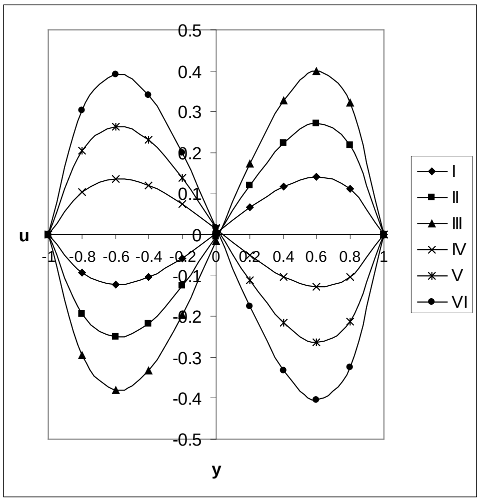

Fig. 1 : Variation of u with G

	I	II	III	IV	V	VI
G	$1x10^3$	$2x10^3$	$3x10^3$	-10^3	$-2x10^3$	$-3x10^3$

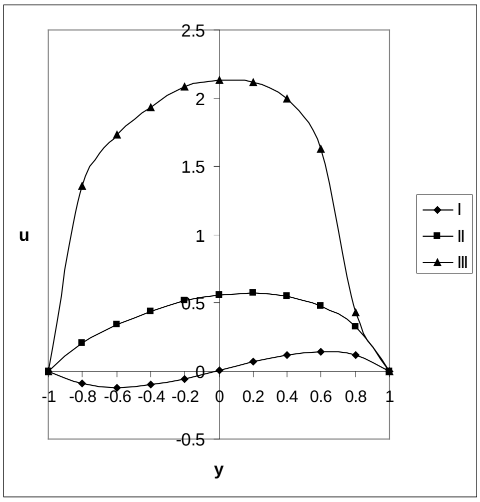

Fig. 2 : Variation of u with M

	I	II	III
M	2	4	6

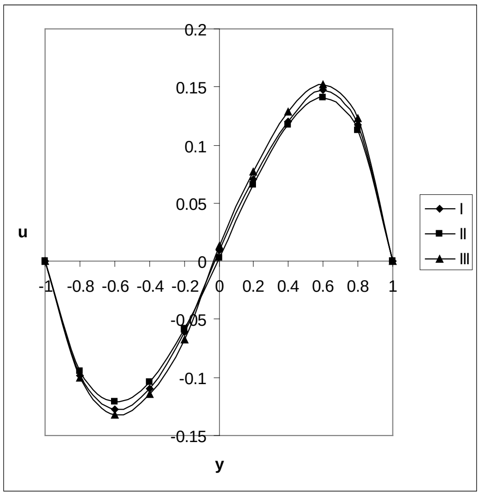

Fig. 3 : Variation of u with α_1

	I	II	III
α_1	0.1	0.3	0.5.

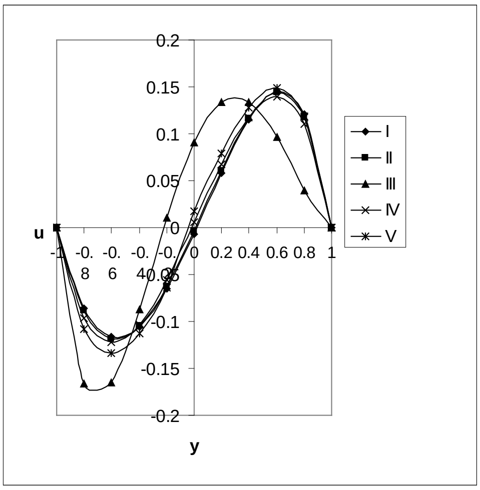

Fig. 4 : Variation of u with N_1

	I	II	III	IV	V
N_1	0.4	1	4	10	100

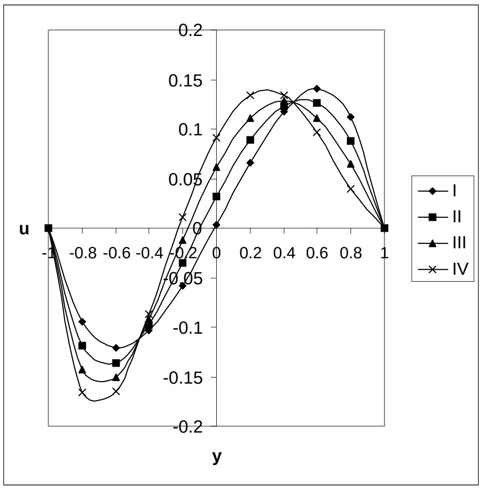

Fig. 5 : Variation of u with Ec

	I	II	III	IV
Ec	0.01	0.03	0.05	0.07

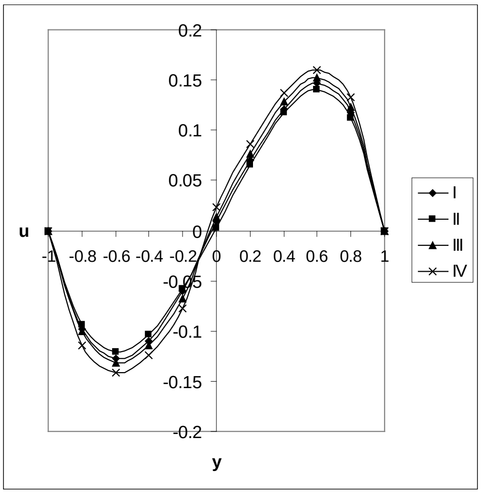

Fig. 6 : Variation of u with x

	I	II	III	IV
x	$\pi/4$	$\pi/2$	π	2π

It is found that the reversal flow which appears in the left half of the channel enlarges with $N_1 \le 4$ and depreciates at $N_1 = 10$ and again enlarges with $N_1 = 100$. Higher the radiative heat flux larger $|u|$ and further higher N_1 smaller $|u|$ and for still higher N_1 larger $|u|$ in the flow region (fig.4). Fig.5 represents the variation of u with Eckert number Ec.

It is found that the reversal flow which appears in left half enlarges with increase in Ec. $|u|$ experiences an enhancement with Ec except in narrow region adjacent to y = +1. Fig.6 represents u with axial distance x. Moving along the axial direction of the channel walls $|u|$ enhances with $x \le \pi$ and depreciates with $x \ge 2\pi$.

