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ONSET OF SURFACE TENSION DRIVEN CONVECTION IN A FLUID LAYER WITH A BOUNDARY SLAB OF FINITE CONDUCTIVITY AND DEFORMABLE FREE SURFACE

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ABSTRACT

In the present study, the effect of a non-uniform basic temperature gradient on the linear stability of the Marangoni convection in a horizontal fluid layer. The upper surface of a fluid layer is deformably free and lower boundary is a considered to be a thin slab of finite conductivity instead of a regular rigid surface. At the contact surface between the thin slab and the fluid layer the thermal boundary conditions are used. The depth ratio, thermal conductivity ratio of solid plate play a decisive role on the stability characteristics of the system. In addition, the influence of the Biot number Bi, the Bond number B_0 and Crispation number Cr arising due to of upper deformable free surface is also emphasized on the stability of the system.

Key words: *Marangoni convection: boundary slab: depth ratio: conductivity ratio.*

1. INTRODUCTION

The onset of convection in a fluid layer is a problem of great importance to many industrial applications and has attracted scientific attention for many decades since Benard earliest experiments in 1900. His name was given to Benard cells, the hexagonal roll cells produced in heated molten spermaceti with a free surface. Convective instabilities claimed to be driven by either buoyancy (Benard) or thermocapillary (Marangoni) effects have been the subject of a great deal of theoretical and experimental investigation since the pioneering theoretical works of Rayleigh (1916) and Pearson (1958).

Rayleigh (1916) developed the theoretical foundations to account for the results of Benard's experiments. He theorized that the convection observed in Benard experiments is driven by buoyancy. However, an experimental study conducted by Block (1956) led him (Block) to theorize that the Benard cells are driven by surface tension instead of buoyancy. Pearson (1958) then proposed a theoretical model in which surface tension was actually causing the observed convective cells. This convection mechanism is usually named Marangoni convection in recognition of his previous work in 1871. The first theoretical study of Benard–Marangoni convection in a planar horizontal fluid layer with a non-deformable free surface was performed by Nield (1964) who showed that for steady convection the two destabilizing mechanisms are both necessary and reinforce one another.

The onset of Marangoni convection in a layer of fluid with free upper surface and heated from below has been investigated by several authors because of its relevance and importance in material science processing during sustained space flight, aircraft structures and automobile industries, geophysics, bio convection, nuclear reactors, solid-matrix heat exchangers, crystal growth, directional solidification of alloys, aerosol production and electronics cooling to mention a few (Pearson 1958, Nield 1964,Scriven and Sterling 1964, Nield 1968,Vidal et al.1981, Idris et al. 2009 Takashima 2009, Rudraiah *et al.* 1985,Char and Chen(1997,1999), Ching2005, Rudraiah and Siddheshwar 2000, Awang and Hashim 2008).

However, the convective instabilities in much more realistic and complicated dynamical systems, instead of single fluid layer is of challenging one due to rapid development of modern techniques. Many authors have studied theoretically and experimentally by considering multilayer of fluid or a fluid layer separated at the middle or bounded from the above or below by a slab. Scriven and Sternling (1965) have considered a two layer fluid model in which each layer has infinite depth and examined the local behaviour of the system near the interface. Even though single layer systems and double layers systems heated from below have received a great deal of attention in the past, there have been very few studies related to the thermal instability and heat transfer phenomena in a system with fluid layer bounded by finite thickness slab.

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Yang (1992) considered the lower boundary to be a solid plate where it is a perfect insulating boundary condition for thermal disturbances, which is difficult compared to conducting boundary condition. It is found that the solid plate with a higher thermal conductivity tends to stabilize the system. The role of the plate thickness is minor in most of the Benard-Marangoni experiments, while the conductivity of the plate has a significant impact on the stability of the system. Char and Chen (1999) focused on Benard-Marangoni instability with a boundary slab of finite conductivity. They solved the problem numerically and later compared to the asymptotic of the long wavelength. It is shown that the critical Rayleigh number, R_c increases with thickness of solid layer to the thickness of fluid and thermal conductivity of solid layer to the thermal conductivity of fluid increases but decrease with $\Gamma = M/R$ provided the viscosity parameter, B is large. The effect of viscosity variation to the thermal conductivity and thickness of the boundary slab is also discussed in detail. When the viscosity parameter, B is small, it will raise the critical Rayleigh number, R_c provided the Biot number, Bi is small but behave oppositely when the viscosity parameter, B is large.

Most of the previous studies were concerned with a uniform vertical temperature gradient in the fluid layer, and as well it is of interest to know the influence of a non-uniform basic temperature gradient on the onset of convective instability. The problem of the effect of a non-uniform temperature gradient on the onset of Marangoni convection has also received considerable attention in the recent past (Nield 1975, Friedrich and Rudraiah 1984, Chen and Char 2003, Ching 2005). Friedrich and Rudraiah (1984) used the Galerkin technique to investigate the effects of rotation and non-uniform basic temperature gradient on the onset of steady Marangoni convection with different types of thermal boundary conditions in the absence of magnetic field. Rudraiah *et al.* (1985) have further studied the combined effect of magnetic field and non-uniform basic temperature gradient on steady Marangoni convection in the absence of rotation. The Marangoni stability in a temperature dependent viscosity fluid layer with a deformable free surface is investigated by Awang and Hashim (2008) and they have shown that the destabilizing effect of exponential viscosity variation can be effectively suppressed through small controlled perturbation in the thermal boundary data.

