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# GENERALIZED $\zeta^*$ -CLOSED SETS IN TOPOLOGICAL SPACES

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# ABSTRACT

In this paper, we introduce the notion of  $g\zeta^*$ -closed sets in topological spaces and investigate some of their properties and we construct a group of  $g\zeta^*c$ - homeomorphisms which contains the group of all homeomorphisms as a subgroup.

*Keywords:*  $g\zeta^*$ *-closed set,*  $g\zeta^*$ *-continuous function,*  $g\zeta^*$ *-irresolute and*  $g\zeta^*$ *-homeomorphism.* 

# 1. INTRODUCTION

Levine [1, 2] introduced the concept of generalized closed sets and semi-closed sets in topological spaces. Maki *et al.*, introduced generalized  $\alpha$ -closed sets (brieflyg $\alpha$ -closed sets) [3] and  $\alpha$ -generalized closed sets (briefly  $\alpha$ g-closed sets) [4] respectively. In section 3 and 4 of this paper, we introduce the concept of  $g\zeta^*$ -closed set and obtain some properties of this set. In section 5 and 6, we introduce the concept of  $g\zeta^*$ -continuous functions and  $g\zeta^*$ -irresolute functions. For a topological space( $X, \tau$ ), we define groups  $g\zeta^*h(X, \tau), g\zeta^*ch(X, \tau)$  and they contain the group  $h(X, \tau)$  whose elements are all homeomorphisms from ( $X, \tau$ ) into itself.

Throughout this paper( $X, \tau$ ), ( $Y, \sigma$ ) and ( $Z, \eta$ ) represent non-empty topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of a space( $X, \tau$ ), cl (A) and int(A) denote the closure of A and interior of A.

# 2. PRELIMINARIES

In this section we recall some of the basic definitions.

**Definition 2.1:** A subset *A* of space  $(X, \tau)$  is called

- (i) Semi-open set [1] if  $A \subseteq cl$  (*int* (A)).
- (ii) Pre-open set [5] if  $A \subseteq int (cl(A))$ .
- (iii)  $\alpha$ -open set [6] if  $A \subseteq int (cl (int (A)))$ .

The complement of a semi-open (resp.pre-open,  $\alpha$ -open) set is called semi-closed (resp.pre-closed,  $\alpha$ -closed).

# **Definition 2.2:** A subset A of $(X, \tau)$ is called

- (i) generalized closed (briefly *g*-closed)set[2] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open set in  $(X, \tau)$ .
- (ii) generalized semi-closed (briefly *gs*-closed) set [7] if *scl* (*A*)  $\subseteq U$  whenever  $A \subseteq U$  and *U* is open set in(*X*,  $\tau$ ).
- (iii)  $\alpha$ -generalized closed (briefly  $\alpha g$ -closed) set [4] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open set in  $(X, \tau)$ .
- (iv) generalized  $\alpha$ -closed (briefly  $g\alpha$ -closed) set [3] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is  $\alpha$ -open set in( $X, \tau$ ).
- (v) generalized pre-closed set (briefly *gp*-closed) set [8] if *pcl* (A)  $\subseteq U$  whenever  $A \subseteq U$  and U is open set in(X,  $\tau$ ).
- (vi) a generalized <sup>#</sup> $\alpha$ -closed set (briefly g<sup>#</sup> $\alpha$ -closed) [16] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is g-open in  $(X, \tau)$ .
- (vii) a <sup>#</sup>generalized  $\alpha$ -closed set (briefly <sup>#</sup>g $\alpha$ -closed) [17] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is a g<sup>#</sup> $\alpha$ -open in  $(X, \tau)$ .

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The complements of the above sets are called their respective open sets.

**Definition2.3:** A function  $f: (X, \tau) \to (Y, \sigma)$  is called (i)  $\alpha$ -continuous [9] if  $f^{-1}(V)$  is  $\alpha$ -closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ . (ii) g-continuous [10] if  $f^{-1}(V)$  is g-closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ . (iii) gs-continuous [11] if  $f^{-1}(V)$  is gs-closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ . (iv)  $g\alpha$ -continuous [3] if  $f^{-1}(V)$  is  $g\alpha$ -closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ . (v)  $\alpha g$ -continuous [4] if  $f^{-1}(V)$  is  $\alpha g$ -closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ . (vi) g''-continuous [12] if  $f^{-1}(V)$  is g''-closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ . (vii) gp-continuous [8] if  $f^{-1}(V)$  is  $g^{\#}\alpha$ -closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ . (viii)  $g^{\#}\alpha$ -continuous [16] if  $f^{-1}(V)$  is  $g^{\#}\alpha$ -closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ . (ix)  ${}^{\#}g\alpha$ -continuous [17] if  $f^{-1}(V)$  is  ${}^{\#}g\alpha$ -closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ .

### 3. $g\zeta^*$ -CLOSED SETS IN TOPOLOGICAL SPACES

We introduce the following definition.

**Definition 3.1:** [13] A subset A of a space  $(X, \tau)$  is called  $g\zeta^*$ -closed if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is<sup>#</sup> $g\alpha$ open set in  $(X, \tau)$ 

**Proposition 3.2:** Every closed set is  $g\zeta^*$ -closed set but not conversely.

**Proof:** Let *A* be a closed set and *U* be a  ${}^{\#}g\alpha$ -open set containing A. Since *A* is closed, we have  $\alpha cl(A) \subseteq cl(A) = A \subseteq U$ . Therefore  $\alpha cl(A) \subseteq U$  and hence *A* is  $g\zeta^*$ -closed set.

Remark 3.3: The converse of the above theorem is not true as shown in the following example.

**Example 3.4:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X, \{a, b\}\}$ . The set  $\{a, c\}$  is  $g\zeta^*$ -closed set but not closed set.

**Proposition 3.5:** Every  $\alpha$ -closed set is  $g\zeta^*$ -closed set but not conversely.

**Proof:** Let A be an  $\alpha$ -closed set and U be a <sup>#</sup> $g\alpha$ -open set containing A. Since A is  $\alpha$ -closed, we have  $\alpha cl(A) = A \subseteq U$ . Therefore  $\alpha cl(A) \subseteq U$  and hence A is  $g\zeta^*$ -closed set.

**Example 3.6:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X, \{a, b\}\}$ . The set  $\{b, c\}$  is  $g\zeta^*$ -closed set but not  $\alpha$ -closed set.

**Proposition 3.7:** Every  $g\zeta^*$ -closed set is  $\alpha g$ -closed set is but not conversely.

**Proof:** Let *A* be a  $g\zeta^*$ -closed set and *U* be any open set containing *A*. Since every open set is  $\alpha$ -open and every  $\alpha$ -open set is  ${}^{\#}g\alpha$ -open. Therefore every open set is  ${}^{\#}g\alpha$ -open [14]. We have  $\alpha cl(A) \subseteq U$ . Hence *A* is  $\alpha g$ -closed set.

**Example 3.8:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ . The set  $\{b\}$  is  $\alpha g$ -closed but not  $g\zeta^*$ -closed set.

**Proposition 3.9:** Every  $g\zeta^*$ -closed set is  $g\alpha$ -closed set is but not conversely.

**Proof:** Let *A* be a  $g\zeta^*$ -closed set and *U* be every  $\alpha$ -open set containing *A*. Since every  $\alpha$ -open set is  ${}^{\#}g\alpha$ -open. We have  $\alpha cl(A) \subseteq U$ . Hence *A* is  $g\alpha$ -closed set.

**Example 3.10:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ . The set  $\{a, b\}$  is  $g\alpha$ -closed but not  $g\zeta^*$ -closed set.

**Proposition 3.11:** Every  $g\zeta^*$ -closed set is *gs*-closed set is but not conversely.

**Proof:** Let *A* be a  $g\zeta^*$ -closed set and *U* be an open set containing *A*. Since every open set is  ${}^{\#}g\alpha$ -open, we have  $scl(A) \subseteq \alpha cl(A) \subseteq U$ . Therefore  $scl(A) \subseteq U$ . Hence *A* is gs-closed set.

**Example 3.12:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ . The set  $\{a, c\}$  is *gs*-closed but not  $g\zeta^*$ -closed set.

**Proposition 3.13:** Every  $g\zeta^*$ -closed set is gp-closed set is but not conversely.

**Proof:** Let A be a  $g\zeta^*$ -closed set and U be an open set containing A. Since every open set is  ${}^{\#}g\alpha$ -open, we have  $pcl(A) \subseteq \alpha cl(A) \subseteq U[15]$ . Therefore  $pcl(A) \subseteq U$ .Hence A is gp-closed set.

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**Example 3.14:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X, \{a, b\}\}$ . The set  $\{b\}$  is *gp*-closed but not  $g\zeta^*$ -closed set.