The secondary velocity (v) which is due to the non – uniform boundary temperature is shown in figs. 7 – 12 for different variations of the governing parameters. Fig.7 represents v with Grashof number G. It is found that for G>0, the secondary velocity is towards the boundary in the left half and is towards the mid region in the right half of the channel. $|u|$ experiences an enhancement with increase in $|G|$ with maximum attained at y = -0.6. The variation of v with M shows that higher the Lorentz force larger $|v|$ in the flow region (fig.8). The variation of v with amplitude α_1 of the non-uniform boundary temperature shows that the secondary velocity depreciates in magnitude with increase in $\alpha_1 \le 0.3$ and enhances with higher $\alpha_1 \ge 0.5$(fig.9). Fig.10 represents the variation of v with radiation parameter N_1. It is found that an increase in $N_1 \le 1$ depreciates $|v|$ in the vicinity of y = ± 1 and enhances it in the central region. For higher $N_1 \ge 4$ it enhances $|v|$. The variation of v with Eckert number Ec is shown in fig.11. Higher the dissipative heat (Ec$\le$0.03) larger $|v|$ and for further higher the dissipative heat (Ec = 0.05) smaller $|v|$ and for still higher dissipative heat (Ec=0.07) larger $|v|$. Moving along the axial direction of the channel walls $|v|$ depreciates in the region (-0.8 & 0.2) and enhances in the region (0.4 & 0.8) with $x \le \pi$ and for higher $x = 2\pi$, $|v|$ enhances in the region(-0.8 & 0) and depreciates in the region (0.2 & 0.8) (fig.12).

The non – dimensional temperature (θ) is shown in figs. 13-18 for different parametric values. Fig.13 represents θ with Grashof number G. It is found that the temperature enhances in the flow region for G>0 and depreciates with G<0 with maximum attained at y = 0. The variation of θ with Hartmann number M shows that the higher the Lorentz force larger the temperature and for further higher Lorentz force smaller the temperature in the flow region (fig.14). Fig.15 represents θ with amplitude α_1 of the boundary temperature. It is found that an increase in $\alpha_1 \le 0.3$, enhances the temperature while it reduces with higher $\alpha_1 \ge 0.5$. The variation of θ with radiation parameter N_1 shows that higher radiative heat flux larger the actual temperature in the flow region (fig.16). The effect of dissipation on θ is shown in fig.17. It is found that higher the dissipative heat (Ec$\le$ 0.03) larger the actual temperature and for further higher dissipative heat (Ec$\ge$ 0.05) lesser the actual temperature everywhere in the flow region. The actual temperature enhances with axial distance $x \le \dfrac{\pi}{2}$ and reduces at $x = \pi$ and again enhances with higher $x = 2\pi$ (fig.18).

