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EFFECT OF POROUS LINING ON THE PERISTALTIC TRANSPORT OF A CASSON FLUID IN AN INCLINED CHANNEL

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ABSTRACT

Peristaltic transport of a Casson fluid in an inclined channel is investigated. The effect of porous lining has been studied under long wavelength and low Reynolds number assumptions. The equations of motion are solved analytically and the expressions for velocity, stream function and flow rate are obtained. The pumping and trapping phenomena are analyzed. The effect of porous lining, yield stress, amplitude ratio, permeability parameter and angle of inclination on the pumping characteristics are discussed graphically. Trapping limits are obtained for the stream function of the fluid flow. The results are interesting and warrant further study on the peristaltic transport of yield stress fluids.

Keywords: Peristalsis, Porous Lining, Casson Fluid, yield stress and permeability.

1. INTRODUCTION

Peristaltic pumping is a mechanism of the fluid transport in a flexible tube by a progressive wave of contractions or expansion from a region of lower pressure to higher pressure. Peristalsis is one of the major mechanisms for many biological systems. In addition, peristaltic pumping occurs in many practical applications involving biomechanical systems. The major industrial applications of this principle is in designing the roller pumps which are useful in pumping fluids without being contaminated due to the contact with the pumping machinery. The problem of the mechanism of peristaltic transport has attracted the attention of many investigators since the first investigation of Latham [1]. The application of peristaltic motion as a means of transporting fluid has also aroused interest in engineering fields (Hanin [2], Ayukawa *et al.* [3],). The important studies of the recent years includes the investigations of El Hakeem and El Misery [4], Haroun [5], Muthu *et al.* [6], Medhavi [7], Medhavi and Singh [8], Hayat *et al.* [9], Kothandapani and Srinivas [10], Hayat *et al.* [11], and a few others.

The dynamics of flow through porous medium has been a topic of considerable interest for the last one and half centuries, since Darcy formulated his famous law describing the motion of a viscous fluid through a porous medium. The study of flow through and past porous media has important applications in various branches of Science, Engineering and Technology. It is well known that porous medium had practical applications especially in geophysical fluid dynamics. It is applicable in the field of energy extraction from geothermal region and the heat removal from nuclear fuel debris. The study of flow of immiscible fluids through and past porous media is useful in improving oil recovery from the underground oil reservoirs. In some pathological situations, the distribution of fatty cholesterol and artery clogging blood clots in the lumen of coronary artery can be considered as equivalent to porous medium. El Shehawey *et al.* [12], El Shehawey and Husseny [13] studied the peristaltic mechanism of a Newtonian fluid through a porous medium. Afifi and Gad [14 and 15] studied the interaction of peristaltic flow with pulsatile fluid (respectively Magneto field) through a porous medium when the ratio between these two frequencies is equal to the wave number of the imposed pressure gradient wave.

Unsteady flow of a viscous incompressible fluid through a circular naturally permeable tube surrounded by a porous material was studied by Verma and Chauhan [16]. Peristaltic transport in a two-dimensional channel filled with a porous medium in the peripheral region and a Newtonian fluid in the core region under the assumption of long wave length and low Reynolds number was studied by Manoranjan Mishra and Ramachandra Rao [17]. Sreenadh *et al.* [18], studied the peristaltic flow of Herschel-Bulkley fluid in an inclined flexible channel lined with porous material under long wave length and low Reynolds number. Hemadri Reddy *et al.* [19], studied the effect of thickness of the porous material on the peristaltic pumping when the tube wall is provided with non-erodible porous lining.

Recently there has been an increasing interest in the flow of time-independent non-Newtonian fluids through tubes/channels possessing a definite yield value because of their applications in polymer process industries and biofluid dynamics. The most popular among these fluids is the Casson fluid [20]. Casson fluids are found to be applicable in developing models for blood oxygenators and haemodialysers. Oka [21], studied blood flow in capillaries with permeable walls using the Casson fluid model. Peristaltic transport of blood by modelling blood as a Casson fluid is studied by Srivastava and Srivastava [22]. They have represented the blood as a two layered fluid model, consisting of a central layer of suspension of all erythrocytes assumed to be a Casson fluid, and a pheripheral layer of plasma as a Newtonian fluid. Mernone and Mazumdar [23 and 24] studied the peristaltic transport of a Casson fluid in two dimensional axisymmetric channel using the generalized form of the constitutive equations for Casson fluid.