The present analysis aims to study the onset of Marangoni convective instability in a horizontal layer with the solid plate at the bottom surface and deformable upper free surface, subject to a non-uniform temperature gradient. The linear stability theory and the normal mode analysis are applied and the resulting eigen value problem is solved analytically. Of interest are the effects of the depth ratio and conductivity ratio, a free surface deformation, Bond number and the Biot number on the onset of Marangoni instability. Three different types of non-uniform temperature gradients are considered and their influences on the onset of Marangoni convection are also discussed in detail.

2. MATHEMATICAL FORMULATION

We consider an incompressible horizontal fluid layer of depth d, overlying a slab of thickness d_s. The lower hot rigid boundary $z = -d_s$ is kept at constant temperature T_0 and while the upper surface has a deflection $\Omega(x, y, t)$ from mean (see Fig.5.1). The upper surface $z = d$ is free to the atmosphere of constant temperature T_l. A Cartesian co-ordinate system (x, y, z) is chosen with origin above the slab of finite thickness and z-axis vertically upward. The surface tension σ is assumed to vary linearly with temperature in the form

$$\sigma = \sigma_0 - \sigma_T (T - T_0) \tag{1}$$

where σ_T is the rate of change of surface tension with temperature and σ_0 is a constant reference value.

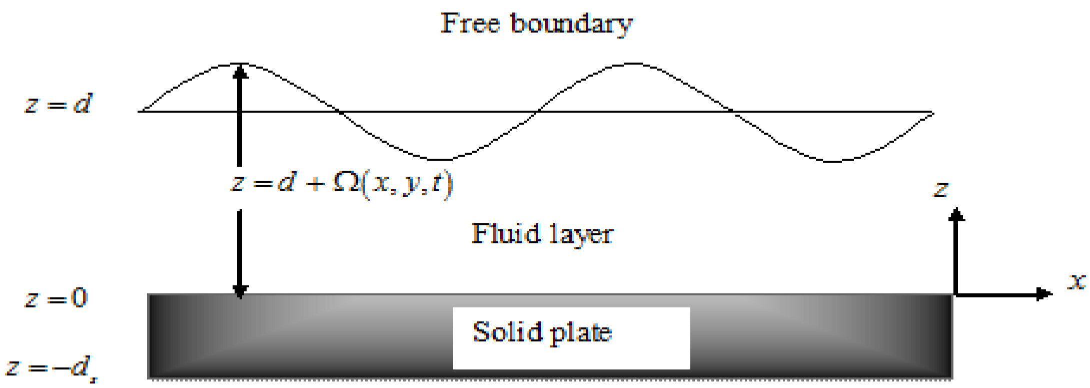

Fig. 1: Physical configuration

The governing equations for the fluid and the solid layers are:

Fluid layer:

$$\nabla \cdot \vec{q} = 0 \tag{2}$$

$$\rho_0 \left(\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \nabla \right) \vec{V} \right) = -\nabla p + \mu \nabla^2 \vec{V} + \rho_0 g \tag{3}$$

$$\frac{\partial T}{\partial t} + \left(\vec{V} \cdot \nabla \right) T = \kappa \nabla^2 T \tag{4}$$

Solid layer:

$$\frac{\partial T_s}{\partial t} = D_s \nabla^2 T_s \; . \tag{5}$$

Here $\vec{V}$ is the velocity vector, p is the pressure, T is the temperature, κ is the thermal diffusivity, while T_s is the temperature in the solid layer, μ is the fluid viscosity and ρ_0 is the fluid density and D_s is the thermal diffusivity of the solid plate.

The upper free surface of fluid layer is free of deformities with its position being $z = d + \Omega\left(x, y, t\right)$. The boundary conditions of velocity, heat flux and tangential and normal stresses at the free upper surface are

$$\frac{\partial \Omega}{\partial t} + u \frac{\partial \Omega}{\partial x} + v \frac{\partial \Omega}{\partial y} = w, \; k_t \nabla T . \hat{n} + HT = 0 \tag{6}$$

$$2\mu D_{nt} = \frac{\partial \sigma}{\partial T} \nabla T . \hat{t}, \;\; p_a - p + 2\mu D_{nn} = \sigma \nabla . \hat{n} \tag{7}$$

where k and H are the thermal conductivity and heat transfer coefficient of the fluid layer respectively. $\left\{ D_{ij} \right\}$ is the rate of strain in the fluid, and $\hat{t}$ and $\hat{n}$ are the tangential and outward normal unit vectors, respectively, at the free upper surface.

The governing equations admit a steady-state solution

$$V_b = 0, \quad -\frac{d}{\Delta T} \frac{dT_b}{dz} = f(z) \tag{8}$$

where $f(z)$ is the non-uniform basic temperature gradient due to the existence of the internal heat generation, which satisfies the condition

$$\int_0^1 f(z)\, dz = 1 \tag{9}$$

To investigate the effect of the non-uniform temperature gradient on the convective instability, the following three different basic temperature profiles are considered

i. $f(z) = 1$ represents a linear temperature profile;
ii. $f(z) = 2\left(1 - z\right)$ represents an inverted parabolic temperature profile;
iii. $f(z) = 2z$ represents a parabolic temperature profile.

