International Journal of Mathematical Archive-4(6), 2013, 151-154
Available online through www.ijma.info ISSN 2229 – 5046

GENERALIZED PRE-SEMI CLOSED SETS IN TOPOLOGICAL SPACES

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(Received on: 24-04-13; Revised & Accepted on: 24-05-13)

ABSTRACT

In this paper we introduce a new class of sets called generalized pre-semi closed sets (briefly gps-closed) in a topological space and discuss some of the basic properties of generalized pre semi-closed sets.

***Mathematics Subject Classification:** 54A05.*

***Keywords:** semi-open, gps-closed.*

1. INTRODUCTION

Levine [6] introduced the concept of semi-open sets and semi-continuity in topological spaces in 1963. In 1970 Levine [5] introduced the notion of generalized closed (briefly g-closed) set. Sundaram and Shiek John [10] introduced the concepts of weakly-closed set and Maki et al[7] introduced the notion of generalized pre-closed sets and investigated some of their basic properties. In 1995 Dontchev introduced the concept of generalized semi pre-open sets in topological spaces. Gnanambal [4] introduced the concept of gpr-closed sets and studied its properties in topological spaces in 1997.

In this paper we define and study the properties of generalized pre semi-closed sets (gps-closed) in a topological space, which is properly placed between pre-closed sets and generalized pre-closed sets.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space (X,τ) is called

i) a semi-open set[6] if $A \subseteq cl(int(A))$ and a semi-closed. if $int(cl(A)) \subseteq A$,
ii) a regular open set[11] if $A = int(cl(A))$ and a regular closed set. if $A = cl(int(A))$,
iii) a pre-open set[8] if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$,

Definition 2.2: A subset A of a topological space (X, τ) is called regular semi-open [2] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$. The family of all regular semi-open sets of X is denoted by RSO(X).

Definition 2.3: A subset A of a topological space (X,τ) is called

i) a generalized closed set (briefly g-closed)[5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,
ii) a weakly closed set (briefly ω-closed)[10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open,
iii) a generalized preclosed set(briefly gp-closed)[7] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,
iv) a generalized semi- preclosed set(briefly gsp-closed)[3] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,
v) a generalized pre regular weakly closed set (briefly gprw-closed)[9] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open,
vi) a strongly generalized closed set (briefly g^*-closed)[12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open,
vii) *g-closed set[12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ω-open,
viii) a generalized pre regular closed set (briefly gpr-closed) [4] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open,
ix) a regular weakly closed set (briefly rw-closed)[1] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open.

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The semi-closure (pre-closure, semi-pre-closure and α-closure respectively) of a subset of a space (X,τ) is the intersection of all semi-closed (pre-closed, semi-pre-closed and α-closed respectively) sets that contain A and is denoted by scl(A) (pcl(A), spcl(A) and αcl(A) respectively).

3. GENERALIZED PRE-SEMI CLOSED SETS

Definition 3.1: A subset A of a topological space (X,τ) is called generalized pre-semi closed (briefly gps-closed) if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open.

Theorem 3.2: Every closed set is gps-closed.

Proof: Since every closed set is pre-closed and pcl(A)=A, the result follows from the definitions.

The following example shows that the above theorem need not be true.

Example 3.3: Let X= {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, X}. Then {c} is gps-closed but not closed.

Theorem 3.4: Every gps-closed set is gpr-closed

Proof: Let A be gps-closed and A $\subseteq$U where U is regular open. Since every regular open set is semi-open and A is gps-closed we have pcl(A) $\subseteq$ U. Hence A is gpr-closed.

The following example shows that the above theorem need not be true.

Example 3.5: Let X= {a, b, c} and τ = {ϕ, {a}, {b}, {a, b}, X}. Then {a, b} is gpr-closed but not gps-closed in (X, τ).

Theorem 3.6: Every gps-closed set is gp-closed.

Proof: Let A be gps-closed and A $\subseteq$U where U is open. Since every open is semi-open, pcl(A) $\subseteq$ U. Hence A is gp-closed.