In view of these, the peristaltic transport of a bio-fluid in an inclined channel is analyzed by modeling the fluid as a Casson fluid lined with porous material is studied under long wavelength and low Reynolds number assumptions. This model can also be applied to blood flow in the sense that erythrocytes region and the plasma regions may be described as plug flow and non- plug flow regions. It is observed that for a Casson fluid the pressure difference and the mechanical efficiency of pumping depend on the thickness of the porous lining. Both increases with increase in thickness of the porous lining.

2. MATHEMATICAL FORMULATION AND SOLUTION

We consider the peristaltic transport of a Casson fluid in a two dimensional channel having width 2a and inclined at an angle β to the horizontal. We assume an infinite wave train travelling with velocity *c* along the wall. We choose a rectangular coordinate system for the channel *X* along the centerline in the direction of wave propagation and *Y* transverse to it and the channel is assumed axisymmetric. The channel is bounded by flexible walls which are lined with non-erodible porous material of thickness ε . For simplicity we restrict our discussion to the half width of the channel as shown in Fig. 1.



Fig. 1: Schematic diagram of the inclined channel

The wall deformation is given by

$$H(X,t) = a + b \sin \frac{2\pi}{\lambda} (X - ct)$$
⁽¹⁾

where *b* is amplitude of the wave and λ is the wavelength and *c* is the wave speed.

Under the assumption of infinite wavelength and neglecting the inertial terms the equations of motion is given by

$$\rho \frac{\partial U}{\partial t} = -\frac{\partial P}{\partial X} - \frac{\partial \tau}{\partial Y} + \rho g \sin \beta$$
(2a)

$$\frac{\partial P}{\partial Y} = 0 \tag{2b}$$

where ρ is the density, U is the axial velocity, t is the time, P is the pressure and τ is the shear stress and g is the acceleration due to gravity. The Casson's constitutive equation corresponding to the flow is given by

$$\tau^{\frac{1}{2}} = \tau_y^{\frac{1}{2}} + \left(-\mu \frac{\partial U}{\partial Y}\right)^{\frac{1}{2}} \quad \text{if} \quad \tau \ge \tau_y \tag{3a}$$

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$$\frac{\partial U}{\partial Y} = 0 \text{ if } \tau \le \tau_y$$
(3b)

where τ_y is the yield stress and μ is the viscosity coefficient of the fluid.

The corresponding boundary conditions are given by

$$U(Y = H - \varepsilon) = -h' \frac{\partial U}{\partial Y}$$
(Saffman slip condition) (4a)

$$-\tau (Y = -Y_p) = \tau_y = \tau (Y = Y_p)$$
^(4b)

$$U(Y = Y_p) = U_p \tag{4c}$$

where U_p is the plug flow velocity.

Under the assumptions that the tube length is an integral multiple of the wavelength and the pressure difference between the ends is constant, the flow becomes steady in the wave frame.

The transformations between fixed frame to moving frame are given by

$$x = X - ct; \ y = Y; \ u(x, y) = U(X - ct, Y) - c; \ v(x, y) = V(X - ct, Y); \ p(x) = P(X, t)$$
(5)

where U and V are velocity components in the laboratory frame, u and v are velocity components in the wave frame and p and P are pressures in wave and fixed frame of references respectively. The pressure remains constant across any axial station of the channel under the assumption that the wavelength is large and the curvature effects are negligible.