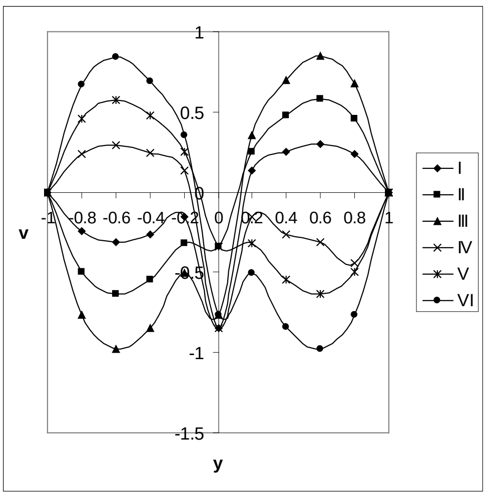

Fig. 7 : Variation of v with G

	I	II	III	IV	V	VI
G	$1x10^3$	$2x10^3$	$3x10^3$	-10^3	$-2x10^3$	$-3x10^3$

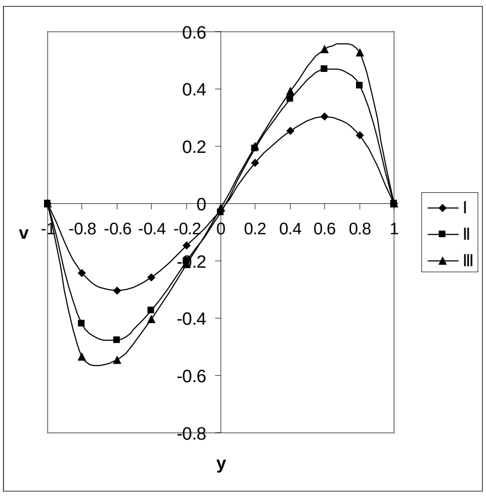

Fig. 8 : Variation of v with M

	I	II	III
M	2	4	6

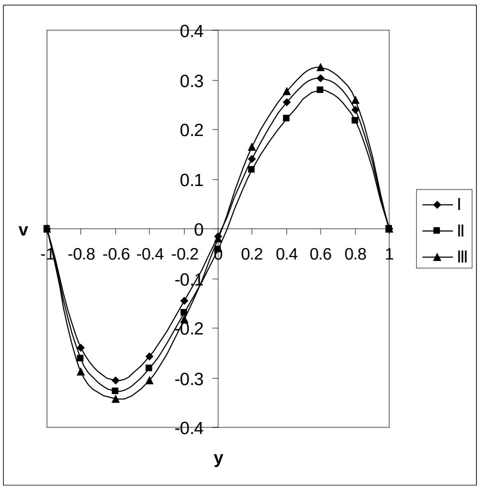

Fig. 9 : Variation of v with α_1

	I	II	III
α_1	0.1	0.3	0.5

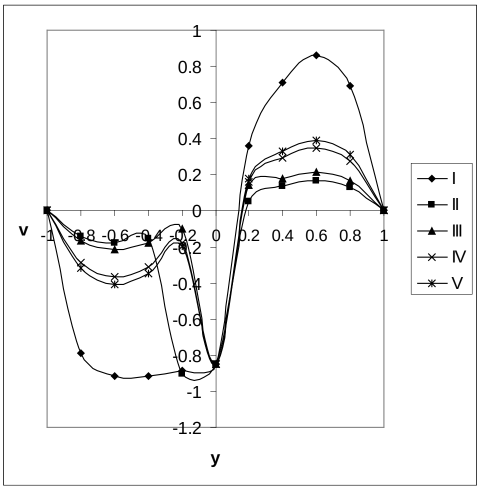

Fig. 10 : Variation of v with N_1

	I	II	III	IV	V
N_1	0.4	1	4	10	100

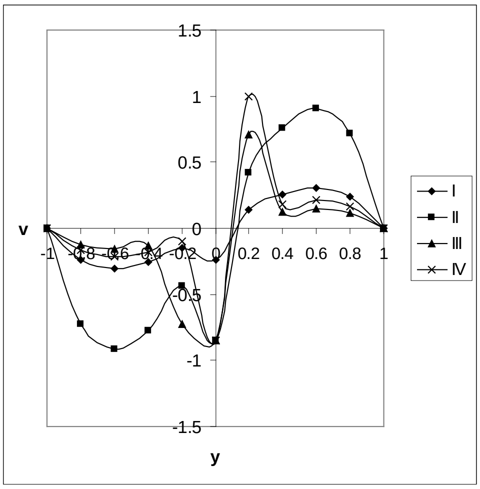

Fig. 11 : Variation of v with Ec

	I	II	III	IV
Ec	0.01	0.03	0.05	0.07

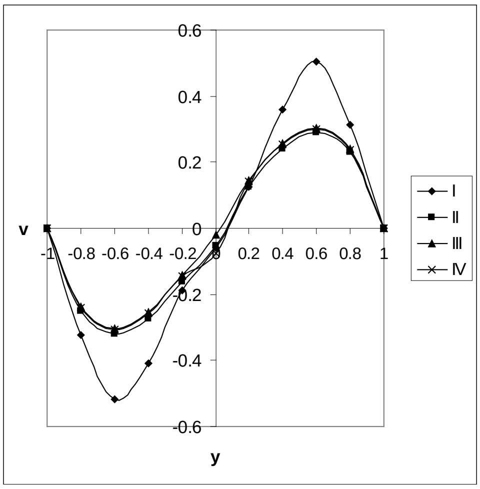

Fig. 12 : Variation of u with x

	I	II	III	IV
x	$\pi/4$	$\pi/2$	π	2π

The resultant velocity is shown in figs.13-17 for different values of G, M, α_1, N_1, Ec and x. In all the cases the profiles for resultant velocity are M shaped curves with a dip at y =0. From fig.13 we find that the resultant velocity enhances with increase in $|G|$. From fig.14 we find that for M=2 the resultant velocity is M shaped curve with a dip at y = 0 and as M increases the profiles of Rt are parabolic in nature with maximum attained at y = 0. An increase in the amplitude α_1 at of the non – uniform boundary temperature enhances Rt in the vicinity of the boundaries y = ± 1 and its change in the neighborhood of y = 0 is marginal(fig.15). From figs. 16 & 17 we note that the resultant velocity depreciates with $N_1 \leq 1$, enhances with N_1 = 4 and again depreciates with higher N_1=10. Also higher the dissipative heat larger the resultant velocity in the vicinity of y = -1. In the region (0, 0.4) the resultant velocity experience an enhancement while at the region (0.4, 1) the resultant velocity depreciates.

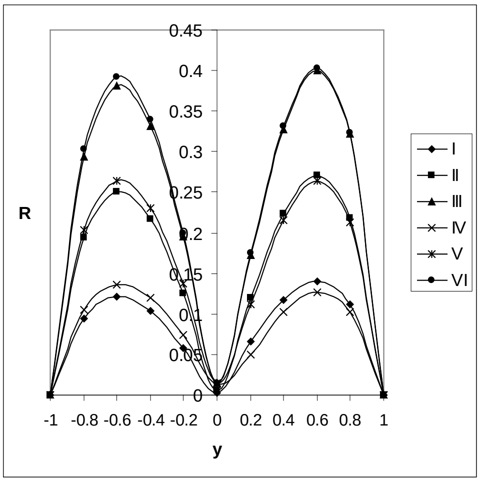

Fig. 13 : Variation of R with G

	I	II	III	IV	V	VI
G	$1x10^3$	$2x10^3$	$3x10^3$	-10^3	$-2x10^3$	$-3x10^3$

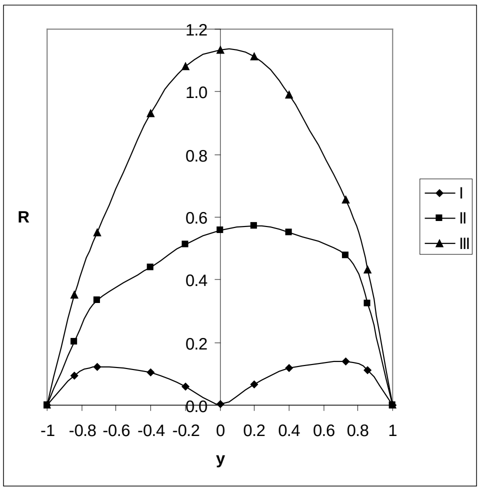

Fig. 14 : Variation of R with M

	I	II	III
M	2	4	6

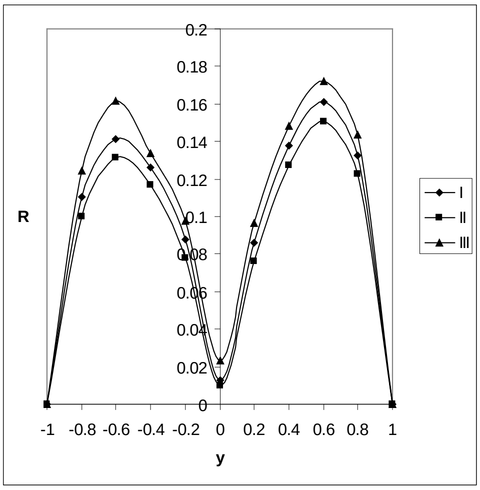

Fig. 15 : Variation of R with α_1

	I	II	III
α_1	0.1	0.3	0.5.

