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CHEMICAL REACTION EFFECTS ON MHD FREE CONVECTION FLOW IN AN IRREGULAR CHANNEL WITH POROUS MEDIUM

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ABSTRACT

The objective of this paper is to study the effects of chemical reaction and heat source on two dimensional free convection MHD flow of a viscous incompressible fluid through a finitely long vertical wavy wall and a smooth flat wall. A uniform magnetic field is assumed to be applied normal to the insulating walls of the channel. The equations governing the flow filed have been solved by using regular perturbation technique by subjecting to a set of appropriate boundary conditions. The solution of the mean part and the total solution of the problem have been evaluated analytically for several sets of values of the parameters pertaining to the problem and are shown graphically.

Key words: *MHD, viscous incompressible fluid, chemical reaction, heat source/ sink.*

1. INTRODUCTION

Massive amount of works on heat and mass transfer have focused mainly on regular geometries, such as a vertical flat plate [1-3], flat plate with inclination [4-6], parallel-plate channel [7-9], and rectangular ducts [10-11], etc. However, it is necessary to study the heat and mass transfer for complex geometries because the prediction of heat and mass transfer for irregular surfaces is a topic of fundamental importance and irregular surfaces often appeared in many applications. For example, flat-plate solar collectors, flat-plate condensers in refrigerators and medical operations in order to increase mass transfer (blood oxy-generator) Eldabe *et al.* [12]. In view of these applications, several authors [13-16] have made investigations of the fluid flows over a wavy wall.

Magnetohydrodynamics convection plays a significant role in various industrial applications. In a broader sense, MHD has applications in three different subject areas, such as astrophysical, geophysical and engineering problems. Examples include high temperature plasmas, cooling of nuclear reactors, liquid metal fluids, MHD accelerators and power generation systems etc. Several scholars [17-20] have shown their interest in MHD flows because of varied applications.

The combined effect of heat and mass transfer with chemical reaction is of great importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering and hence received a considerable amount of attention in recent years. There are two types of reactions such as (i) homogeneous reaction and (ii) heterogeneous reaction. A homogeneous reaction occurs uniformly through the given phase, where as heterogeneous reaction takes place in a restricted region / with in the boundary of a phase. The effect of chemical reaction depends on whether the reaction is heterogeneous or homogeneous. Seddek [21] studied the effects of chemical reaction and variable viscosity over a heat source. Champkha [22] analyzed the MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/ absorption and chemical reaction. The effect of chemical reaction on free convective flow of a polar fluid through porous medium in the presence of internal heat generation was investigated by Patil and Kulkarni [23]. Ananda Rao [24] studied the chemical reaction effects on an unsteady MHD free convective flow past an infinite vertical porous plate with constant suction and heat source. Sudheer Babu and Satyanarayana [25] discussed effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field. Alam et al. [26] analyzed the transient MHD free convective heat and mass transfer flow with thermophoresis past a radiate inclined permeable plate with chemical reaction and temperature dependent viscosity.

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The objective of present study is to analyze the chemical reaction effects on free convection MHD flow through a porous media in a vertical wavy channel. The dimensional less equations of continuity, linear momentum, energy and diffusion, which govern the flow field, are solved analytically by using perturbation technique. The solution is made on two parts, the mean part and perturbed part. These two parts are obtained separately. The closed form of solutions for velocity, temperature, concentration, skin friction, Nusselt number as well as Sherwood number are discussed and presented graphically.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider the two dimensional steady laminar free convective heat and mass transfer in an incompressible electrically conducting viscous boussinesq liquid through porous media in a vertical channel bounded by a long flat wall $y = d$ and a parallel wavy wall at $y = \in^{*} \cos K_w X$. The x – axis is taken parallel to the flat wall, the y- axis in perpendicular to it as shown in Fig (1). A uniform Magnetic field B_0 is applied to the fluid normal to the insulating walls of the channel. The flow is generated by the temperature difference at the walls so that the wavy and the flat walls are mainted at constant temperature T_w and T_1 respectively. The Boussinesq's approximation is used so that the density variations will be considered only in the buoyancy force. Assuming that the flow takes place at low concentration we neglect Soret and Doufer effects, the following assumptions are made.

(i) All the fluid properties except density in the buoyancy force are constant.
(ii) The viscous and magnetic dissipative effects are neglected in the energy equation.
(iii) The volumetric heat source/sink term in the energy equation is constant.
(iv) The magnetic Reynolds number is small so that the induced magnetic field can be neglected.
(v) The wave length of the wavy wall is large such that K_w is small.
(vi) The viscous dissipation and work done by pressure are sufficiently small in comparision with both heat flow by conduction and the wall temperatures.
(vii) The electric field is assumed to be zero.

Under these assumptions the appropriate governing equations of continuity momentum and energy and concentration equations are given by

$$\frac{\partial u'}{\partial X} + \frac{\partial v'}{\partial Y} = 0 \tag{1}$$

$$\rho\left(u'\frac{\partial u'}{\partial X} + v'\frac{\partial u'}{\partial Y}\right) = -\frac{\partial P'}{\partial X} + \mu\left(\frac{\partial^2 u'}{\partial X^2} + \frac{\partial^2 u'}{\partial Y^2}\right) - \rho g x - \frac{\mu}{k'}u' - \sigma B_0^2 u' \tag{2}$$

$$\rho\left(u'\frac{\partial v'}{\partial X} + v'\frac{\partial v'}{\partial Y}\right) = -\frac{\partial P'}{\partial Y} + \mu\left(\frac{\partial^2 v'}{\partial X^2} + \frac{\partial^2 v'}{\partial Y^2}\right) \tag{3}$$

$$\rho C_p\left(u'\frac{\partial T}{\partial X} + v'\frac{\partial T}{\partial Y}\right) = K_T\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) + Q \tag{4}$$

$$u'\frac{\partial C'}{\partial X} + v'\frac{\partial C'}{\partial Y} = D\left(\frac{\partial^2 C'}{\partial X^2} + \frac{\partial^2 C'}{\partial Y^2}\right) - K_r'\left(C' - C_s\right) \tag{5}$$