Using the non-dimensional quantities

$$\bar{x} = \frac{x}{\lambda}; \quad \bar{y} = \frac{y}{a}; \quad \bar{u} = \frac{u}{c}; \quad \bar{v} = \frac{v}{c\delta}; \quad \delta = \frac{b}{\lambda}; \quad \phi = \frac{b}{a}; \quad \bar{p} = \frac{pa^2}{\lambda\mu c};$$

$$\bar{\tau} = \frac{\tau}{\frac{\mu c}{a}}; \quad \bar{\tau}_y = \frac{\tau_y}{\frac{\mu c}{a}}; \quad \bar{h} = \frac{h}{a}; \quad \bar{y}_p = \frac{y_p}{a}; \quad F = \frac{\mu c}{\rho g a^2}; \quad \bar{u}_p = \frac{u_p}{a}$$
(6)

The non-dimensional wall equations is given by (dropping the bars)

$$y = h(x) = 1 + \phi \sin 2\pi x \tag{7}$$

The equations of motion in dimensionless form is

$$\frac{\partial \tau}{\partial y} = -\frac{dp}{dx} + \frac{\sin \beta}{F}$$
(8)
where $\tau^{\frac{1}{2}} = \tau^{\frac{1}{2}}_{y} + \left(-\frac{\partial u}{\partial y}\right)^{\frac{1}{2}}$

The corresponding boundary conditions in non-dimensional form are

$$u = -1 - \alpha \frac{\partial u}{\partial y}$$
 at $y = h - \varepsilon$ (9a)

$$-\tau(y = -y_p) = \tau_y = \tau(y = y_p)$$
^(9b)

$$u = u_p$$
 at $y = y_p$ (9c)

where ε is the thickness of the lining and α is permeability parameter.

The instantaneous volume flow rate in fixed frame is given by

$$Q = \int_0^{h-\varepsilon} u \, dy \tag{10}$$

If q is the rate of flow independent at x and t in wave frame, then

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$$q = \int_0^{y_p} u_p \, dy + \int_{y_p}^{h-\varepsilon} u \, dy \tag{11}$$

The average flow rate over one period ($T = \frac{\lambda}{c}$) of the peristaltic wave is defined as $\bar{Q} = \frac{1}{T} \int_0^T Q \, dt = q + 1$

Solving (8) with boundary condition (10), we obtain the expressions for the velocity distribution as

$$u = \frac{1}{2} [P+f] \left\{ \begin{pmatrix} ((h-\varepsilon)^2 + 2\alpha(h-\varepsilon) - y^2) + 2y_p(\alpha + (h-\varepsilon) - y) \\ -4\sqrt{y_p} \left(\alpha(h-\varepsilon)^{\frac{1}{2}} + \frac{2}{3}(h-\varepsilon)^{\frac{3}{2}} - \frac{2}{3}y^{\frac{3}{2}} \right) \\ \begin{pmatrix} ((\mu-\varepsilon)^2 + \frac{2}{3}(h-\varepsilon)^{\frac{1}{2}} + \frac{2}{3}(h-\varepsilon)^{\frac{3}{2}} - \frac{2}{3}y^{\frac{3}{2}} \end{pmatrix} \right\} - 1 \text{ for } y_p \le y \le h-\varepsilon$$
(13a)

$$u_{p} = \frac{1}{2} [P+f] \begin{cases} \left((h-\varepsilon)^{2} + 2\alpha(h-\varepsilon) - \frac{1}{3}y_{p}^{2}\right) + 2y_{p}(\alpha + (h-\varepsilon)) \\ -4\sqrt{y_{p}} \left(\alpha(h-\varepsilon)^{\frac{1}{2}} + \frac{2}{3}(h-\varepsilon)^{\frac{3}{2}}\right) \end{cases} - 1 \text{ for } 0 \le y \le y_{p}$$
(13b)
where $y_{p} = \frac{\tau_{y}}{[P+f]}; P = -\frac{dp}{dx} \text{ and } f = \frac{\sin\beta}{F}$