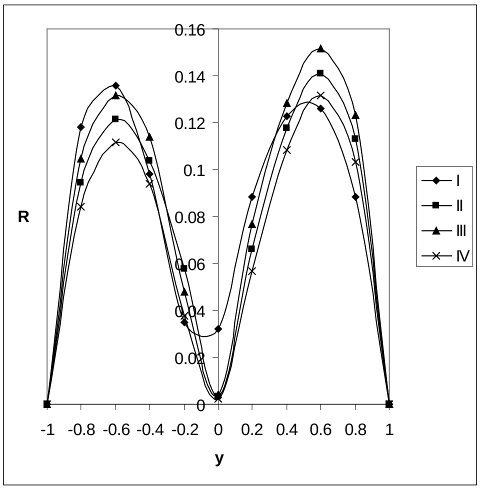

Fig. 16 : Variation of R with N_1

	I	II	III	IV	V
N_1	0.4	1	4	10	100

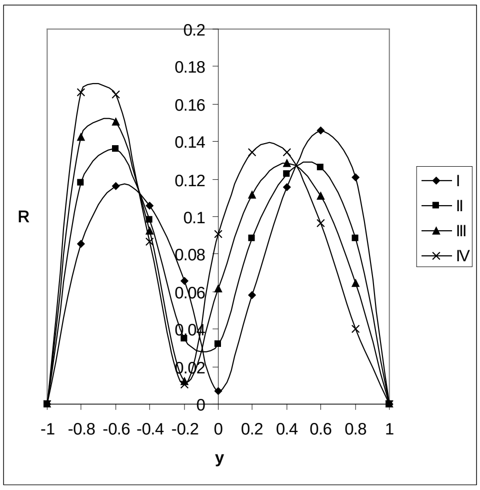

Fig. 17 : Variation of R with Ec

	I	II	III	IV
Ec	0.01	0.03	0.05	0.07

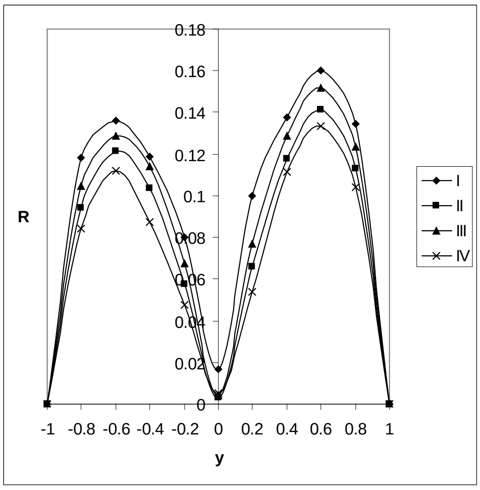

Fig. 18 : Variation of R with x

	I	II	III	IV
x	$\pi/4$	$\pi/2$	π	2π

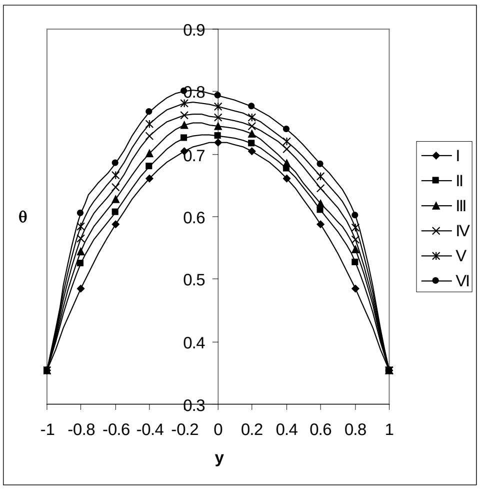

Fig. 19 : Variation of θ with G

	I	II	III	IV	V	VI
G	$1x10^3$	$2x10^3$	$3x10^3$	-10^3	$-2x10^3$	$-3x10^3$

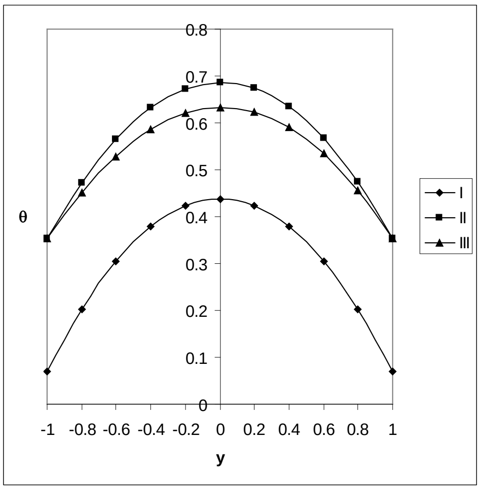

Fig. 20 : Variation of θ with M

	I	II	III
M	2	4	6

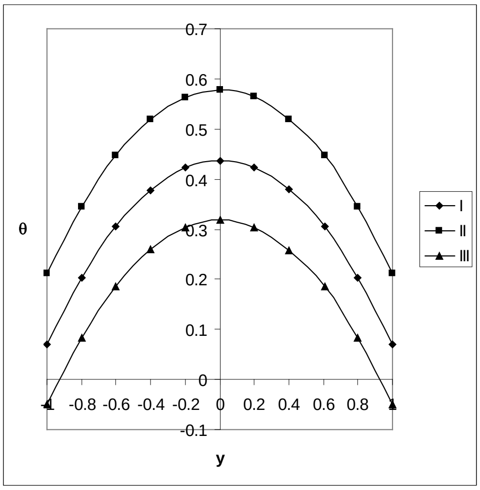

Fig. 21 : Variation of θ with α_1

	I	II	III
α_1	0.1	0.3	0.5.