In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced in the form

$$\vec{V} = \vec{V}', \quad T = T_b(z) + T', \quad p = p_b(z) + p' \tag{10}$$

where the primed quantities denotes the perturbed quantities over their equilibrium counterparts. Substituting (10) into $(2)-(5)$ and linearizing the equations in the usual manner, eliminating the pressure from the momentum equations by operating curl twice and only the vertical component is retained. The variables are then nondimensionalized using $d, d^2/\kappa, \kappa/d$ and ΔT as the units of length, time, velocity, and temperature, the non-dimensional disturbance equations are now given by

$$\left(\frac{1}{\Pr}\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2 w = 0 \tag{11}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T = f(z)\,w \tag{12}$$

$$\frac{\partial \theta_s}{\partial t} = \frac{D_s}{D_f}\nabla^2\theta_s \tag{13}$$

The boundary conditions are:

$$\frac{\partial \Omega}{\partial t} = w, \quad \frac{\partial T}{\partial z} + Bi\left(T - \Omega\right) = 0 \quad \text{at} \quad z = 1 \tag{14}$$

$$\left(\frac{\partial^2}{\partial z^2} - \nabla_h^2\right)w = M\,\nabla_h^2\left(T - \Omega\right) \quad \text{at} \ z = 1 \tag{15}$$

$$Cr\left[\frac{1}{\Pr}\frac{\partial}{\partial t} + \left(\frac{\partial^2}{\partial z^2} + 3\nabla_h^2\right)\right]\frac{\partial w}{\partial z} + \left(B_0 - \nabla_h^2\right)\nabla_h^2\Omega = 0 \quad \text{at} \ z = 1 \tag{16}$$

Here, $M = \sigma_T\,\Delta T\,d/\mu\kappa$ is the Marangoni number, $B_0 = \rho_0 g\,d^2/\sigma_T$ is the Bond number, which is measure of the flattening of the upper free surface by gravity and forming meniscus by the surface tension, $Bi = Hd/k_t$ is the Biot number representing the heat flux through upper free surface and $Cr = \mu\kappa/\sigma_T d$ is the Crispation number which show the idea of the rigidity of upper free surface of the fluid layer.

At the interface $z = 0$:

$$w = 0, \qquad Dw = 0 \tag{17}$$

$$T = \theta_s, \qquad DT = k_r\,D\theta_s \tag{18}$$

and at $z = -d_s$:

$$D\theta_s = 0. \tag{19}$$

Here, $k_r = k_s/k_f$ is the ratio of the thermal conductivity of the solid plate to that of the fluid layer, and $d_r = d_s/d$ is the ratio of the solid plate thickness to the liquid layer thickness. The operator $D = d/dz$ denotes differentiation with respect to z. Since the principle of exchange instability holds for surface tension driven convection in fluid layer (see Pearson 1958 and Vidal 1981), it is reasonable to assume that it holds good even for the present configuration as well. Hence, the time derivatives are dropped conveniently from Eqs. $(11)-(13)$. Then performing a normal mode expansion of the dependent variables in fluid layer as

$$(w, T, \theta_s) = \left[W(z), \Theta_f(z), \Theta_s(z)\right] \exp\left[i(lx + my)\right] \tag{20}$$

and substituting this in Eqs. $(11)-(13)$ (with $\partial/\partial t = 0$), we obtain the following ordinary differential equations:

$$\left(D^2 - a^2\right)^2 W = 0 \tag{21}$$

$$\left(D^2 - a^2\right)\Theta_f = -f(z)W \tag{22}$$

$$\left(D^2 - a^2\right)\Theta_s = 0. \tag{23}$$

The boundary conditions are

at $z = 1$:

$$W = 0,\ D\Theta_f + Bi(\Theta_f - Z) = 0 \tag{24}$$

$$\left(D^2 + a^2\right)W + Ma^2\left(\Theta_f - Z\right) = 0 \tag{25}$$

$$-Cr\left(D^2 - 3a^2\right)DW + \left(B_0 + a^2\right)a^2 Z = 0 \tag{26}$$

at $z = 0$:

$$W = 0, \qquad DW = 0 \tag{27}$$

$$\Theta_f = \Theta_s,\ D\Theta_f = k_r\, D\Theta_s \tag{28}$$

and at $z = -d_s$:

$$D\Theta_s = 0. \tag{29}$$

The perturbation equations $(21)-(23)$, subject to the boundary conditions $(24)-(29)$, constitute an eigen value system of eighth order. Solving the perturbation equation (23) for the solid layer, together with the boundary conditions (28) and (29), the thermal boundary condition at the solid-fluid interface, at $z = 0$ becomes

$$D\Theta_f = k_r\, a \tanh\left(a\, d_r\right)\Theta_f. \tag{30}$$

3. METHOD OF SOLUTION

It is possible to solve Eqs. (5.21) directly, we get the general solution in the form

$$W = A\cosh(az) + B\sinh(az) + C\, z\cosh(az) + D\, z\sinh(az) \quad (31)$$

where A, B, C *and* D are constants determined using the boundary conditions given by Eqs. (24) and (27) to obtain W as

$$W = B\left[\sinh(az) - a\,z\cosh(az) + (a\coth a - 1)\,z\sinh(az)\right]. \tag{32}$$

Equation (22) is solved using boundary conditions (24),(25) and (30), evaluated for various temperature profiles and expression for Marangoni number M are given in the following cases.

3.1 Linear Temperature Profile $f(z) = 1$

The expression for the Marangoni number M is given by

$$M = \frac{\lambda_5 + a^2\lambda_4}{a^2(\Delta_1 - \lambda_6)}. \tag{33}$$

From the above expression we note that in the absence of surface deflection and as $k_r \to 0$ and $d_r \to 0$, $M_c \to 48$. This is the known exact value (Pearson [76]).