The following example shows that the above theorem need not be true.

Example 3.7: Let X= {a, b, c} and τ = { ϕ, {c}, {a, c}, X}. Then {b, c} is gp-closed but not gps-closed in (X, τ).

Theorem 3.8: Every ω-closed set is gps-closed.

Proof: Let A be ω-closed and A $\subseteq$U where U is semi-open. Since every closed is pre-closed set, pcl(A) $\subseteq$ cl(A), so pcl(A) $\subseteq$U. Hence A is gps-closed.

The following example shows that the above theorem need not be true.

Example 3.9: Let X = {a, b, c} and τ = {ϕ, {b}, X}. Then {a} is gps-closed but not ω-closed.

Theorem 3.10: Every gps-closed set is gprw-closed:

Proof: Let A be a gps-closed set and A $\subseteq$U where U is regular semi-open. Every regular semi-open set is semi-open. Hence pcl(A)$\subseteq$U. Thus A is gprw-closed.

The following example shows that the above theorem need not be true.

Example 3.11: Let X = {a, b, c, d} and τ = {ϕ, {b}, X}. Then {a, b} is gprw-closed but not gps-closed.

Theorem 3.12: Every gps-closed is gsp-closed.

Proof: Let A be gps-closed and A$\subseteq$U where U is open. Since every open is semi-open and spcl(A) $\subseteq$ pcl(A), pcl(A) $\subseteq$U, we have spcl(A) $\subseteq$U where U is semi-open. Hence A is gsp-closed.

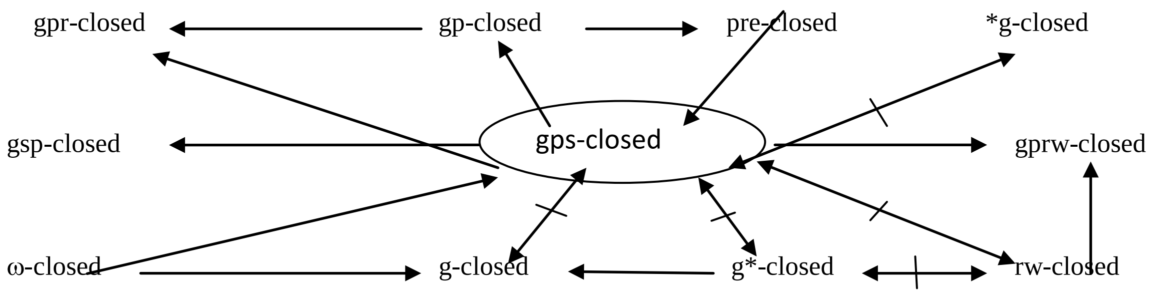
The following examples show that the above theorem need not be true.

Example 3.13: Let X = {a, b,c} and τ={ϕ, {a}, {c}, {a, c}, X}. Here {a} is gsp-closed but not gps-closed.

Remark 3.14: The following example shows that gps-closed is independent of g-closed, g*-closed, rw-closed and *g-closed.

Example 3.15: Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. Then the collection of
i) closed sets in (X, τ) is {ϕ, {d}, {c, d}, {a, c, d}, {b, c, d}, X}.
ii) gps-closed sets in (X, τ) is { ϕ, {c}, {d}, {c, d}, {b, c, d}, {a, c, d}, X}
iii) g-closed sets in (X, τ) is { ϕ, {d}, {c, d}, {a, d}, {b, d}, {b, c, d}, {a, b, d}, {a, c, d}, X}
iv) g*-closed sets in (X, τ) is { ϕ, {d}, {c, d}, {a, d}, {b, d}, {a, b, d}, {a, c, d}, {b, c, d}, X}
v) rw-closed sets in (X, τ) is {ϕ, {d}, {a, b}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X}
vi) *g-closed sets in (X, τ) is { ϕ, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, X}.

From the above discussion and known results we have the following implications.



Remark 3.16: Union of two gps-closed sets need not be gps-closed.