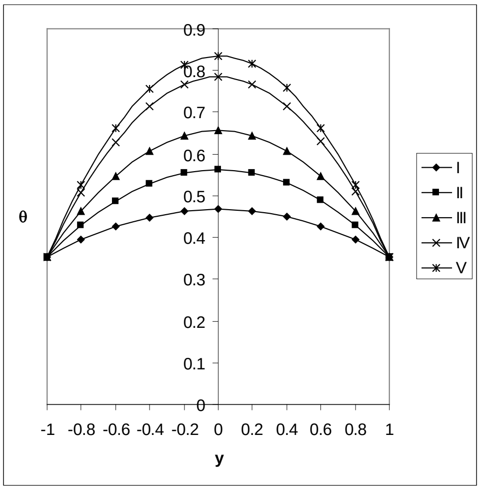

Fig. 22 : Variation of θ with N_1

	I	II	III	IV	V
N_1	0.4	1	4	10	100

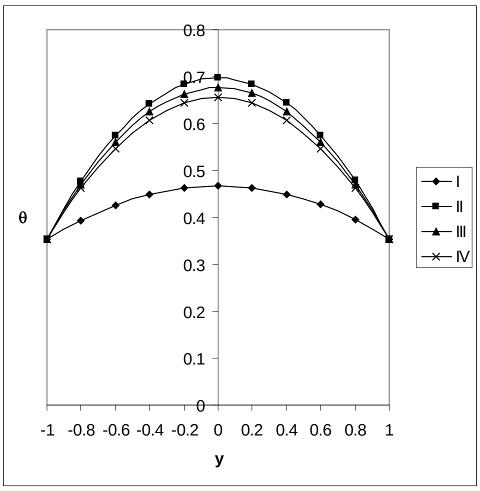

Fig. 23 : Variation of θ with Ec

	I	II	III	IV
Ec	0.01	0.03	0.05	0.07

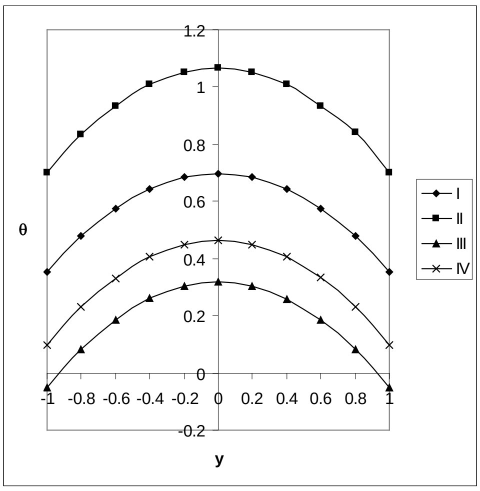

Fig. 24 : Variation of θ with x

	I	II	III	IV
x	π/4	π/2	π	2π

The shear stress at wall y = ± 1 is shown in tables 1- 4 for different values of G, M, α_1, N_1, Ec and x. It is found that stress enhances with increase in $|G|$ at y = ± 1. Higher the Lorentz force larger $|\tau|$ at both the walls. At y = +1 the magnitude of the stress dissipates with the amplitude $\alpha_1 \leq 0.3$ and enhances with higher $\alpha_1 \geq 0.5$. While at y = -1 it dissipates with α_1 for all G. An increase in the radiation parameter N_1 results in depreciation in $|\tau|$ at y = +1 and enhances at y=-1. (tables 1&3). From tables 2 & 4 we find that higher the dissipative heat lesser $|\tau|$ at y = +1 and larger $|\tau|$ at y = -1. The magnitude of τ enhances with increase in $x \le \frac{\pi}{2}$ and depreciates with $x \ge \pi$. and at y = -1, $|\tau|$ enhances with lower and higher values of x and depreciates with intermediate value of x.

TABLE – 1
SHEAR STRESS (τ) AT y = +1

G	I	II	III	IV	V	VI	VII	VIII	IX
$1x10^3$	0.1536	-0.3468	-1.3427	0.2107	0.1875	0.2105	0.199043	0.1391	0.1276
$2x10^3$	0.2843	-0.7272	-2.7953	0.4226	0.3621	0.3982	0.380200	0.2553	0.2324
$3x10^3$	-1.3120	-6.2890	-4.2930	-2.3930	-2.7510	-1.3120	-2.53150	0.3417	-1.3120
$-1x10^3$	-0.1974	0.2977	1.2189	-0.2361	-0.2250	-0.2544	-0.23972	-0.1829	-0.1715
$-2x10^3$	-0.4178	0.5620	2.3281	-0.4710	-0.4628	-0.5317	-0.49731	-0.3887	-0.3658
$-3x10^3$	-0.6680	0.7875	3.3228	-0.7136	-0.7218	-0.8389	-0.78037	-0.6244	-0.5901
M	**2**	**4**	**6**	**2**	**2**	**2**	**2**	**2**	**2**
α_1	**0.5**	**0.5**	**0.5**	**0.1**	**0.3**	**0.5**	**0.5**	**0.5**	**0.5**
N_1	**4**	**4**	**4**	**4**	**4**	**0.4**	**1**	**10**	**100**

TABLE –2
SHEAR STRESS (τ) AT y =+1

G	I	II	III	IV	V	VI
$1x10^3$	0.153628	0.143608	0.133626	0.216397	0.146961	0.116468
$2x10^3$	0.284384	0.276384	0.272384	0.441202	0.331850	0.205846
$3x10^3$	-1.312000	-1.302020	-1.300000	-1.312000	-1.311000	-1.311000
$-1x10^3$	-0.197480	-0.196460	-0.195452	-0.234960	- 0.125303	-0.160083
$-2x10^3$	-0.417832	-0.406836	-0.396836	-0.461513	- 0.212678	-0.347257
$-3x10^3$	-0.668049	-0.662049	-0.648249	-0.688648	- 0.267549	-0.567028
Ec	**0.01**	**0.03**	**0.05**	**0.01**	**0.01**	**0.01**
x	**π/4**	**π/4**	**π/4**	**π/2**	**π**	**2π**

TABLE –3
SHEAR STRESS (τ) AT y = -1

G	I	II	III	IV	V	VI	VII	VIII	IX
$1x10^3$	0.361987	0.7631	1.6514	0.4455	0.4183	0.305034	0.361670	0.376505	0.387966
$2x10^3$	0.746799	1.5598	3.4126	0.8898	0.8494	0.632893	0.741165	0.775834	0.798757
$3x10^3$	2.152000	7.7230	11.230	2.6310	2.1160	2.152000	2.556500	1.204982	2.152000
$-1x10^3$	-0.3181	-0.7141	-1.2273	-0.4201	-0.3808	0.261229	0.281600	-0.332700	-0.344161
$-2x10^3$	-0.6135	-1.3947	-2.9454	-0.8415	0.7488	0.499634	0.524900	-0.642575	-0.665498
$-3x10^3$	-0.8790	-2.0365	-4.2488	-1.2552	-1.0957	-0.708221	-0.75850	-0.922600	-0.957016
M	**2**	**4**	**6**	**2**	**2**	**2**	**2**	**2**	**2**
α_1	**0.5**	**0.5**	**0.5**	**0.1**	**0.3**	**0.5**	**0.5**	**0.5**	**0.5**
N_1	**4**	**4**	**4**	**4**	**4**	**0.4**	**1**	**10**	**100**