Here

$$\Delta_1 = \frac{Cr}{a^2\left(B_0 + a^2\right)}\left(L_1 - 3a^2 L_2\right)$$

$$L_1 = \left(-a^4 + 3a^2 + (a\coth a - 1)\right)\sinh a + \left(a^3(a\coth a - 3)\right)\cosh a$$

$$L_2 = \left(a\coth a - 1 - a^2\right)\sinh a + (a^2\coth a - a)\cosh a$$

$$\lambda_1 = \left(\frac{a^2}{4} + \frac{a^2(a\coth a - 1)}{4}\right)\sinh a + \left(\frac{a}{2} - \frac{a^3}{4} + \frac{a^2\coth a}{4}\right)\cosh a + \lambda_2$$

$$\lambda_2 = \left[\left(-\frac{1}{4} - \frac{(a\coth a - 1)}{4a^2}\right)\sinh a + \left(\frac{1}{4a}(a\coth a - 2)\right)\cosh a + \Delta_1\right]$$

$$\lambda_3 = \frac{3a^3\sinh a + Bi\cosh a + 4\lambda_1 k_r\tanh(a\,d_r)}{4a^2\left(a^3\sinh a + Bi\cosh a + k_r\left(a^3\sinh a + Bi\cosh a\right)\tanh(a\,d_r)\right)}$$

$$\lambda_4 = \frac{4a^2\lambda_3 - 3}{4a^2 k_r\tanh(a\,d_r)}$$

$$\lambda_5 = \left(a^3\coth a - 2a^2\right)\sinh a + \left(2a^2\coth a - a^3 - 2a\right)\cosh a$$

$$\lambda_6 = \left(\lambda_3 a^2 + \frac{a^2}{4} + \frac{(a\coth a - 1)}{4}\right)\sinh a + \left(\lambda_4 a^2 - \frac{3a}{4} + \frac{a(a\coth a - 1)}{4}\right)\cosh a$$

3.2 Inverted Parabolic Temperature Profile $f(z) = 2(1 - z)$

The expression for the Marangoni number M is given by

$$M = \frac{\lambda_{11} + a^2\lambda_9}{a^2(\Delta_1 - \lambda_{10})} \tag{34}$$

where

$$\lambda_7 = \left(6a + 3a^3 + a^3(a\coth a - 1)\right)\sinh a + \left(3a^2 - a^4 + 3(a\coth a - 1)\right)\cosh a$$

$$\lambda_8 = (a\coth a - 1) - 3a^2 + \frac{1}{4}\left(2a^3 - 3a\right)(a\coth a - 3) + \Delta_1$$

$$\lambda_9 = \frac{2\lambda_7 + 6\lambda_8\cosh a}{12a^4\left(\sinh a + k_r\cosh a\tanh(a\,d_r)\right)}$$

$$\lambda_{10} = \left(\lambda_7 a^2 + \frac{a^2}{6} - 1\right)\sinh a + \left(\lambda_8 a^2 - \frac{a}{2} + \frac{\left(a^2 - 3\right)\left(a\coth a - 1\right)}{4}\right)\cosh a$$

$$\lambda_{11} = \frac{\left(2\lambda_8 k_r a^4 \tanh\left(a d_r\right) - \lambda_7\right)}{\sinh a + Bi\cosh a + k_r\left(a^3 \sinh a + Bi\cosh a\right)\tanh\left(a d_r\right)}$$

3.3. Parabolic Temperature Profile $f(z) = 2z$

The expression for the Marangoni number M is given by

$$M = \frac{\lambda_{14} + a^2\lambda_{15}}{a^2\left(\Delta_1 - \lambda_{16}\right)} \qquad (35)$$

where

$$\lambda_{12} = \left(\left(a\coth a - 1\right) - 3a^2\right)\sinh a + \left(3a \ \ - 3a^4 + 2\left(a\coth a - 1\right)\right)\cosh a + \Delta_1$$

$$\lambda_{13} = \frac{\left(-6a + \left(2a^3 - 3a\right)\left(a\coth a - 1\right)\right)\sinh a + \left(2a^2 + \left(3a^2 + 3\right)\left(a\coth a - 1\right)\right)\cosh a}{Bi\cosh a + k_r\left(a^3\sinh a + Bi\cosh a\right)\tanh\left(ad_r\right)}$$

$$\lambda_{14} = \frac{\lambda_{12} - 3\left(a\coth a - 1\right)\cosh a}{12a^4\left(\sinh a + k_r\cosh a\tanh\left(a d_r\right)\right)}$$

$$\lambda_{15} = \frac{\lambda_{12} - 6a^4\sinh a}{6a^4\cosh a} + \left(\frac{a^2 \ \cosh a}{2} - \frac{1}{a}\right)$$

$$\lambda_{16} = \frac{\lambda_{12} - 6a^4\sinh a}{6a^4\cosh a} + \left(6a + 3a^3 + a^3\left(a\coth a - 1\right)\right)\sinh a$$

The expression for Marangoni number is evaluated for different values of various physical parameters and the results are discussed in detail in the next section.