The boundary conditions are given by

$$\begin{aligned} &u' = 0,\ v' = 0,\ T = T_w,\ C' = C_w \quad \text{on} \quad Y = \in^{*} \cos K_w X \\ &u' = 0,\ v' = 0,\ T = T_1,\ C' = C_1 \quad \text{on} \quad Y = d \end{aligned} \tag{6}$$

The non-dimensional quantities are

$$\left(x, y\right) = \frac{1}{d}\left(X, Y\right),\ \left(u, v\right) = \frac{d}{\nu}\left(u', v'\right),\ \theta = \frac{T - T_s}{T_w - T_s}\ T_w \neq T_s,$$

$$C = \frac{C' - C_s}{C_w - C_s}\ C_w \neq C_s,\ P' = \frac{p'd^2}{\rho \nu^2},\ a^2 = \frac{d^2}{k'},\ \lambda = K_w d,\ M^2 = \frac{\sigma B_0^2 d^2}{\rho \nu},$$

$$\in=\frac{\in^*}{d},\ P_r=\frac{\mu C_p}{K_T},\ S_c=\frac{\nu}{d},\ \alpha=\frac{Qd^2}{K_T\left(T_w-T_s\right)},\ m_T=\frac{\left(T_1-T_s\right)}{\left(T_w-T_s\right)},\ m_C=\frac{\left(C_1-C_s\right)}{\left(C_w-C_s\right)},K_r=\frac{K_r'd^2}{\nu}.$$

The non dimensional equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{gxd^3}{\nu^2}-\left(M^2+a^2\right)u \tag{8}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2} \tag{9}$$

$$P_r\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+\alpha \tag{10}$$

$$S_c\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-K_rS_cC \tag{11}$$

The dimensionless boundary conditions are

$$u=0,\ v=0,\ \theta=1,\ C=1,\qquad \text{on}\quad y=\in\cos\lambda x$$

$$u=0,\ v=0,\ \theta=m_T,\ C=m_{C,}\quad \text{on}\quad y=1 \tag{12}$$

In the static fluid we have

$$0=-\frac{\partial P_s}{\partial x}-\frac{\rho_s gxd^3}{\rho\nu^2} \tag{13}$$

where $\rho_s=\rho\left\{1+\beta_T\left(T_w-T_s\right)\theta+\beta_m\left(C_w-C_s\right)C\right\}$ is the well-known Boussinesq approximation

In view of equation (13), equation (8) becomes

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial}{\partial x}\left(P-P_s\right)+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+G_r\theta+G_mC-\left(M^2+a^2\right)u \tag{14}$$

where $G_r=\dfrac{d^3gx\beta_T\left(T_w-T_s\right)}{\nu^2},\ G_m=\dfrac{d^3gx\beta_m\left(C_w-C_s\right)}{\nu^2}$

All the physical variables are defined in the nomenclature.

3. METHOD OF SOLUTION

We assume that the solution consists of a mean part and perturbed part so that the velocity, temperature and concentration distributions are

$$u(x,y)=u_0\left(y\right)+u_1\left(x,y\right),\ v(x,y)=v_1\left(x,y\right),P(x,y)=P_0\left(y\right)+P_1\left(x,y\right),$$

$$\theta(x,y)=\theta_0\left(y\right)+\theta_1\left(x,y\right),\ C(x,y)=C_0\left(y\right)+C_1\left(x,y\right), \tag{15}$$

where the perturbed quantites $u_1,\ v_1,\ p_1,\ \theta_1$ and C_1 are small compared with the mean or the zeroth order quantites.

Substituting the above Equation (15) into the Equations (7)-(11) and equating the harmonic and non-harmonic terms and neglecting the higher order terms of $O(\varepsilon^2)$, we obtain the following set of equations:

The zeroth order

$$\frac{d^2u_0}{dy^2}+G_r\theta_0+G_mC_0-\left(M^2+a^2\right)u_0=K_p,$$

$$\frac{d^2\theta_0}{dy^2}=-\alpha,$$

$$\frac{d^2C_0}{dy^2}=K_rS_cC_0,$$

$$u_0=0,\quad \theta_0=1,\quad C_0=1 \qquad \text{on} \quad y=0 \tag{16}$$

$$u_0=0,\quad \theta_0=m_T,\quad C_0=m_C \quad \text{on} \quad y=1 \tag{17}$$

The first order

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0 \tag{18}$$

$$u_0\frac{\partial u_1}{\partial x}+v_1\frac{du_0}{dy}=-\frac{\partial P_1}{\partial x}+\frac{\partial^2u_1}{\partial x^2}+\frac{\partial^2u_1}{\partial y^2}+G_r\theta_1+G_mC_1-M^2u_1-a^2u_1 \tag{19}$$

$$u_0\frac{\partial v_1}{\partial x}=-\frac{\partial P_1}{\partial y}+\frac{\partial^2v_1}{\partial x^2}+\frac{\partial^2v_1}{\partial y^2} \tag{20}$$

$$P_r\left(u_0\frac{\partial \theta_1}{\partial x}+v_1\frac{d\theta_0}{dy}\right)=\frac{\partial^2\theta_1}{\partial x^2}+\frac{\partial^2\theta_1}{\partial y^2} \tag{21}$$

$$S_c\left(u_0\frac{\partial C_1}{\partial x}+v_1\frac{dC_0}{dy}\right)=\frac{\partial^2C_1}{\partial x^2}+\frac{\partial^2C_1}{\partial y^2}-K_rS_cC_1 \tag{22}$$

$$\begin{aligned}&u_1=-u_0',\quad v_1=0,\quad \theta_1=\theta_0',\quad C_1=-C_0' \qquad \text{on} \quad y=0\\ &u_1=0,\qquad v_1=0,\quad \theta_1=0,\qquad C_1=0 \qquad\ \ \text{on} \quad y=1\end{aligned} \tag{23}$$

In deriving the first equation in (16) the constant pressure gradiant term

$K_p=\frac{\partial}{\partial x}\left(P_0-P_s\right)$, has been taken equal to zero. (See ref [21])

To solve the equations (18)-(22) we introduce the following similarity transformations

$$u_1=-\frac{\partial \psi_1}{\partial y},\qquad v_1=\frac{\partial \psi_1}{\partial x} \tag{24}$$