TABLE –4
SHEAR STRESS (τ) AT y = -1

G	I	II	III	IV	V	VI
$1x10^3$	0.361987	0.372986	0.381980	0.446468	0.252913	0.289651
$2x10^3$	0.746799	0.752799	0.759799	0.884528	0.467950	0.606340
$3x10^3$	2.152000	2.192000	2.282000	21.152000	21.151000	21.151000
$-1x10^3$	-0.318183	-0.328684	-0.338284	-0.427906	-0.274520	-0.246087
$-2x10^3$	-0.613541	-0.620541	-0.633542	-0.864220	-0.586915	-0.465136
$-3x10^3$	-0.879081	-0.889081	-0.899081	-1.299953	-0.931964	-0.651638
Ec	**0.01**	**0.03**	**0.05**	**0.01**	**0.01**	**0.01**
x	**π/4**	**π/4**	**π/4**	**π/2**	**π**	**2π**

The rate of heat transfer (Nusselt number) at y = ± 1 is shown in tables 5 – 8 for different parametric values. The variation of Nu with Grashof number G shows that the rate of heat transfer enhances at y +1 and depreciates at y = -1 with increase in $|G|$. With respect to Hartmann number M it is found that the Nu at y+1 enhances with $M \le 4$ and depreciate with $M \ge 6$, and at y = -1, $|Nu|$ depreciates with M for all G. The variation of Nu with amplitude α_1 shows that $|Nu|$ enhances with increase in α_1 in the heating case and depreciates with α_1 in the cooling case at both the walls.

TABLE – 5
NUSSELT NUMBER (Nu) AT y = +1

G	I	II	III	IV	V	VI	VII	VIII	IX
$1x10^3$	0.1536	-1.1939	-0.4694	-2.3854	-2.814766	-4.631198	-3.722982	-3.217467	-3.065760
$2x10^3$	0.2843	-0.5267	-0.2192	-1.1468	-1.204118	-1.569897	-1.387007	-1.207341	-1.163774
$3x10^3$	-1.3120	0.0007	0.0007	0.0006	0.000686	0.000678	0.000682	-0.723476	0.000678
$-1x10^3$	-0.1974	0.8717	0.4179	2.1474	1.888042	1.939139	1.913590	1.629270	1.588129
$-2x10^3$	-0.4178	0.4830	0.2233	1.1138	1.064792	1.210938	1.137865	0.979977	0.950585
$-3x10^3$	-0.6680	0.3393	0.1553	0.7558	0.753839	0.910343	0.832091	0.719578	0.695936
M	**2**	**4**	**6**	**2**	**2**	**2**	**2**	**2**	**2**
α_1	**0.5**	**0.5**	**0.5**	**0.1**	**0.3**	**0.5**	**0.5**	**0.5**	**0.5**
N_1	**4**	**4**	**4**	**4**	**4**	**0.4**	**1**	**10**	**100**

The variation of Nu with radiation parameter N_1 shows that the rate of heat transfer enhances at y = +1 and depreciates at y = -1 with increase in N_1 (tables 5 & 7). The effect of dissipation on Nu is shown in tables 6 & 8. It is find that higher the dissipative heat larger $|Nu|$ at y = +1 and smaller $|Nu|$ at y = -1. Moving along axial direction of the channel walls we notice that the rate of heat transfer at y=± 1 increases at $x = \frac{\pi}{2} \,\&\, 2\pi$ and depreciates at $x = \pi$ in heating case and in the cooling case a reversed effect is observed in the behavior of $|Nu|$.

TABLE – 6
NUSSELT NUMBER (Nu) AT y = +1

G	I	II	III	IV	V	V
$1x10^3$	-3.431846	-3.442846	-3.452840	-5.025386	-2.220998	-2.342420
$2x10^3$	-1.267228	-1.269228	-1.272228	-1.559028	-1.230760	-1.057165
$3x10^3$	0.000678	0.000686	0.000698	0.000678	0.000678	0.000678
$-1x10^3$	1.684350	1.694351	1.784352	1.457046	2.297302	2.185926
$-2x10^3$	1.019781	1.029781	1.012786	0.885587	1.043532	1.216455
$-3x10^3$	0.751820	0.762820	0.774822	0.636045	0.651652	0.811530
Ec	**0.01**	**0.03**	**0.05**	**0.01**	**0.01**	**0.01**
x	**π/4**	**π/4**	**π/4**	**π/2**	**π**	**2π**

TABLE – 7
NUSSELT NUMBER (Nu) AT y = -1

G	I	II	III	IV	V	VI	VII	VIII	IX
$1x10^3$	3.4208	1.1867	0.459727	2.3838	2.8093	4.616516	3.712943	3.207171	3.055935
$2x10^3$	1.2591	0.5204	0.210272	1.1454	1.1995	1.559963	1.379735	1.199629	1.156330
$3x10^3$	-0.0006	-0.0006	-0.0007	-0.0006	-0.0006	-0.000624	-0.000628	0.716559	-0.000624
$-1x10^3$	-1.6897	-0.8769	-0.4267	-2.1488	-1.8916	-1.945312	-1.918482	-1.634505	-1.593240
$-2x10^3$	-1.0263	-0.4888	-0.2328	-1.1152	-1.0684	-1.218663	-1.143778	-0.986287	-0.956715
$-3x10^3$	-0.7590	-0.3455	-0.1653	-0.7573	-0.7582	-0.919072	-0.838637	-0.726543	-0.702682
M	**2**	**4**	**6**	**2**	**2**	**2**	**2**	**2**	**2**
α_1	**0.5**	**0.5**	**0.5**	**0.1**	**0.3**	**0.5**	**0.5**	**0.5**	**0.5**
N_1	**4**	**4**	**4**	**4**	**4**	**0.4**	**1**	**10**	**100**