Solving (13) using the conditions $\psi_p = 0$ at y = 0 and $\psi = \psi_p$ at $y = y_p$ we obtain the stream function as

$$\psi = \frac{1}{2} [P+f] \left\{ \begin{pmatrix} (h-\varepsilon)^2 y + 2\alpha(h-\varepsilon)y - \frac{y^3}{3} \end{pmatrix} + 2y_p \left(\alpha y + (h-\varepsilon)y - \frac{y^2}{2} \right) \\ -4\sqrt{y_p} \left(\alpha(h-\varepsilon)^{\frac{1}{2}}y + \frac{2}{3}(h-\varepsilon)^{\frac{3}{2}}y - \frac{4}{15}y^{\frac{5}{2}} \right) - \frac{1}{15}y_p^3 \end{pmatrix} - y \text{ for } y_p \le y \le h-\varepsilon$$
(14a)

$$\psi_{p} = \frac{1}{2} [P+f] y \begin{cases} \left((h-\varepsilon)^{2} + 2\alpha(h-\varepsilon) - \frac{1}{3}y_{p}^{2} \right) + 2y_{p} \left(\alpha + (h-\varepsilon) \right) \\ -4\sqrt{y_{p}} \left(\alpha(h-\varepsilon)^{\frac{1}{2}} + \frac{2}{3}(h-\varepsilon)^{\frac{3}{2}} \right) \end{cases} - y \text{ for } 0 \le y \le y_{p}$$
(14b)

The flux in the wave frame is obtained as

$$q = \frac{1}{2}[P+f](h-\varepsilon)^3 s(x) - (h-\varepsilon)$$

The pressure gradient obtained is

$$\frac{dp}{dx} = -\frac{2[q+(h-\varepsilon)]}{(h-\varepsilon)^3 s(x)} + f$$
(15)
where $s(x) = \frac{2}{3} + \frac{2\alpha y_p}{(h-\varepsilon)^2} + \frac{1}{(h-\varepsilon)} \left[2\alpha + y_p \right] - \frac{4\sqrt{y_p}}{(h-\varepsilon)^2} \left[\alpha + \frac{2}{5}(h-\varepsilon) \right] - \frac{1}{15} \left(\frac{y_p}{h-\varepsilon} \right)^3$

5. PUMPING CHARACTERISTIC

The pressure rise per wavelength is given $\Delta P = \int_0^1 \frac{dp}{dx} dx = f - 2[qI_1 + I_2]$ (16) where $I_1 = \int_0^1 \frac{1}{(h-\varepsilon)^3 s(x)} dx$ and $I_2 = \int_0^1 \frac{1}{(h-\varepsilon)^2 s(x)} dx$

The average flow rate is given by

$$\bar{Q} = \frac{f - \Delta P - 2[I_2 - I_1]}{2I_1} \tag{17}$$

The dimensionless time mean flow \bar{Q}_0 for zero pressure is

$$\bar{Q}_0 = \frac{f - 2[l_2 - l_1]}{2l_1} \tag{18}$$

Also the dimensionless pressure rise for zero mean flow is obtained as

$$(\Delta P)_0 = f - 2[I_2 - I_1] \tag{19}$$

The frictional force F_{λ} at the wall is obtained as

$$F_{\lambda} = \int_{0}^{1} -(h-\varepsilon) \frac{dp}{dx} dx$$

= 2[qI₂ + I₃] - f(1-\varepsilon) (20)

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(12)

where
$$I_3 = \int_0^1 \frac{1}{(h-\varepsilon)s(x)} dx$$

6. RESULTS AND DISCUSSIONS

The objective of this analysis is to study the flow characteristics of a Casson fluid in an inclined channel with the effect of porous lining ε and with permeability α .

To study the behavior of axial velocity u, numerical calculations for several values of permeability parameter α , thickness of the porous lining ε , yield stress τ_y angle of inclination β and amplitude ratio ϕ are carried out. Fig (2) shows that the increase in permeability α results in increase of velocity distribution. Fig (3) depicts that velocity decreases as yield stress τ_y increases. The effect of thickness of porous lining ε on the velocity distribution can be seen through Fig (4). It reveals that the axial velocity decreases with increase in ε . From Fig (5) it is observed that velocity decreases with the decrease in the angle of inclination β . It is observed that velocity increases with increase in amplitude ratio ϕ from Fig (6)

The variation of dimensionless pressure drop is analyzed graphically for the variation of permeability parameter α , yield stress τ_y , angle of inclination β , thickness of the porous lining ε and amplitude ratio ϕ and are plotted in Figs (7-11).