4. RESULT AND DISCUSSIONS

Effect of non-uniform basic temperature gradients on Marangoni convection with a boundary slab of finite conductivity is investigated theoretically. The resulting eigen value problem is solved exactly; the marginal curves in the plane (M, a) are obtained by using the expression given by Eqs. (33), (34) and (35), where M is a function of the parameters a, Cr, Bi, B_0, d_r and k_r. For a given set of parameters, the critical Marangoni number for the onset of convection is defined as the minimum of the global minima of marginal curve. We denote this critical value by M_c and the corresponding critical wave number by a_c.

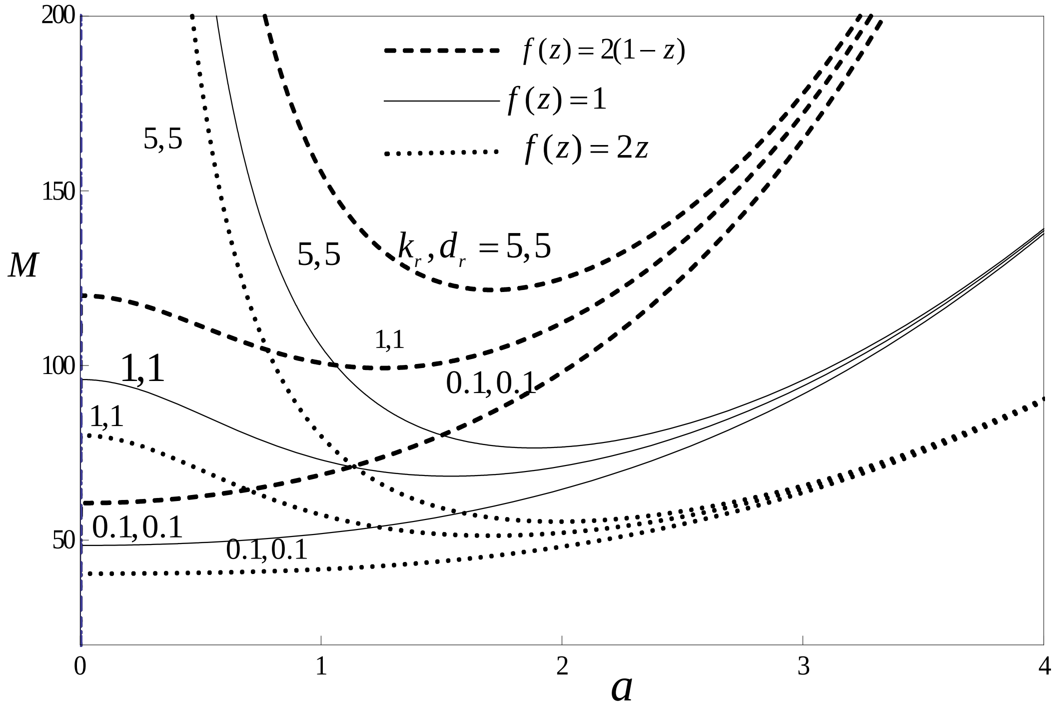

Fig. 2: Marginal stability curves M versus a for different values of k_r and d_r for $B_0 = 0.1$, $Cr = 10^{-5}, Bi = 0$ with different temperature profile.

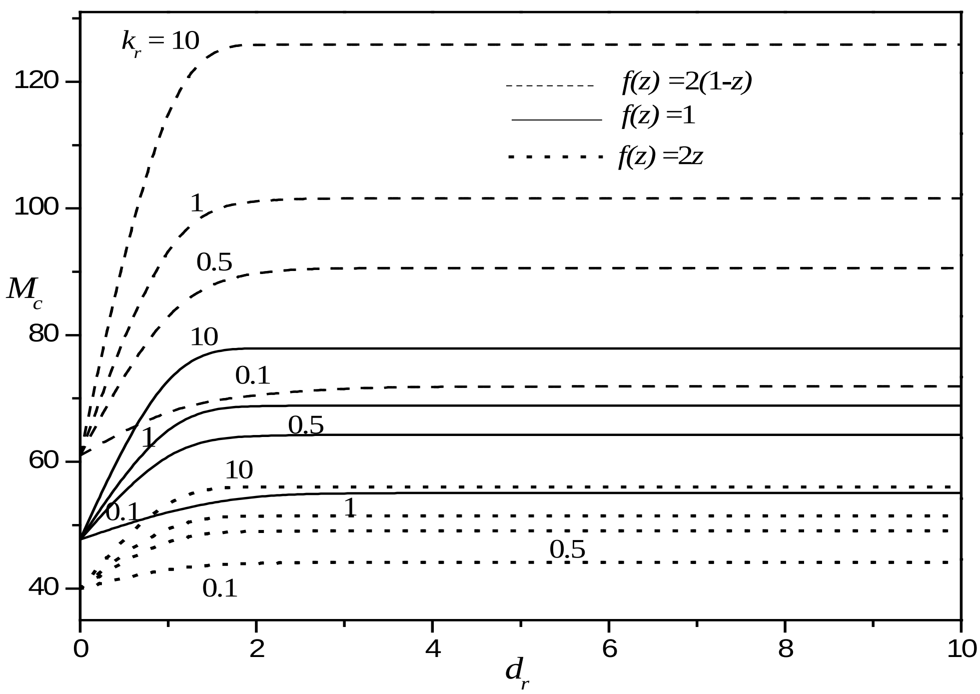

Fig. 3: Critical Marangoni number versus d_r for different values of k_r for $B_0 = 0.1$, $Cr = 10^{-5}$ with different temperature profile.