**Remark 3.15:** The following examples show that  $g\zeta^*$ -closeness is independent on g-closeness.

**Example 3.16:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ , The set  $\{b\}$  is  $g\zeta^*$ -closed but not g-closed.



**Remark 3.17:** The following diagram shows that the relationships of  $g\zeta^*$ -closed sets with other known existing sets.  $A \rightarrow B$  represents *A implies B* but not conversely.

#### 4. BASIC PROPERTIES OF $g\zeta^*$ CLOSED SETS

**Theorem 4.1:** If *A* and *B* are  $g\zeta^*$ -closed in *X* then,  $A \cup B$  is  $g\zeta^*$  closed in *X*.

**Proof:** Let *A* and *B* be any two  $g\zeta^*$ -closed in *X* and *U* be any  ${}^{\#}g\alpha$  –open set containing *A* and *B*. we have  $\alpha cl(A) \subseteq U$  and  $\alpha cl(B) \subseteq U$ . Thus  $\alpha cl(A) \cup \alpha cl(B) \subseteq U$ . Hence  $A \cup B$  is  $g\zeta^*$ -closed in *X*.

**Theorem 4.2:** If a set A is  $g\zeta^*$ -closed, then  $\alpha cl(A) - A$  contains no non-empty  ${}^{\#}g\alpha$  –closed set.

**Proof:** Suppose that *A* is  $g\zeta^*$ -closed set. Let *U* be a  ${}^{\#}g\alpha$  –closed set contained in  $\alpha cl(A) - A$ . Now  $U^c$  is  ${}^{\#}g\alpha$  –open set of  $(X, \tau)$  such that  $A \subseteq U^c$ . Since *A* is  $g\zeta^*$  closed set of  $(X, \tau)$ , then  $\alpha cl(A) \subseteq U^c$ . Thus  $U \subseteq (\alpha cl(A))^c$ . Also  $U \subseteq \alpha cl(A) - A$ . Therefore  $U \subseteq (\alpha cl(A))^c \cap (\alpha cl(A)) = \emptyset$  and hence  $U = \emptyset$ .

**Theorem 4.3:** If A is  ${}^{\#}g\alpha$  –open and  $g\zeta^*$ -closed subset of  $(X, \tau)$  then A is an  $\alpha$ -closed subset of X.

**Proof:** Since *A* is  ${}^{\#}g\alpha$  –open and  $g\zeta^*$ -closed, $\alpha cl(A) \subseteq A$ . Then *A* is  $\alpha$ -closed.

**Theorem 4.4:** Let *A* be a  $g\zeta^*$ -closed subset of *X*. If  $A \subseteq B \subseteq \alpha cl(A)$ , then *B* is also an  $g\zeta^*$ -closed subset of *X*.

**Proof:** Let *U* be a  ${}^{\#}g\alpha$  –open set of *X* such that  $B \subseteq U$ . Then  $A \subseteq U$ . Since *A* is an  $g\zeta^*$ -closed set  $\alpha cl(A) \subseteq U$ . Also  $B \subseteq \alpha cl(A), \alpha cl(B) \subseteq \alpha cl(A) \subseteq U$ . Hence *B* is also an  $g\zeta^*$ -closed subset of *X*.

**Theorem 4.5:** Let A be an  $g\zeta^*$ -closed set in X.Then A is  $\alpha$ -closed iff  $\alpha cl(A) - A$  is closed.

**Proof:** Necessity: Let *A* be an  $g\zeta^*$ -closed subset of *X*. Then  $\alpha cl(A) = A$  and so  $\alpha cl(A) - A = \emptyset$  which is closed.

**Sufficiency:** Since A is  $g\zeta^*$ -closed, by theorem 4.2,  $\alpha cl(A) - A$  contains no non-empty  ${}^{\#}g\alpha$ -closed set. But  $\alpha cl(A) - A$  is closed. This implies  $\alpha cl(A) - A = \emptyset$ . That is  $\alpha cl(A) = A$ . Hence A is  $\alpha$ -closed.

#### 5. GENERALIZED $\zeta^*$ -CONTINUOUS MAP

**Definition 5.1:** A function  $f: (X, \tau) \to (Y, \sigma)$  is called  $g\zeta^*$ -continuous if  $f^{-1}(V)$  is a  $g\zeta^*$ -closed set of  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ .

**Theorem 5.2:** Every  $\alpha$ -continuous map is  $g\zeta^*$ -continuous.

**Proof:** Let *V* be a closed set of  $(Y, \sigma)$ . Since *f* is a  $\alpha$ -continuous map,  $f^{-1}(V)$  is  $\alpha$ -closed in  $(X, \tau)$ .