TABLE – 8
NUSSELT NUMBER (Nu) AT y = - 1

G		I	II	III	IV	V
$1x10^3$	-3.431846	3.420885	3.416865	5.025168	2.231012	2.332052
$2x10^3$	-1.267228	1.259149	1.250149	1.558893	1.241868	1.047830
$3x10^3$	0.000678	-0.000624	-0.000614	-0.000624	-0.000624	-0.000624
$-1x10^3$	1.684350	-1.689752	-1.691752	-1.457109	-2.286963	-2.195649
$-2x10^3$	1.019781	-1.026335	-1.030235	-0.885664	-1.034148	-1.227305
$-3x10^3$	0.751820	-0.769283	-0.772286	-0.636128	-0.642870	-0.893353
Ec	**0.01**	**0.03**	**0.05**	**0.01**	**0.01**	**0.01**
x	**π/4**	**π/4**	**π/4**	**π/2**	**π**	**2π**

7. CONCLUSIONS

In this paper we briefly discussed the effect of radiation and dissipation on convective heat transfer flow of a viscous electrically conducting fluid in a non – uniformly heated vertical channel. The important conclusions are following:

1) The reversal flow which appears in the left half of the channel enlarges with $N_1 \leq 4$ and depreciates at $N_1 = 10$ and again enlarges with $N_1 = 100$. Higher the radiative heat flux larger $|u|$ and further higher N_1 smaller $|u|$ and for still higher N_1 larger $|u|$ in the flow region. It is found that the reversal flow which appears in left half enlarges with increase in Ec. $|u|$ experiences an enhancement with Ec except in narrow region adjacent to y = +1.

2) The secondary velocity depreciates in magnitude with increase in $\alpha_1 \leq 0.3$ and enhances with higher $\alpha_1 \geq 0.5$. It is found that an increase in $N_1 \leq 1$ depreciates $|v|$ in the vicinity of y = ± 1 and enhances it in the central region. For higher $N_1 \geq 4$ it enhances $|v|$. Higher the dissipative heat (Ec$\leq$0.03) larger $|v|$ and for further higher the dissipative heat (Ec = 0.05) smaller $|v|$ and for still higher dissipative heat (Ec=0.07) larger $|v|$.

3) An increase in $\alpha_1 \leq 0.3$, enhances the temperature while it reduces with higher $\alpha_1 \geq 0.5$. The variation of θ with radiation parameter N_1 shows that higher radiative heat flux larger the actual temperature in the flow region. It is found that higher the dissipative heat (Ec$\leq$ 0.03) larger the actual temperature and for further higher dissipative heat (Ec$\geq$ 0.05) lesser the actual temperature everywhere in the flow region.

4) At y = +1 the magnitude of the stress dissipates with the amplitude $\alpha_1 \leq 0.3$ and enhances with higher $\alpha_1 \geq 0.5$. While at y = -1 it dissipates with α_1 for all G. An increase in the radiation parameter N_1 results in depreciation in $|\tau|$ at y = +1 and enhances at y=-1. We find that higher the dissipative heat lesser $|\tau|$ at y = +1 and larger $|\tau|$ at y = -1. $|Nu|$ enhances with increase in α_1 in the heating case and depreciates with α_1 in the cooling case at both the walls.

8. REFERENCES

1. Ajay Kumar Singh: MHD free convection and mass transfer flow with Hall effect, viscous dissipation, Joule heating and thermal diffusion, Ind. J. pure & Application Physics V.41.PP.24-35(2003).
2. Anug W: Fully developed laminar free convection between vertical plates heated asymmetrically. *Int. J. Heat and Mass Transfer*, 15, pp.1577-1580, (1972).
3. Barletta A and Rossidi Schi, E., Int. Comms. Heat Mass Transfer, V.28, and PP.449- 459 (2001).
4. Barletta, A, Int. J Heat Mass Transfer, V.40, PP.15-26 (1997).
5. Barletta, A., Int. Heat Mass Transfer, V.41, PP.3501-3513 (1998).
6. Bejamin Gebhort, Yogesh Jaluria, Roop-L Mahjan and Bhagat Samnokia : Buoyancy induced flows and transfport, Springer –Verlog, berling (1988).
7. Bejan, A and Khair, K.R: Heat and Mass transfer by natural convection in a porous medium, Int. J. Heat and Mass transfer, V.28, PP.908-918 (1985).
8. Bejan ,A, Convection heat transfer, New York; John wiley (1984).
9. Bird, R.S : J. Soc. Plastic enginnering,V.11,P.7 (1955).
10. Bulent Yesilata , Int. Comns. J. Heat Mass transfer, V.29, PP.589-600 (2002).