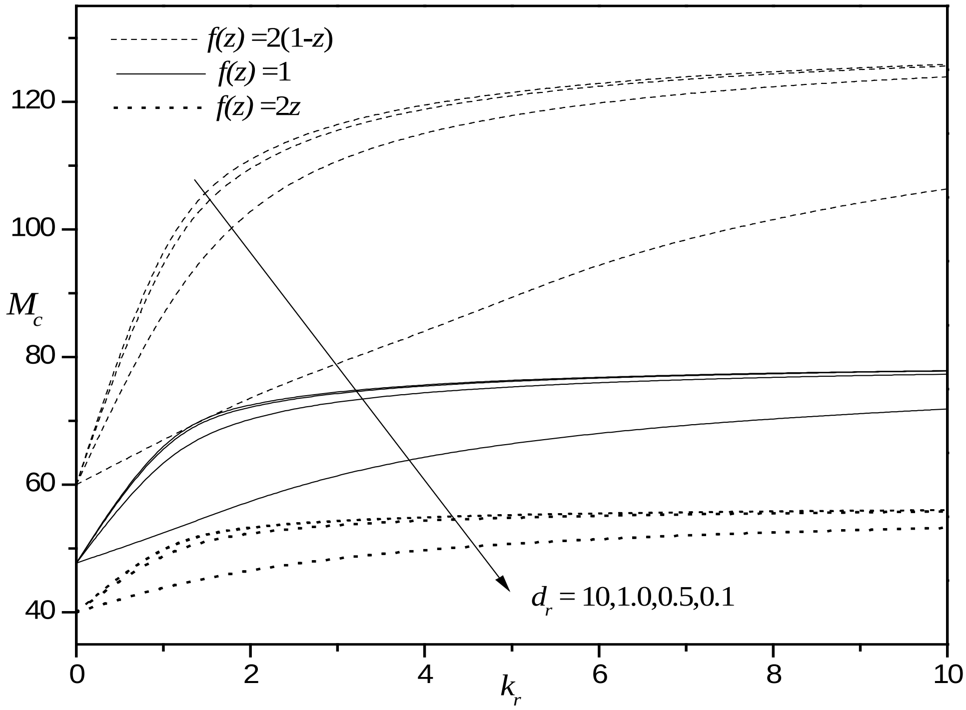

Fig. 4: Critical Marangoni number versus k_r for different values of d_r for $B_0 = 0.1,\ Cr = 10^{-5}$ with different temperature profile.

Numerically calculated values of M and the corresponding values of a are shown in Fig.2 for a range values of k_r and d_r respectively with different temperature profiles for $Bi = 0,\ B_0 = 0.1$ and $Cr = 10^{-5}$. From the Fig.2 it is seen that (i) the Marangoni number M reaches a maximum with the larger depth ratio d_r and thermal conductivity ratio k_r (ii) with the larger depth ratio dr and thermal conductivity ratio k_r, the global minimum occurs at short wavelength $(a \neq 0)$, and (iii) in comparison to the linear temperature profile, the inverted parabolic temperature profile shows higher Marangoni number, while the parabolic temperature profile lower ones.

Figure 3 shows the variation of M_c as a function of depth ratio d_r for different values of k_r with different temperature profile for $Bi = 0, B_0 = 0.1$ and $Cr = 10^{-5}$. It is noted that M_c increases with increase of k_r, the larger depth ratio d_r is stabilizing, since the fluid-solid interface tends to be isothermal instead, the critical Marangoni number M_c increases with d_r. In comparison to the linear temperature profile, the inverted parabolic temperature profile shows higher critical Marangoni number, while the parabolic temperature profile lower ones. Hence the inverted parabolic temperature profile is the most stabilizing basic temperature distribution, while the parabolic profile is the most destabilizing one among these three types of non-uniform basic temperature gradients.

Figure 4 shows the variation of M_c as a function of conductivity ratio k_r for different values of d_r with different temperature profiles for $Bi = 0,\ B_0 = 0.1$ and $Cr = 10^{-5}$. It is noted that M_c increases initially but the variation is negligible for further increase in the values of k_r. An increase in the thermal conductivity ratio k_r results in a stabilizing state, since thermal disturbances are easily dissipated deep in to the solid layer, and critical Marangoni number M_c increases.

In Fig.5 the critical Marangoni number M_c is plotted against the Crispation number Cr for different non-uniform basic temperature gradients for values of $k_r = 0 = d_r$ and $k_r = 1 = d_r$. It is observed that the critical Marangoni number M_c decreases with an increase of the Crispation number and thus making system more unstable. The reason being that an increase in Cr is to increase the deflection of the upper free surface, which in turn, promotes instability much faster. It is also evident from the figure that M_c decreases with increase of the k_r and d_r.

The effect of the Bond number B_0 on the Marangoni number M in the case of linear temperature profile for $Cr = 0.001,\ B_i = 0$ and $d_r = 1 = k_r$ is shown in Fig.6. It is observed that increase in the value of B_0 makes the system more stable. The reason for this may be attributed to the fact that an increase in the gravity effect, which keep the free surface flat against the effect of surface tension, which forms a meniscus on the free surface, and hence an increase in B_0 makes the system more stable.