Every  $\alpha$ -closed set is  $g\zeta^*$ -closed set. Therefore  $f^{-1}(V)$  is  $ag\zeta^*$ -closed set of  $(X, \tau)$ .

Hence *f* is a  $g\zeta^*$ -continuous map.

The converse of the above theorem need not be true by the following example.

**Example 5.3:** Let  $X = Y = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$  and  $\sigma = \{Y, \phi, \{a, c\}\}$ .

Let  $f: (X, \tau) \to (Y, \sigma)$  be defined by f(a) = a, f(b) = b, f(c) = c and f(d) = d.

Therefore *f* is not  $\alpha$ -continuous. However *f* is  $g\zeta^*$ -continuous.

#### Theorem 5.4:

(i) Every g''-continuous map is  $g\zeta^*$ -continuous.

(ii) Every  $g\zeta^*$ -continuous map is  $g\alpha$ -continuous, gp-continuous, gs-continuous,  $\alpha g$ -continuous.

**Proof:** It is obvious.

The converses of the above theorems need not be true by the following example.

**Example 5.5:** (i) Let  $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{b\}\}$  and  $\sigma = \{Y, \phi, \{a\}\}$ . Let  $f: (X, \tau) \to (Y, \sigma)$  be defined by f(a) = b, f(b) = a, and f(c) = c. Therefore f is not g''-continuous.

However *f* is  $g\zeta^*$ -continuous.

(ii) Let  $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}\}$  and  $\sigma = \{Y, \phi, \{b\}\}$ . Let  $f: (X, \tau) \to (Y, \sigma)$  be defined by f(a) = a, f(b) = b and f(c) = c. Therefore f is not  $g\zeta^*$ -continuous.

However *f* is *gp*-continuous, *gs*-continuous and  $\alpha g$ -continuous.

#### 6. GENERALIZED $\zeta^*$ -IRRESOLUTE MAP

**Definition 6.1:** A function  $f: (X, \tau) \to (Y, \sigma)$  is called  $g\zeta^*$ -irresolute  $f^{-1}(V)$  is a  $g\zeta^*$ -closed set of  $(X, \tau)$  for every  $g\zeta^*$ -closed set V of  $(Y, \sigma)$ .

**Theorem 6.2:** Every  $g\zeta^*$ -irresolute map is  $g\zeta^*$ -continuous.

**Proof:** Let V be a closed set of  $(Y, \sigma)$  and hence it is  $g\zeta^*$ -closed set.Since f is  $g\zeta^*$ -irresolute,  $f^{-1}(V)$  is  $ag\zeta^*$ -closed set of  $(X, \tau)$ .

Hence *f* is a  $g\zeta^*$ -continuous map.

The converse of the above theorem need not be true by the following example.

**Example 6.3:** Let  $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{b, c\}\}$  and  $\sigma = \{Y, \phi, \{a\}\}$ . Let  $f: (X, \tau) \to (Y, \sigma)$  be defined by f(a) = b, f(b) = a and f(c) = c. Then f is not  $g\zeta^*$ -irresolute, since  $\{c\}$  is a  $g\zeta^*$ -closed set of  $(Y, \sigma)$ , but  $f^{-1}(\{c\})$  is not a  $g\zeta^*$ -closed set of  $(X, \tau)$ .

However *f* is  $g\zeta^*$ -continuous.

**Theorem 6.4:** If  $f:(X,\tau) \to (Y,\sigma)$  and  $g:(Y,\sigma) \to (Z,\eta)$  are  $g\zeta^*$ -irresolute, then the composition  $g \circ f:(X,\tau) \to (Z,\eta)$  is  $g\zeta^*$ -irresolute.

**Proof:** It is obvious.

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#### 7. GENERALIZEDZ\*c-HOMEOMORPHISM AND THEIR GROUP STRUCTURE

**Definition 7.1:** A function  $f: (X, \tau) \to (Y, \sigma)$  is said to be

(i)  $q\zeta^*$ -open if the image f(U) is  $q\zeta^*$ -open in  $(Y, \sigma)$  for every open set U of  $(X, \tau)$ .

(*ii*) $g\zeta^*$ -closed if the image f(U) is  $g\zeta^*$ -closed in  $(Y, \sigma)$  for every open set U of  $(X, \tau)$ .

(*iii*)  $g\zeta^*c$ -homeomorphism if f is bijective and f and  $f^{-1}$  are  $g\zeta^*$ -irresolute.