From Fig (7) we can observe the variation of ΔP with \bar{Q} for the variation of permeability parameter α . It is interesting to note that all the curves are intersecting in the free pumping region at $(\Delta P > 0)$ at $\bar{Q} = 0.6$. For $0 \le \bar{Q} \le 0.6$ we observe that ΔP decreases with increase in α , and in the rest of the region ΔP increases with increase in α . The variation of ΔP with \bar{Q} for different values of τ_y is shown in Fig (8). ΔP increases with increase in shear stress for $0 \le \bar{Q} \le 0.5$, the pumping is slow between $\bar{Q} = 0.5$ and $\bar{Q} = 0.65$ and ΔP decreases with increase in τ_y for the rest of the region. Fig (9) is plotted to see the effect of porous lining ε on pressure rise ΔP with \bar{Q} . It seems to be noted that ΔP increases with increase in ε , if $0 \le \bar{Q} \le 0.75$ and from $\bar{Q} = 0.75$ to $\bar{Q} = 0.85$, ΔP is having the same behavior if $\varepsilon = 0.1 \& 0.2$, but if $\varepsilon = 0.3$, the flow is more, if $0.85 \le \bar{Q} \le 1$, ΔP increases as ε decreases. The variation of ΔP with \bar{Q} for different values of β is shown in Fig (10). It is observed that pumping region increases as the angle of inclination β increases with increase in ϕ . ΔP increases with increase in ϕ through Fig (11). Pumping region increases with increase in ϕ . ΔP increases with increase in ϕ , if $0.83 \le \bar{Q} \le 1$, ΔP increases as ϕ decreases. We observe that ΔP decreases as \bar{Q} increases as ϕ decreases. We observe that ΔP decreases as \bar{Q} increases in ϕ , if $0.83 \le \bar{Q} \le 1$, ΔP increases in ϕ , if $0.83 \le \bar{Q} \le 1$, ΔP increases as ϕ decreases. We observe that ΔP decreases as \bar{Q} increases.

The non-dimensional frictional force F_{λ} verses \bar{Q} is shown in Figs (11-14) for different values of permeability parameter α , plug radius y_p , angle of inclination β and ε the thickness of the porous lining. In all these graphs a reversal behavior is observed with the case of ΔP . In Fig (12) it is observed that the curves are intersecting at $\bar{Q} = 0.45$. For $0 \leq \bar{Q} \leq 0.45$ we observe that frictional force increases with increase in α and in the rest of the region F_{λ} decreases with increase in α . The variation of F_{λ} with \bar{Q} for different values of y_p is shown in Fig (13). It is observed that the curves are intersecting at $\bar{Q} = 0.55$. F_{λ} decreases as y_p increase for $0 \leq \bar{Q} \leq 0.5$ and F_{λ} increases as y_p increase for $0.5 \leq \bar{Q} \leq 1$. Fig (14) shows the variation of F_{λ} for variation in the values of β . Frictional force F_{λ} decreases with decrease in angle of inclination β . Fig (15) shows that frictional force F_{λ} decreases as the thickness of the porous lining ε decreases. It is also observed that for $0 \leq \bar{Q} \leq 0.65$ F_{λ} increases as ε increases and for $0.65 \leq \bar{Q} \leq 1$ F_{λ} decreases as ε increases.

7. TRAPPING PHENOMENA

An interesting phenomenon of peristalsis is trapping. The formation of an internally circulating bolus of the fluid by closed streamlines is called trapping and this trapped bolus is pushed ahead along with the peristaltic waves.