Eliminating the pressure from (19) and (20) we can express equations (19)-(22) in terms of the stream function ψ_1 in the form

$$u_0\left(\psi_{1,xxx}+\psi_{1,xyy}\right)-u_0''\psi_{1,x}=2\psi_{1,xxyy}+\psi_{1,xxxx}+\psi_{1,yyyy}-G_r\theta_{1,y}-G_mC_{1,y}-M^2\psi_{1,yy}-a^2\psi_{1,yy} \tag{25}$$

$$P_r\left(u_0\theta_{1,x}+\psi_{1,x}\theta_0'\right)=\theta_{1,xx}+\theta_{1,yy} \tag{26}$$

$$S_c\left(u_0 C_{1,x}+\psi_{1,x}C_0'\right)=C_{1,xx}+C_{1,yy}-K_r S_c C_1 \tag{27}$$

We assume ψ_1, θ_1 and C_1 in the form

$$\psi_1\left(x,y\right)=e^{i\lambda x}\overline{\psi}\left(\lambda,y\right),\quad \theta_1\left(x,y\right)=e^{i\lambda x}\overline{t}\left(\lambda,y\right),\quad C_1\left(x,y\right)=e^{i\lambda x}\overline{\phi}\left(\lambda,y\right) \tag{28}$$

(Perturbation series expansion for small wave lenth λ in which terms of exponential order arise)

From which we infer that

$$u_1(x,y)=-\in e^{i\lambda x}\psi'(\lambda,y),\qquad v_1(x,y)=\in i\lambda e^{i\lambda x}\psi(\lambda,y)$$

In view of equation (28), the equations (25) to (27) becomes

$$\overline{\psi}^{iv}-i\lambda\left[u_0\left(\overline{\psi}''-\lambda^2\overline{\psi}\right)+u_0''\overline{\psi}\right]-\lambda^2\left(2\overline{\psi}''-\lambda^2\overline{\psi}\right)=G_r\overline{t}'+G_m\overline{\phi}'+M^2\overline{\psi}''+a^2\overline{\psi}'' \tag{29}$$

$$\overline{t}''-\lambda^2\overline{t}=P_r i\lambda\left[u_0\overline{t}+\overline{\psi}\theta_0'\right] \tag{30}$$

$$\overline{\phi}''-\lambda^2\overline{\phi}-K_r S_c\overline{\phi}=S_c i\lambda\left(u_0\overline{\phi}+\overline{\psi}C_0'\right) \tag{31}$$

where the primes denote differentiation with respect to y.

The boundary condition (23) can now be written in terms of ψ_1 as

$$\begin{aligned}&\psi_{1,y}=u_0',\quad \psi_{1,x}=0 \qquad \text{on}\quad y=0\\ &\psi_{1,y}=0,\quad \psi_{1,x}=0 \qquad \text{on}\quad y=1\end{aligned} \tag{32}$$

For small values of λ (or K_w), we can expand $\overline{\psi}\left(\lambda,y\right)$, $\overline{t}\left(\lambda,y\right)$, $\overline{\phi}\left(\lambda,y\right)$ in terms of λ so that

$$\begin{aligned}&\overline{\psi}\left(\lambda,y\right)=\sum_{j=0}^{\infty}\lambda^j\psi_j,\quad \overline{t}\left(\lambda,y\right)=\sum_{j=0}^{\infty}\lambda^j t_j,\\ &\overline{\phi}\left(\lambda,y\right)=\sum_{j=0}^{\infty}\lambda^j\phi_j,\quad \left(j=0,1,2,.........\right)\end{aligned} \tag{33}$$

Substituting these results into (29)-(32), we obtain the following sets of ordinary differential equations

$$\psi_0^{iv}-(M^2+a^2)\psi_0''=G_r t_0'+G_m\phi_0', \tag{34}$$

$$t_0''=0, \tag{35}$$

$$\phi_0''-K_r S_c\phi_0=0, \tag{36}$$

$$\psi_1^{iv}-(M^2+a^2)\psi_1''=G_r t_1'+G_m\phi_1'+i(u_0\psi_0''-u_0''\psi_0), \tag{37}$$

$$t_1''=P_r i\left(u_0 t_0+\psi_0\theta_0'\right), \tag{38}$$

$$\phi_1''-K_r S_c\phi_1=S_c i\left(u_0\phi_0+\psi_0 C_0'\right), \tag{39}$$

$$\psi_2^{iv}-(M^2+a^2)\psi_2''=G_r t_2'+G_m\phi_2'+i(u_0\psi_1''-u_0''\psi_1)+2\psi_0'', \tag{40}$$

$$t_2''=P_r i(u_0 t_1+\psi_1\theta_0')+t_0, \tag{41}$$

$$\phi_2''-K_r S_c\phi_2=S_c i(u_0\phi_1+\psi_1 C_0')+\phi_0, \tag{42}$$

With the boundary conditions

$$\psi_0' = u_0', \quad \psi_0 = 0, \quad t_0 = -\theta_0', \quad \phi_0 = -C_0' \quad on \quad y = 0$$
$$\psi_0' = 0, \quad \psi_0 = 0, \quad t_0 = 0, \quad \phi_0 = 0 \quad on \quad y = 1 \tag{43}$$

$$\psi_j' = 0, \quad \psi_j = 0, \quad t_j = 0, \quad \phi_j = 0 \quad on \quad y = 0$$
$$\psi_j' = 0, \quad \psi_j = 0, \quad t_j = 0, \quad \phi_j = 0 \quad on \quad y = 1, \quad for \ j \geq 0 \tag{44}$$