 $(iv)q\zeta^*$ -homeomorphism if f is bijective and f and  $f^{-1}$  are  $q\zeta^*$ -continuous.

#### Theorem 7.2:

(i) Suppose that f is a bijection, then the following conditions are equivalent.

- (a) f is a  $g\zeta^*$ -homeomorphism.
- (b) f is a  $q\zeta^*$ -open and  $q\zeta^*$ -continuous.
- (c) *f* is a  $q\zeta^*$ -closed and  $q\zeta^*$ -continuous.
- (ii) If f is a homeomorphism, then f and  $f^{-1}$  are  $q\zeta^*$ -irresolute.

(iii) Every  $q\zeta^*c$ -homeomorphism is a  $q\zeta^*$ -homeomorphism.

#### **Proof:**

(i) It is obvious.

(ii) First we prove that  $f^{-1}$  is  $g\zeta^*$ -irresolute. Let A be a  $g\zeta^*$ -closed set of  $(X, \tau)$ . To show  $(f^{-1})^{-1}(A) = f(A)$  is  $g\zeta^*$ closed set in  $(Y, \sigma)$ . Let U be a  $g\zeta^*$ -open set such that  $f(A) \subseteq U$ .

Then  $A = (f^{-1}(f(A))) \subseteq f^{-1}(U)$  and  $f^{-1}(U)$  is  ${}^{\#}g\alpha$  -open. Since A is  $g\zeta^*$ -closed,  $\alpha cl(A) \subseteq f^{-1}(U)$ , we have  $\alpha cl(f(A)) = f(\alpha cl(A)) \subseteq f(f^{-1}(U)) \subseteq U$  and so f(A) is  $g\zeta^*$ -closed. Thus  $f^{-1}$  is  $g\zeta^*$ -irresolute. Since  $f^{-1}$  is also a homeomorphism $(f^{-1})^{-1} = f$  is  $g\zeta^*$ -irresolute. (iii) It is proved by Theorem6.2.

**Theorem 7.3:** For a topological space  $(X, \tau)$  we define the following three collections of functions.

- $g\zeta^*ch(X,\tau) = \{f/f: (X,\tau) \to (X,\tau) \text{ is a } g\zeta^*c\text{-homeomorphism}\}.$ (i)
- (ii)  $g\zeta^*h(X,\tau) = \{f/f: (X,\tau) \to (X,\tau) \text{ is a } g\zeta^*\text{-homeomorphism}\}.$
- (iii)  $h(X,\tau) = \{f/f: (X,\tau) \to (X,\tau) \text{ is a homeomorphism} \}.$

**Theorem 7.4:** For a topological space  $(X, \tau)$  the following properties hold.

- (i)  $h(X,\tau) g\zeta^* ch(X,\tau) g\zeta^* h(X,\tau)$ .
- (ii) The set  $g\zeta^*ch(X,\tau)$  forms a group under composition of functions.
- (iii) The group  $h(X,\tau)$  is subgroup of  $g\zeta^*ch(X,\tau)$ .
- (iv) If f:  $(X, \tau) \to (Y, \sigma)$  is a  $g\zeta^*c$ -homeomorphism then it induces an isomorphism  $f^*: g\zeta^*ch(X, \tau) \to g\zeta^*ch(Y, \sigma)$ .

#### **Proof:**

(i) It is proved by using Theorem 5.2, Theorem 6.2 and a fact that every *continuous* map is  $\alpha$ -*continuous*.

(ii) It is proved by using Theorem 6.4; for any element,  $a, b \in g\zeta^* ch(X, \tau)$ , the following binary operation  $w: q\zeta^* ch(X,\tau) \times q\zeta^* ch(X,\tau) \rightarrow q\zeta^* ch(X,\tau)$  is well defined  $w(a,b) = b \circ a$ .

(iii) By (i),  $h(X,\tau) \subseteq q\zeta^*ch(X,\tau)$  and  $h(X,\tau) \neq \emptyset$ . For any elements  $a, b \in h(X,\tau)$  and the binary operation w in

(ii), it is shown that  $w(a, b^{-1}) = b^{-1}a \in h(X, \tau)$ .

(iv)We define  $f: g\zeta^* ch(X, \tau) \to g\zeta^* ch(Y, \sigma) by f * (h) = fohof^{-1}$ . Then using Theorem 6.4, we have that  $(h) \in$  $g\zeta^*ch(X,\tau)$ . It is shown that f \* is a required group isomorphism.

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