By analysis, one gets the trapping when \bar{Q} lies between \bar{Q}_{min} and \bar{Q}_{max} , i.e. one gets trapping when

$$\bar{Q}_{min} \leq \bar{Q} \leq \bar{Q}_{max},$$

where

$$\bar{Q}_{min} = \varepsilon - 1 - \phi$$
 and $\bar{Q}_{max} = \varepsilon - 1 + \phi$

The streamline patterns in the wave frame for different values of α , ε , and β by taking P = 1 and F = 0.1 are shown in Figs (16-18). To see the effect of permeability parameter α , when $y_p = 0.1$, $\varepsilon = 0.3$, $\phi = 0.2$, $\beta = \frac{\pi}{6}$ on the trapping, we prepared Fig (16). It shows the formation and variation of trapped bolus for different values of α . No trapped bolus is seen for $\alpha = 0$ and $\alpha = 0.05$ and it shows that the number of trapped bolus increases with increase in

 α . The effect of the porous lining ε on trapping is analyzed through Fig (17) when $y_p = 0.6$, $\alpha = 0.2$, $\phi = 0.8$, $\beta = \frac{\pi}{6}$. It is noted that for increasing ε , the size of the trapped bolus decreases and finally disappears for $\varepsilon \ge 0.15$. Fig (18)

shows the outcome of β , the angle of inclination when $y_p = 0.4$, $\varepsilon = 0.3$, $\alpha = 0.2 \phi = 0.7$, on trapping. We observe that bolus appears when $\beta = \frac{\pi}{6}$ and it is observed that for both $\beta = \frac{\pi}{12} \& \frac{\pi}{8}$ that there is no formation of trapped bolus.

8. CONCLUSIONS

In this article, the effect of porous lining on the peristaltic transport of a Casson fluid in an inclined channel has been studied. Analytical solutions have been developed for velocity distribution, stream function, pressure rise and frictional force. We find different interesting observations as follows:

- trapped bolus increases with increase in permeability parameter and decreases with increase in thickness of the porous lining,
- 2. increase in permeability α results in increase of velocity distribution,
- 3. velocity decreases as yield stress τ_y increases,
- 4. velocity decreases with the decrease in the angle of inclination β ,
- 5. velocity increases with increase in amplitude ratio ϕ ,
- 6. Δp decreases as \overline{Q} increases,
- 7. pumping region increases as the angle of inclination β increases,
- 8. Pumping region increases with increase in ϕ



Fig. 2: Velocity Profiles for different α



Fig. 3: Velocity Profiles for different τ_v

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Fig. 4: Velocity Profiles for different ε



Fig. 5: Velocity Profiles for different β



Fig. 6: Velocity Profiles for different ϕ







Fig.8: The variation Δp with \overline{Q} for different τ_y



Fig. 9: The variation Δp with \overline{Q} for different ε



Fig. 10: The variation Δp with \overline{Q} for different β



Fig. 11: The variation Δp with \overline{Q} for different ϕ



Fig. 12: The variation F_{λ} with \overline{Q} for different α

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Fig. 13: The variation of F_{λ} with \overline{Q} for different y_p



Fig. 14: The variation of F_{λ} with \overline{Q} for different β



Fig. 15: The variation of F_{λ} with \overline{Q} for different ε

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Fig.16: Streamline profile when P = 1, $y_p = 0.1$, $\beta = \frac{\pi}{6}$, $\phi = 0.2$, F = 0.1, $\varepsilon = 0.3$ $(a)\alpha = 0$, $(b)\alpha = 0.05$ $(c)\alpha = 0.1$ $(d)\alpha = 0.15$



Fig. 17: Streamline profile when P = 1, $y_p = 0.6$, $\beta = \frac{\pi}{6}$, $\phi = 0.8$, F = 0.1, $\alpha = 0.2$ (a) $\varepsilon = 0.05$ (b) $\varepsilon = 0.1$ (c) $\varepsilon = 0.15$ (d) $\varepsilon = 0.2$



Fig.18: Streamline profile when P = 1, $y_p = 0.4$, $\phi = 0.7$, F = 0.1, $\alpha = 0.2$, $\varepsilon = 0.3$ $(a)\beta = \frac{\pi}{12}$ $(b)\beta = \frac{\pi}{8}$ $(c)\beta = \frac{\pi}{6}$.

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