The solutions of above ordinary differential equations with respect to the boundary conditions (43) and (44) are

$$C_0(y) = \cosh Ay + b_2 \sinh Ay$$
$$\theta_0(y) = -\frac{\alpha y^2}{2} + a_0 y + 1$$
$$u_0(y) = b_3 \cosh\sqrt{a_1}\, y + b_5 \sinh\sqrt{a_1}\, y + A_1 y - A_2 y^2 - A_3 \cosh Ay - A_3 b_2 \sinh Ay + b1$$
$$t_0(y) = a_0(y-1)$$
$$\phi_0(y) = k_3 \cosh Ay + k_4 \sinh Ay$$
$$\psi_0(y) = a_{10} + a_{11} y + a_8 \cosh\sqrt{a_1}\, y + a_9 \sinh\sqrt{a_1}\, y - \frac{A_1}{2} y^2 + b_6 \sinh Ay + b_7 \cosh Ay$$
$$u(y) = \left(b_3 \cosh\sqrt{a_1}\, y + b_5 \sinh\sqrt{a_1}\, y + A_1 y - A_2 y^2 - A_3 \cosh Ay - A_3 b_2 \sinh Ay + b_1\right) -$$
$$\in \left(a_{11} + a_8\sqrt{a_1} \sinh\sqrt{a_1}\, y + a_9\sqrt{a_1} \cosh\sqrt{a_1}\, y - A_1 y + b_6 A \cosh Ay + b_7 A \sinh Ay\right)$$
$$v(y) = -\in \lambda \left(a_{10} + a_{11} y + a_8 \cosh\sqrt{a_1}\, y + a_9 \sinh\sqrt{a_1}\, y - \frac{A_1}{2} y^2 + b_6 \sinh Ay + b_7 \cosh Ay\right)$$
$$\theta(y) = -\frac{\alpha y^2}{2} + a_0 y + 1 + \in \left[a_0(y-1)\right]$$
$$C(y) = \cosh Ay + b_2 \sinh Ay + \in \left(k_3 \cosh Ay + k_4 \sinh Ay\right)$$

The skin friction τ_{xy} at any point in the fluid is given by

$$\tau = u_0'(y) + \in e^{i\lambda x} u_1'(y) + i \in \lambda e^{i\lambda x} v_1(y)$$
$$\tau_{y=0} = \left(b_5\sqrt{a_1} + A_1 - A_3 b_2 A\right) - \in \left(a_8 a_1 - A_1 + b_7 A^2\right)$$
$$\tau_{y=1} = \left(b_3\sqrt{a_1} \sinh\sqrt{a_1} + b_5\sqrt{a_1} \cosh\sqrt{a_1} + A_1 - 2A_2 - A_3 A \sinh A - A_3 b_2 A \cosh A\right) -$$
$$\in \left(a_8 a_1 \cosh\sqrt{a_1} + a_9 a_1 \sinh\sqrt{a_1} - A_1 + b_6 A^2 \sinh A + b_7 A^2 \cosh A\right)$$

The dimensional Nusselt number Nu is given by

$$Nu = \frac{\partial\theta}{\partial y} = \theta_0'(y) + \in e^{i\lambda x} t'(y)$$
$$Nu_{y=0} = a_0 + \in a_0$$
$$Nu_{y=1} = -\alpha + a_0 + \in a_0$$

The dimensional Sherwood number Sh is given by

$$Sh = \frac{\partial C}{\partial y} = C_0'(y) + \in \phi_0'(y)$$
$$Sh_{y=0} = b_2 A + \in k_4 A$$
$$Sh_{y=1} = A \sinh A + b_2 A \cosh A + k_3 A \sinh A + k_4 A \cosh A$$

APPENDIX:

$$a_0 = m_T - 1 + \frac{\alpha}{2}, \qquad b_1 = \frac{G_r}{a_1} - \frac{G_r \alpha}{a_1^2},$$

$$a_1 = M^2 + a^2, \qquad b_2 = \frac{m_C - \cosh A}{\sinh A},$$

$$a_3 = b_5\sqrt{a_1} + A_1 - A_3 b_2 A, \qquad b_3 = A_3 - b_1,$$

$$a_4 = \cosh\sqrt{a_1} - 1, \qquad b_4 = A_2 - A_1 + A_3\cosh A + A_3 b_2 \sinh A - b_1,$$

$$a_5 = \sinh\sqrt{a_1} - \sqrt{a_1}, \qquad b_5 = \frac{b_4 - b_3\cosh\sqrt{a_1}}{\sinh\sqrt{a_1}},$$

$$a_6 = \sqrt{a_1}\sinh\sqrt{a_1}, \qquad b_6 = \frac{G_m k_3}{A^3 - a_1 A},$$

$$a_7 = \sqrt{a_1}\cosh\sqrt{a_1} - \sqrt{a_1}, \qquad b_7 = \frac{G_m k_4}{A^3 - a_1 A}$$

$$a_8 = \frac{(a_7 b_8) - (a_5 b_1) + (b_{07} a_7) - (b_9 a_7) + (b_9 a_5)}{a_4 a_7 - a_5 a_6}, \qquad b_8 = \frac{A_1}{2} - b_6 \sinh A - b_7 \cosh A,$$

$$a_9 = \frac{(a_6 b_8) + (b_7 a_6) - (b_9 a_6) - (a_4 b_{10}) + (b_9 a_4)}{a_5 a_6 - a_4 a_7}, \qquad b_9 = a_3 - b_6 A,$$

$$a_{10} = -(b_7 + a_8), \qquad a_{11} = b_9 - a_9\sqrt{a_1}, \qquad b_{10} = A_1 - b_6 A\cosh A - b_7 A \sinh A,$$

$$A = \sqrt{K_r S_c}, \qquad k_3 = -(b_2 A),$$

$$A_1 = \frac{G_r a_0}{a_1}, \qquad k_4 = -\frac{(k_3\cosh A)}{(\sinh A)},$$

$$A_2 = \frac{G_r \alpha}{2a_1},$$

$$A_3 = \frac{G_m}{A^2 - a_1},$$

$$B_1 = b_3\sqrt{a_1}\sinh\sqrt{a_1} + b_5\sqrt{a_1}\cosh\sqrt{a_1} + A_1 - 2A_2 - A_3 A\sinh A - A_3 b_2 A\cosh A,$$

$$B_2 = a_8 a_1 - A_1 + b_7 A^2,$$

$$B_3 = a_8 a_1 \cosh\sqrt{a_1} + a_9 a_1 \sinh\sqrt{a_1} - A_1 + b_6 A^2 \sinh A + b_7 A^2 \cosh A$$

4. RESULTS AND DISCUSSIONS

In this paper we have analyzed the chemical reaction effects on free convection MHD flow through a porous media between a long vertical wavy wall and a parallel flat wall. The expressions for velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are solved analytically, various parameters are entering in to the problem and presented with help of graphical illustration.