Figure 7shows the plot of critical Marangoni number M_c as a function of depth ratio d_r for different values of thermal conductivity ratio k_r and Biot number Bi, in the case of linear temperature profile with $Cr = 10^{-5}$ and $B_0 = 0.1.$ It is seen that increasing Bi is to increase the critical Marangoni number and hence its effect is to delay the onset of Marangoni convection. This may be due to the fact that with increasing Bi, the thermal disturbances easily dissipate into the ambient surrounding due to a better convective heat transfer coefficient at the top free surface and hence makes the system more stable. Figure 8 shows the plot of critical Marangoni number M_c as a function of depth ratio d_r for different values thermal conductivity ratio k_r and Biot number Bi, in the case of linear temperature profile with $Cr = 10^{-5},\ B_0 = 0.1$ and the results are qualitatively similar to results of Fig. 7.

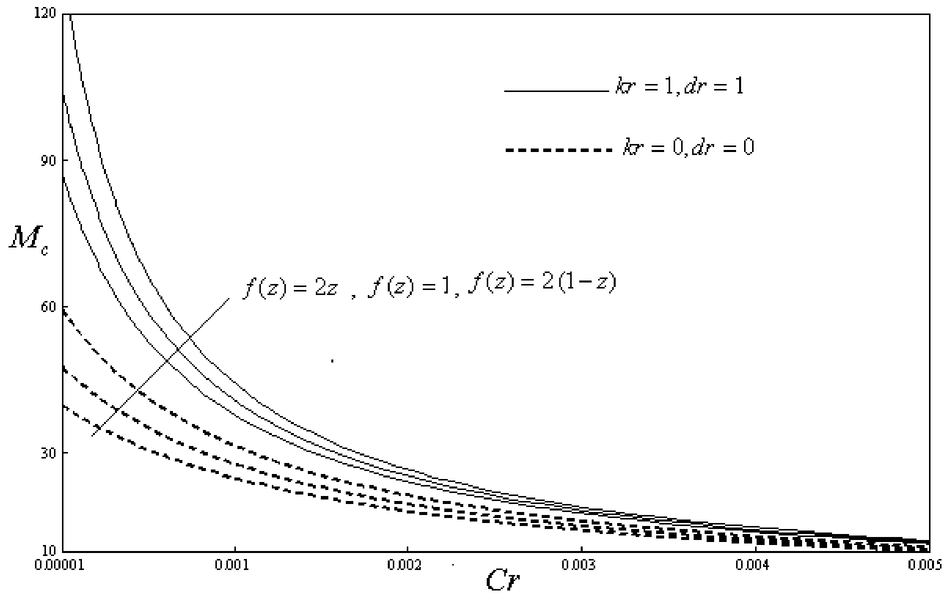

Fig. 5: Critical Marangoni number versus Cr for different values of d_r and k_r for $B_0 = 0.1$ with different temperature profile.

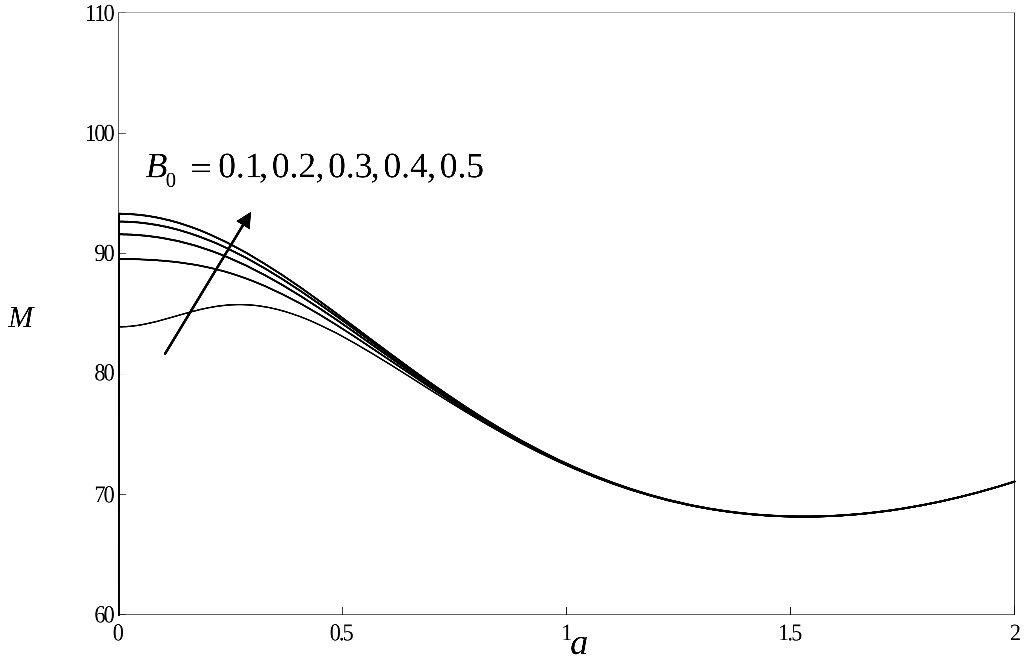

Fig. 6: Marangoni number versus a for different values of B_0 with $Cr = 0.001$, $d_r = 1 = k_r$, for linear temperature profile.