11. Buretta, R, Thermal convection in a fluid filled porous layer with uniform internal heat sources, Ph.D. dissertation, University of Minnesota (1972).
12. Campo A, Manca O, and Marrone B: Numerical investigation of the natural convection flows for low-Prandtl fluid in vertical parallel-plates channels. *ASME Journal of Applied Mechanics,* 73, pp. 6-107 (2006).
13. Chambre, P.L: Application Science Research, A.E, PP.393 (1957).
14. Chamka AJ. MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. Int Comm Heat and Mass transfer: Vol. 30 pp.413-22 (2003).
15. Chandrasekhar, S: Hydrodynamic and Hydromagnetic stability, clarandon press, oxford (1961).
16. Chen, T.s, Yuh, C.F and Moutsoglo, A: Int. Heat and Mass transfer, V.23, PP.527-537 (1980).
17. Cheng, P: Advances in heat transfer, V.14, PP.1-205 (1979).
18. Costa, V.A.F: Thermodynamics of natural convection in enclosures with viscous dissipation, Int.J.Heat and Mass transfer,V.48,pp.2333-2341(2005)
19. Deka R. Das UN, Soundalgekar VM: Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Forschung in Ingenieurwesen, Vol60:pp284-287 (1994).
20. El. Hakein, M.A., Comns. Heat Mass transfer, v.27, PP.581-590 (2000).
21. Fand, R.M and Bruckar, J. Int. J. Heat Mass transfer, PP. 723-732 (1996).
22. Gabor, D, Saroa, E.S, Baker, J.L. and Cassulo, J.L. and Cassulo, J.L. Proc. Fast Reactor Sahly –Mfg California, April 2-4, Conf, 740401 , p.823 (1974).
23. Gebhart , B.J fluid Mech, V.14,pp.225 -232 (1962).
24. Gebhart, B and Mollel dorf, J. Fluid Mech, V.38, pp.97-107 (1969).
25. Gebhart B, Pera L: The nature of vertical natural convection flow resulting from the combined buoyancy effects of thermal and mass diffusion. J. Heat Mass Transfer Vol. 14, pp.2025-2050(1971).
26. Gill, W, M and Casal, A.D: A theoretical investigation of natural convection effects in forced horizontal flows, Amer. Inst. Chem. Engg. Jour.., V.8, pp.513-520(1962).
27. Gill, W.M and Casal, A.D: S.Amer, Inst. Chemical. Engineering, V.13, P.137 (1962).
28. Greosh, R.J and less, R.D: Trans Amer Soc. Mech. Engng., V.8, P.667 (1958).
29. Hardec, H.C: SAND, 74-0329, Sandia Laboratories (1974).
30. Hardee, H.C and Nilson, R.M: Nuclear science and Engn, V, 633, p. 110 (1979).
31. Haritha A: 'Transient free convective Heat transfer of a viscous fluid in channels/circular ducts with heat sources and radiation effects., Ph.D. Thesis, S.P. Mahila University, Tirupathi (2010).
32. Helman, J.K., Babeller, B and Babrov, H : Trans. Amer, Sec, Mech, Engn., V.78,P .32 (1950).
33. Ibrahim FS., Elaiw AM., Bakr AA: Effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction. Communications in Nonlinear Science and Numerical Simulation Vol. 13 pp.1056-1066 (2008).
34. Israel, C, Int. J. Heat Mass transfer, V.40, PP.2305-2311 (2003).
35. Jha BK, Singh AK, and Takhar HS: Transient free convection flow in a vertical channel due to symmetric heating. *International Journal of Applied Mechanics and Engineering,* 8(3), pp.497-502 (2003).
36. Kesavaiah DCh, Satyanarayana PV and Venkataramana S: Effects of the Chemical Reaction and Radiation absorption on an unsteady MHD convective Heat and Mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Int. J. of Appl. Math and Mech.7 (1): pp.52-69 (2011).
37. Krishna, D.V., (1986), Kolposhcinkov, V.L, Martynenko, O.G, Shabrunya, S.I., Shrip, A.I, and Luikov, A.V. Mixed thermal convection of a vertical tube at non uniform temperature. Accepted for pubilication in Int. J. Heat Mass transfer.(2002)
38. Lees, L: Jet Propulsion, V.20, P.259 (1950).
39. Lighthil, M.J : Aero Res. Council, July (1958).
40. Low, G.M: J. Aero Sci, V.22, P.239 (1955).
41. Manca O, Marrone B, Nardini S, and Naso V: Natural convection in open channels. In: Sunden B, Comini G. *Computational Analysis of Convection Heat Transfer.* WIT press, Southempton, pp.235-278 (2000)
42. Muthucumaraswamy R, Ganesan P. Effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate, J. Appl. Mech Tech Phys, 42, pp.665-671 (2001).
43. Nakayama, A and Pop, Int. Comms. Heat Mass Transfer, V.16, P. 173 (1989).
44. Narahari M: Free convection flow between two long vertical parallel plates with variable temperature at one boundary. *Proceedings of International Conference on Mechanical & Manufacturing Engineering (ICME 2008),* Johor Bahru, Malaysia (2008).
45. Narahari M, Sreenadh S, and Soundalgekar VM: Transient free convection flow between long vertical parallel plates with constant heat flux at one boundary. *Thermophysics and Aeromechanics,* 9(2), pp.287-293 (2002).
46. Narahari M: Oscillatory plate temperature effects of free convection flow of dissipative fluid between long vertical parallel plates. *Int. J. of Appl. Math. And Mech.* 5(3). pp 30-46 (2009).
47. Ostrach S : Combined natural and forced convection laminar flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperature. Technical Report 3141, NASA, USA (1954).
48. Ostrach S: Laminar natural-convection flow and heat transfer of fluids with and without heat sources in channels with constant wall temperatures. Technical Report 2863, NASA, USA (1952).

49. Ostrach, S.NACA, TN, P.3141 (1954).
50. Palm,E : Ann –REV: Fluid Mech., V.7,P.39 (1975).
51. Pantokratoras A: Fully developed laminar free convection with variable thermophysical properties between two open-ended vertical parallel plates heated asymmetrically with large temperature differences. *ASME Journal of Heat Transfer*, 128, pp.405-408 (2006).
52. Prasad, P.M.V: Hydromagnetic convective heat and mass transfer through a Porous medium in channels/pipes, Ph.D thesis, S. K. University, Anantapur, India(2006).
53. Raptis A, Perdikis C: Free convection flow of water near 4 C past a moving plate. Forschung Im Ingenieurwesen; Vol. 67: pp.206-208 (2002).
54. Raptis, A. Kafousis, N and Massalas, C. Reg. J, Energy Heat Mass Transfer, V.3, pp.279-283(1995).
55. Simpikins, P.G (1979), Greenberg, Kosiniki, S, and Macchesney, J.B. Thermophoresis. The mass transfer mechanism in modified chemical vapour deposition. J.Appl. Phys. 50 (9) P.5676.
56. Singh AK and Paul T: Transient natural convection between two vertical walls heated/cooled asymmetrically. *International Journal of Applied Mechanics and Engineering.* 11(1), pp. 143-154 (2006).
57. Soundalgekar, V.M and Pop, I. Int, J. Heat mass transfer, V.17, PP.85-90 (1974).
58. Soundalgekar, V.M and Jaiswal, B.S.D, Uplkar, A.G and Takhar, H.S, Appl., Mech and Engg., V.4, PP. 203-218 (1999).
59. Sreevani,M . Mixed convection heat and mass transfer through a porous medium in channels with dissipative effects, Ph.D thesis, S. K. University, Anantapur India (1992).

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