The influence of Schmidt number S_c on the zeroth order velocity and concentration profiles are plotted in figs. 2 and 3 respectively. The Schmidt number embodies the ratio of momentum to the mass diffusitivity. The Schmidt number therefore quantities the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

Figs 4 and 5 depict the variation of zeroth order velocity u_0 and concentration C_0 profiles against y for different values of chemical reaction parameter K_r for fixed values of other parameters. It is observed that, increasing the value of chemical reaction results decreases the velocity and concentration in the boundary layer. This is due to fact that destructive chemical reaction reduces the solutal boundary layer thickness and increase the mass transfer.

In figs 6 and 7 represents the variation of the zeroth order velocity u_0 for different values of Grashof number G_m and thermal Grashof number G_r . As expected it is observed that the velocity u_0 increase with increasing Grashof number and thermal Grashof number. This is due to the fact that buoyancy force enhances fluid velocity and increase the boundary layer thickness with increasing values of G_m or G_r .

For various values of porosity parameter a on velocity u_0 is shown in figure 8. We observed that u_0 decreases for increasing values of a .

Fig. 9 illustrates the influence of magnetic field parameter M on u_0 in the boundary layer. Application of magnetic field to an electrically conducting fluid gives rise to a resistive type force called the lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer.

The variations in the velocity v and concentration C profiles for different values of Schmidt number is clearly observed in figs. 10 and 11. It is obvious that the velocity and concentration decrease with increase in Schmidt number.

Figs 12 and 13 depict the effect of chemical reaction K_r on non dimensional velocity v and concentration C profiles against y for fixed values of other physical parameters. We observe that increasing values of K_r results decrease the velocity and concentration in the boundary layer.

Fig 14 and 15 shows the behavior of dimensionless velocity v and temperature θ for different values of heat source parameter. The analytical results shows that an increase in α results a increase in the velocity and temperature.

Figs. 16 and 17 show the local skin friction coefficient for different values of α and G_r at $y=0$ and $y=1$ keeping all the parameters fixed. From the fig (16) we see that for fixed G_r , at $y=0$, τ increase as α increases. On the other hand as G_r increases skin friction coefficient has no effect at $y=1$, against α (See fig 17).

Fig. 18 and 19 describe the behavior of rate of heat transfer Nu with changes in the values of α and m_C at $y=0$ and $y=1$. It is observed from these figures that the Nusselt number increases due to increases in the heat source parameter α at both the walls.

Fig. 20 and 21 deals with the variation of rate of mass transfer Sh for different values of K_r and S_c at the walls $y=0$ and $y=1$. From fig 20 we observe that Sh decrease with increase of S_c .
whereas reverse trend is observed in fig. 21.

GRAPHS:

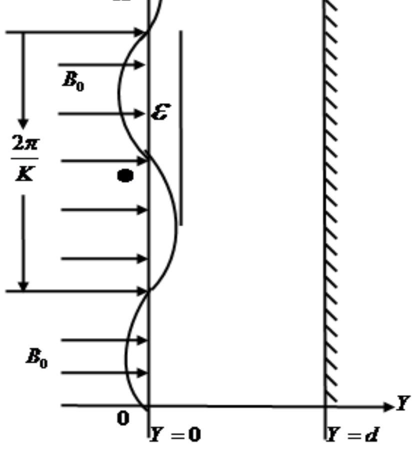

Fig. 1: physical model

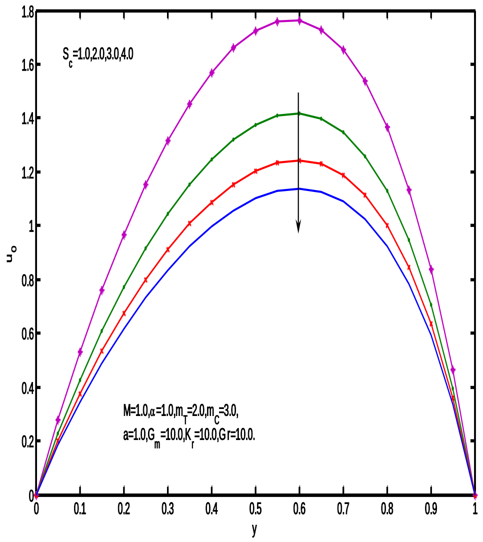

Fig 2: Effect of S_c on zeroth order velocity u_0

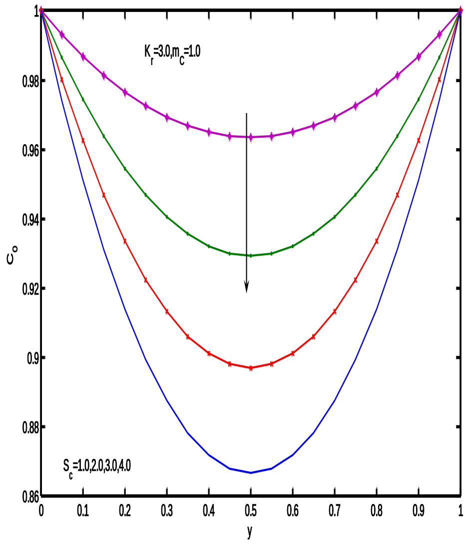

Fig 3: Effect of S_c on zeroth order Concentration C_0

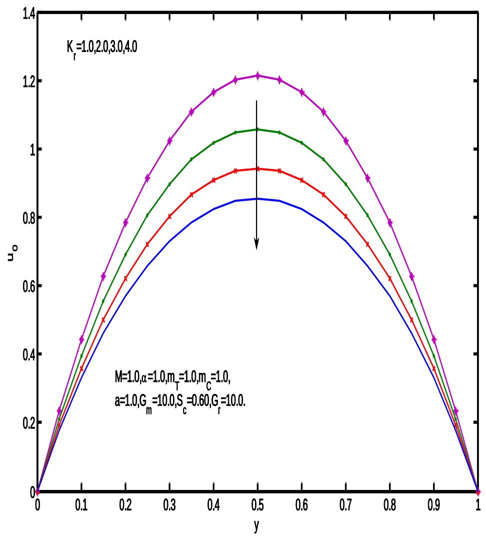

Fig 4: Effect of K_r on zeroth order velocity u_0

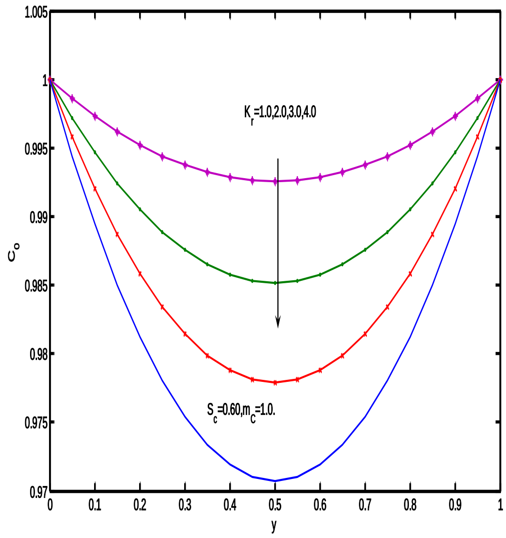

Fig 5: Effect of K_r on zeroth order Concentration C_0

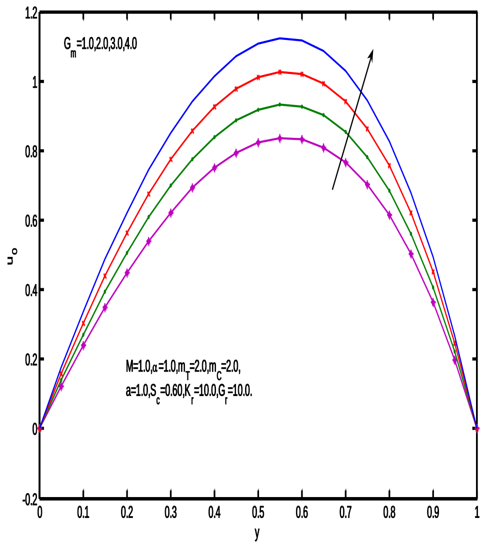

Fig 6: Effect of G_m on zeroth order velocity u_0

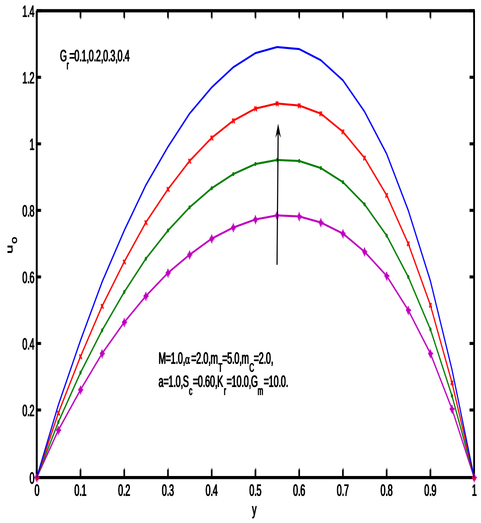

Fig 7: Effect of G_r on zeroth order velocity u_0

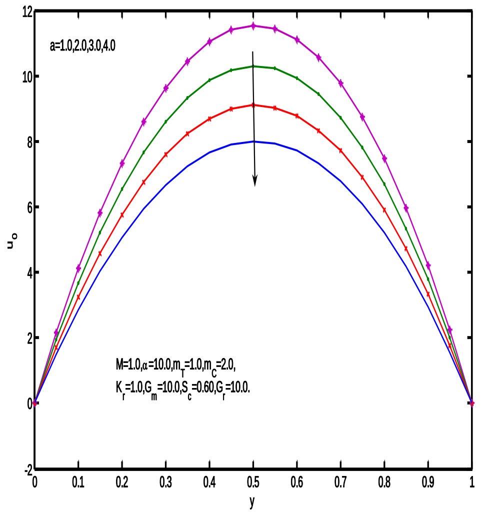

Fig 8: Effect of a on zeroth order velocity u_0

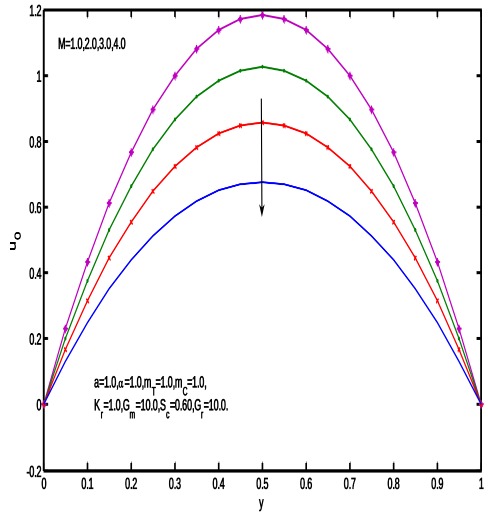

Fig 9: Effect of M on zeroth order velocity u_0

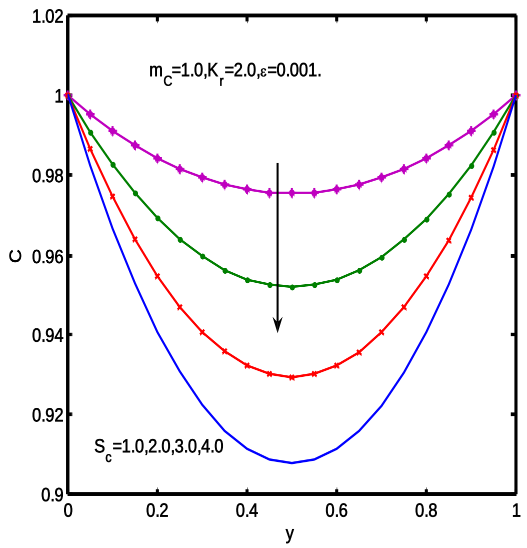

Fig 10: Effect of S_c on v profile

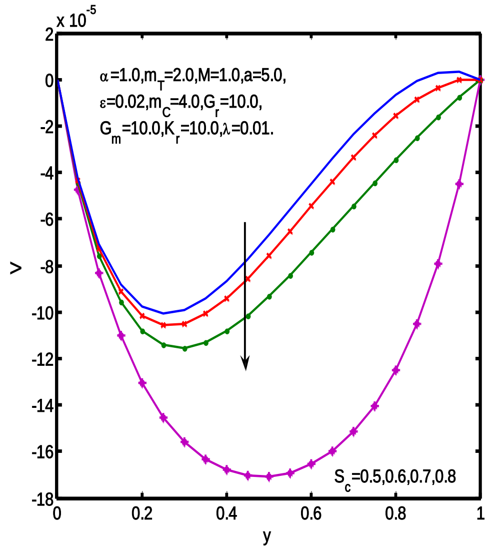

Fig 11: Effect of S_c on C profile

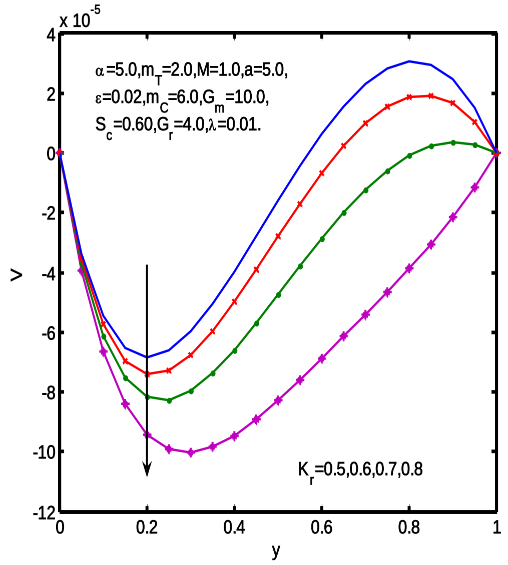

Fig 12: Effect of K_r on v profile

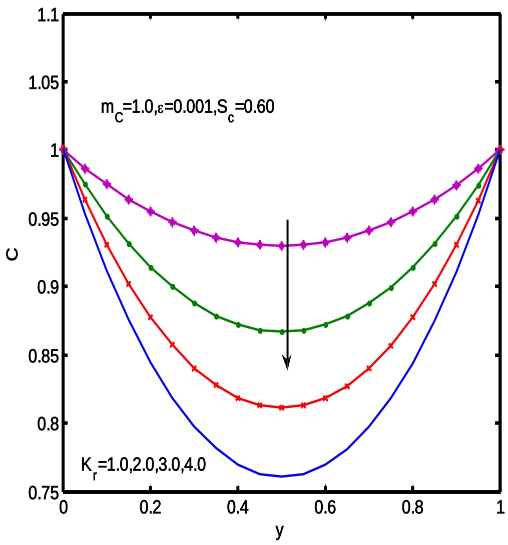

Fig 13: Effect of K_r on C profile