Example 3.17: Let X= {a, b, c} and τ= {ϕ, {a}, {b}, {c}, {b, c}, {a, c}, X}. Then A= {a} and B= {b} are gps-closed but their union A ∪ B= {a, b} is need not gps-closed.

Theorem 3.18: Intersection of two gps-closed sets is gps-closed.

Proof: Let A and B be two gps-closed sets. Let A∩B ⊆ U, where U is semi-open. We have pcl (A) ⊆U and pcl(B)⊆U. Thus pcl(A∩B)⊆ pcl(A) ∩ pcl(B) ⊆ U . Hence A ∩B is gps-closed.

Theorem 3.19: Let A be a gps-closed in (X, τ) then pcl(A)-A does not contain any non empty semi-closed.

Proof: Let F be a semi-closed set such that F ⊆ pcl(A) − A. Then F ⊆ X−A implies A ⊆ X−F. Since A is gps-closed and X−F is semi open. Therefore pcl(A) ⊆X−F. That is F ⊆ X−pcl(A). Hence F ⊆ pcl(A) ∩(X−pcl(A)) = ϕ. This shows F =ϕ.

Theorem 3.20: Let A be gps-closed in (X, τ). Then A is pre-closed if and only if pcl(A) - A is semi-closed.

Proof: (Necessity) Let A be gps-closed. Assume that A is pre-closed. Then pcl(A) = A and so pcl(A) − A= ϕ. Hence pcl(A) - A is semi-closed (Sufficiency): Assume that pcl(A) − A is semi-closed. Then pcl(A)−A =ϕ since A is gps-closed. That is pcl(A)=A. Hence A is pre-closed.

Theorem 3.21: If A is semi-open and gps-closed, then A is pre-closed.

Proof: Since A is semi-open and A ⊆ A. we have pcl(A)⊆A, since A is gps-closed. Therefore pcl(A)=A, which means that A is pre-closed.

Theorem 3.22: If A is gps-closed then $scl(\{x\}) \cap A \neq \phi$, for each $x \in pcl(A)$.

Proof: Let $x \in pcl(A)$. If $scl(\{x\}) \cap A = \phi$, then A $\subseteq scl(\{x\})^c$. So $pcl(A) \subseteq scl(\{x\})^c$. Since $x \in pcl(A)$, which implies $x \in scl(\{x\})^c$ which is a contradiction. Hence $scl(\{x\}) \cap A \neq \phi$.

Theorem 3.23: Let A be a subset of X. If $scl(\{x\}) \cap A \neq \phi$, for each $x \in scl(A)$ then $scl(A) - A$ contains no non empty semi closed set.

Proof: Let F be a semi-closed set such that $F \subseteq scl(A) - A$. If there exists a $x \in F$, then $\phi \neq scl(\{x\}) \cap A \subseteq F \cap A \subseteq scl(A) - A$, a contradiction. Hence $F = \phi$.

Theorem 3.24: If A is a gps-closed subset of X such that A $\subseteq$B $\subseteq$ pcl(A) then B is a gps-closed set in X.

Proof: Let A be gps-closed set of X such that A $\subseteq$B $\subseteq$ pcl(A). Let U be a semi-open set of X such that B $\subseteq$U. Then A $\subseteq$U. Since A is gps-closed, we have pcl(A) $\subseteq$U. Now pcl(B) $\subseteq$ pcl(pcl(A)) = pcl(A) $\subseteq$U. Therefore B is gps-closed set in X.

Theorem 3.25: For every point x of a space X, $X - \{x\}$ is gps-closed or semi-open.

Proof: Suppose $X - \{x\}$ is not semi-open. Then X is the only semi-open set containing $X - \{x\}$. This implies pcl($X - \{x\}$) $\subseteq$X. Hence $X - \{x\}$ is gps-closed set in X.

Theorem 3.26: If A is semi-open and gps-closed, then A is pre-closed.

Proof: Suppose A is semi-open and gps-closed. As A $\subseteq$ A, we have pcl(A) $\subseteq$A. This implies A is pre-closed.

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Source of support: Nil, Conflict of interest: None Declared