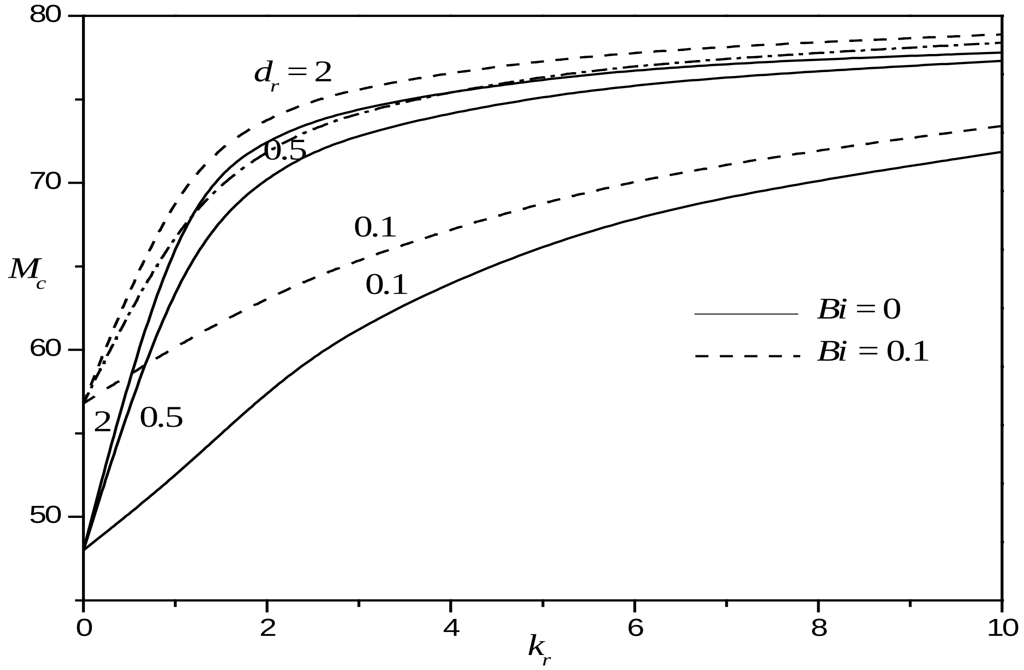

Fig.7: Critical Marangoni number versus k_r for different values of d_r with $B_0 = 0.1$ and $Cr = 10^{-5}$ for linear temperature profile.

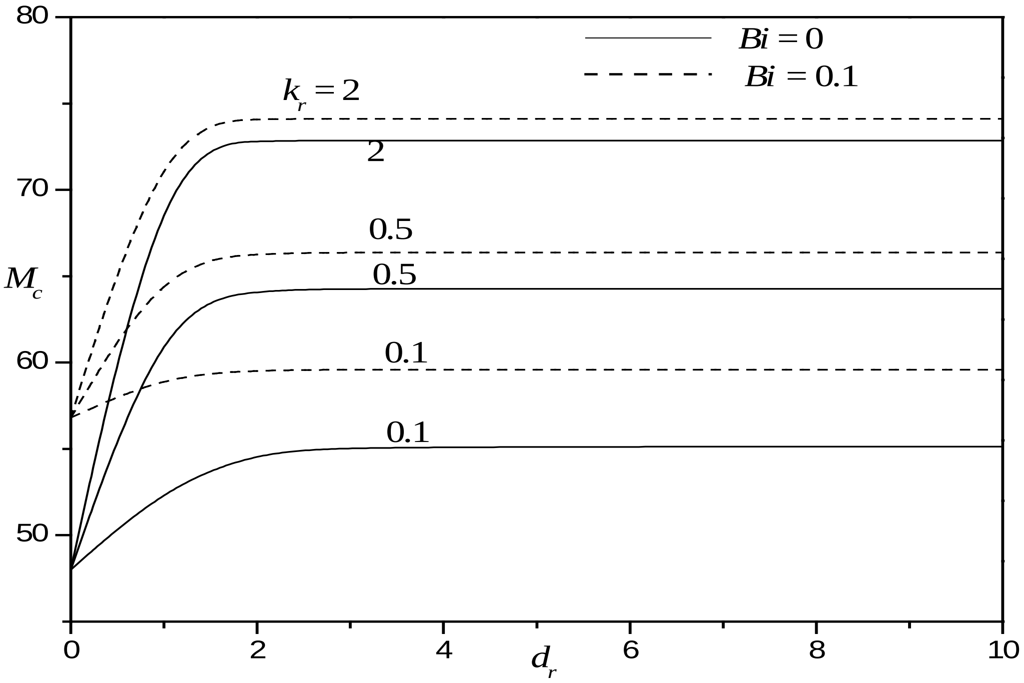

Fig. 8: Critical Marangoni number versus d_r for different values of k_r with $B_0 = 0.1$ and $Cr = 10^{-5}$ for linear temperature profile.

5. CONCLUSIONS

The problem of Marangoni convection in an horizontal fluid layer with deformable to upper free surface and lower boundary is considered to be a thin slab of finite conductivity has been studied theoretically. Of interest are the influences of non-uniform basic temperature gradients, depth ratio, thermal conductivity ratio, the Crispation number, Bond number and the Biot number on the onset of Marangoni instability. The following conclusions may be made from this study.

1. In comparison to the linear temperature profile, the inverted parabolic temperature profile shows higher values of critical Marangoni numbers, while the parabolic temperature profile shows lower ones. Hence the inverted parabolic temperature profile is the most stabilizing basic temperature distribution, while the parabolic profile is the most destabilizing one among these three types of non-uniform basic temperature gradients.

2. Critical Marangoni number increases with the increase of depth ratio and thermal conductivity ratio for all temperature profiles.

3. The effects of the Bond number and the Biot number on the onset of Marangoni convections are more pronounced. The system become more stable as the Biot number and the bond number increases.

4. The critical Marangoni number increase as the Crispation number decreases. Hence system become more unstable as Crispation number increases.

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