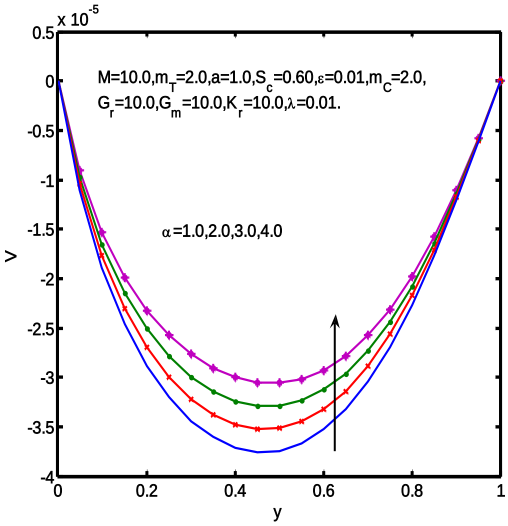

Fig. 14: Effect of α on v profile

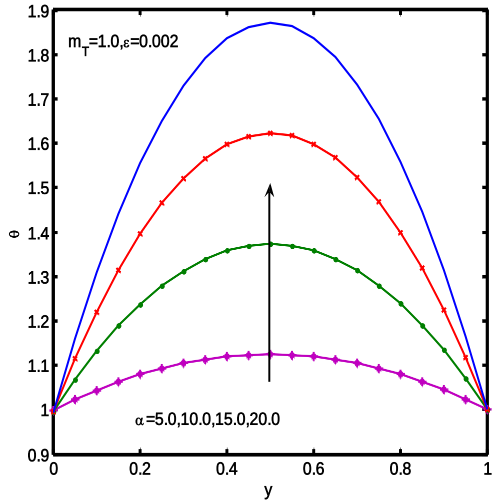

Fig. 15: Effect of α on θ profile

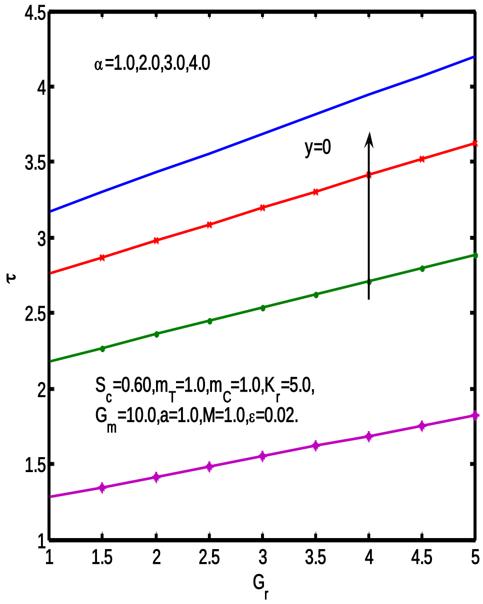

Fig.. 16: Skin friction for α versus G_r at $y=0$

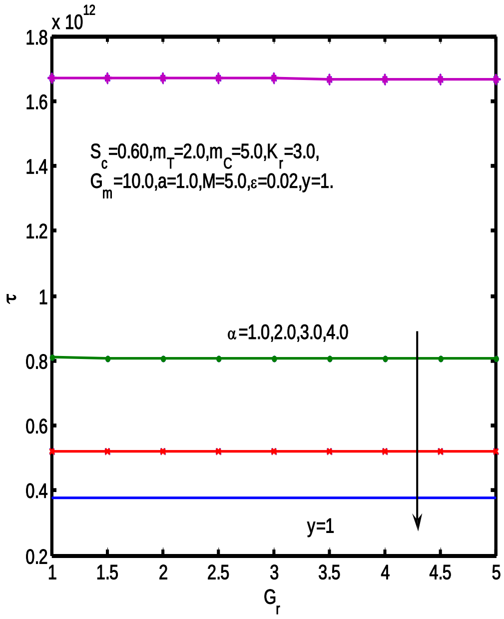

Fig. 17: Skin friction for α versus G_r at $y=1$

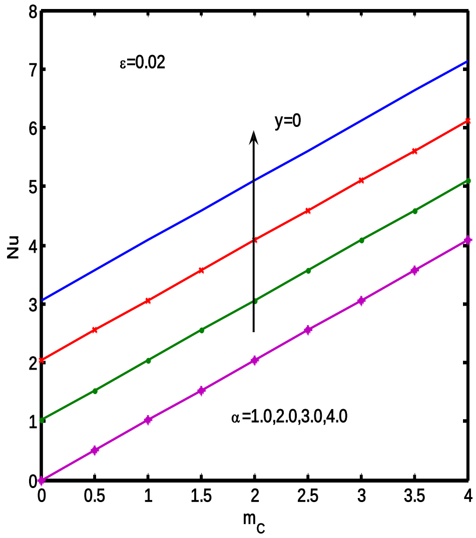

Fig. 18: Nusselt number for α versus m_T at $y=0$

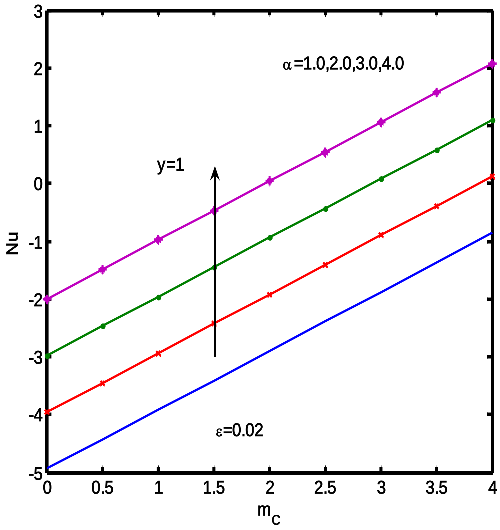

Fig. 19: Nusselt number for α versus m_T at $y=1$

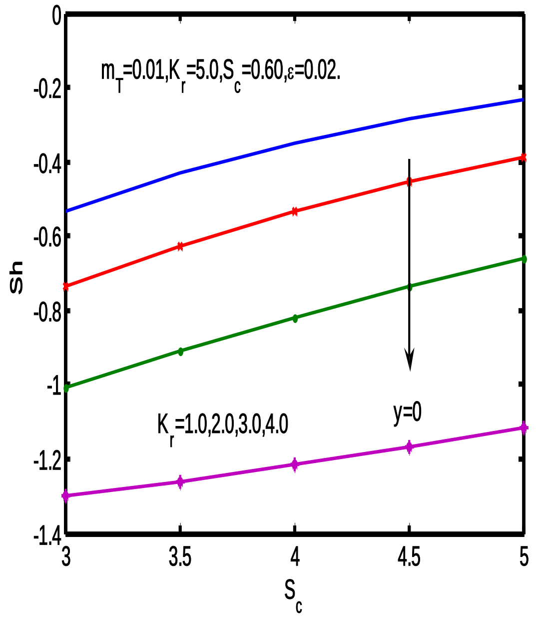

Fig. 20: Sherwood number for K_r versus S_c at $y=0$

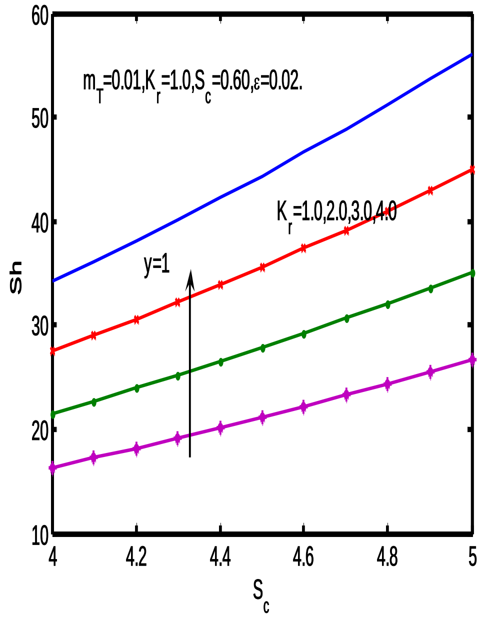

Fig. 21: Sherwood number for K_r versus S_c at $y=1$

NOMENCLATURE

a	Porosity parameter
C'	Species concentration at any point in the fluid (kg. m^{-3})
C	Dimensionless concentration
C_0	Zeroth-order concentration
C_p	Specific heat at constant pressure (KJk g^{-1} K^{-1})
D	Mass diffusion coefficient ($m^2.s^{-1}$)
d	Distance between the walls
G_m	Mass Grashof number
G_r	Thermal Grashof number
gx	Acceleration due to gravity ($m^2.s^{-1}$)
K_T	Fluid Thermal conductivity (W m^{-1} K^{-1})
K_w	Wave number
K_r	Chemical reaction parameter
M	Magnetic field parameter
m_C	Wall-mass ratio (J.k g^{-1} K^{-1})
P	Dimensionless pressure
P'	Dimensional fluid pressure (N. m^{-2})
P_r	Prandtl number
Q	The constant volumetric heat source/sink
S_c	Schmidt number
T	Dimensional temperature of the fluid (K)
(u', v')	Velocity components along the (X, Y) axes ($m.s^{-1}$)
(u, v)	Dimensionless velocity
u_0	Dimensionless zeroth-order velocity profile
(X, Y)	Coordinate system (m)
(x, y)	Dimensionless coordinate system

GREEK SYMBOLS:

α	Dimensionless heat source/sink parameter
β_m	Coefficient of volumetric mass expansion (K^{-1})
β_T	Coefficient of volumetric thermal expansion (K^{-1})
$\in^*$	Amplitude of the irregular wall
$\in$	The dimensionless amplitude
Ω	Pressure drop
λ	Dimensionless wavelength of the irregular wall
μ	Dynamic viscosity (kg. $m^{-1}.s^{-1}$)
ν	Fluid Kinematic viscosity ($m^2.s^{-1}$)
ρ	Fluid density (kg. m^{-3})
θ	Dimensionless temperature of the fluid
θ_0	Dimensionless zeroth-order temperature profiles
ψ_1	The stream function

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