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MAGNETIC FIELD EFFECT ON TRANSIENT FREE CONVECTION FLOW THROUGH POROUS MEDIUM PAST AN IMPULSIVELY STARTED VERTICAL PLATE WITH FLUCTUATING TEMPERATURE AND MASS DIFFUSION

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ABSTRACT

In this paper mass transfer effects on MHD, radiative free convection flow through porous medium past an impulsively started vertical plate with variable temperature and mass diffusion in presence of transverse magnetic field is investigated. The non-dimensional governing equations are solved by the usual Laplace transform technique. The effects of various physical parameters viz., radiation parameter N, Grashof number Gr, modified Grashof number Gm, Prandtl number Pr, Schmidt number Sc, time t, permeability parameter K and magnetic parameter M on velocity field, temperature field and concentration are studied through graphs. Expression for Skin friction is also derived and discussed. It is noticed that velocity decreases with an increase in M, Gm, N and Gr where as it shows reverse effect in case of K.

Key words: *MHD, Mass transfer, Radiation, Porous medium and vertical plate.*

1. INTRODUCTION

The Problem of MHD laminar flow through a porous medium has become very important in recent years because of its possible applications in many branches of Science and Technology, particularly in the field of Agricultural Engineering to study the underground water resources, seepage of water in river beds; In Chemical Engineering for filtration and purification process; In Petroleum Technology to study the movement of natural gas, oil and water through the oil reservoirs. Feike *et al.* [1] first discussed analytical solutions for solute transport in three dimensional semi-infinite porous media. Ling *et al.* [2] discussed steady mixed convection boundary layer flow over a vertical flat plate in a porous medium filled with water at 4^0c: case of variable wall temperature. Later Ling *et al.* [3] discussed steady mixed convection boundary-layer flow over a vertical flat surface in a porous medium filled with water at 4^0c variable surface heat flux.Palani *et al.* [4] studied "Free Convection MHD Flow with Thermal Radiation From An Impulsively-Started Vertical Plate. Muthucumaraswamy *et al.* [5], investigated radiation effects on flow past an impulsively started infinite vertical plate with variable temperature. Prasad *et al.* [6] considered radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate. Muthucumaraswamy and Chandrakala studied MHD Flow Past an Impulsively Started Vertical Plate with. Variable Temperature and Uniform Mass Diffusion. Unsteady MHD free convection oscillatory couette flow through a porous medium with periodic wall temperature was studied by Raju *et al.* [8-9]. Ravikumar *et al.* [10-11], considered heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in presence of temperature dependent heat source. Heat and Mass transfer effects on unsteady free convection boundary layer flow past an impulsively started vertical surface with Newtonian heating was investigated by Raju *et al.* [12]. Recently Ravikumar *et al.* [13], investigated MHD double diffusive and chemically reactive flow through porous medium bounded by two vertical plates. Motivated by the above cited work, in this paper we have considered MHD flow past an impulsivelystarted vertical plate with variable temperature and variablemass diffusion in porous medium analytically using Laplace transform technique.

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2. MATHEMATICAL ANALYSIS

In this paper we have considered the flow of unsteady viscous incompressible fluid. The x-axis is taken along the plate in the upward direction and y-axis is taken normal to the plate. Initially the fluid and plate are at the same temperature. A transverse magnetic field B_0, of uniform strength is applied normal to the plate. The viscous dissipation and induced magnetic field has been neglected due to its small effects. Initially the fluid and plate are at the same temperature T_∞ and concentration C_∞ in the stationary condition. At time t>0, Temperature of the plate is raised to T_w and the concentration level near the plate is raised linearly with respect to time. The flow modal is as under.

$$\frac{\partial u}{\partial t} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu}{k}u \tag{1}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} \tag{2}$$

$$\rho C_p \frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where C_p-specific heat at constant pressure T_∞- temperature of the fluid near the plate; C-species concentration in the fluid; T-temperature of the fluid far away from the plate; C_∞-concentration in the fluid far away from the plate; t-time; ρ-fluid density; g-acceleration due to gravity; β-volumetric coefficient of the thermal expansion; β*- volumetric coefficient of concentration expansion; ν - kinematics viscosity; β_0–external magnetic field; σ - Stefan – Boltzmann constant; u - velocity of the fluid in the x - direction, κ - thermal conductivity of the fluid; y -co-ordinate axis normal to the plate and D- mass diffusion coefficient.

The following boundary conditions have been assumed:

$$t \le 0 \; : u = 0, T = T_\infty, \; C = C_\infty \text{ for all values of y} \tag{4}$$

$$t > 0: \quad u = u_0, T = T_\infty + (T_w - T_\infty)\frac{u_0^2 t}{\nu}, C = C_\infty + (C_w - C_\infty)\frac{u_0^2 t}{\nu} \; at \; y = 0$$

$$u \to 0, \; T \to T_\infty, \; C \to C_\infty \quad as \quad y \to \infty$$

Introducing the following non-dimensional quantities:

$$\bar{y} = \frac{y u_0}{\nu}, \; \bar{u} = \frac{u}{u_0}, \; \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \; Sc = \frac{\nu}{D}, \; \mu = \rho\nu$$

$$P_r = \frac{\mu C_p}{\kappa}, \; G_r = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, \; M = \frac{\sigma B_0^2}{\rho u_0^2}, \quad G_m = \frac{g\beta^*\nu(C_w - C_\infty)}{u_0^3} \tag{5}$$

$$\bar{C} = \frac{(C - C_\infty)}{(C_w - C_\infty)}, \; \bar{t} = \frac{t u_0^2}{\nu}, \quad K = \frac{k u_0^2}{\nu^2}$$

With symbol : $\bar{u}$ - dimensionless velocity; P_r -Prandtl number; M-magnetic field parameter; $\bar{y}$ - dimensionless co-ordinate axis normal to the plate; θ -dimensionless temperature; G_r -thermal Grashof number; G_m - mass Grashof number; S_c - Schmidt number; $\bar{C}$ -dimensionless concentration and μ - coefficient of viscosity.

Equatios (1), (2) and (3) leads to

$$\frac{\partial \bar{u}}{\partial \bar{t}} = G_r\theta + G_m\bar{C} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - M_1\bar{u} \text{ Where } M_1 = M^2 + \frac{1}{k} \tag{6}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c} + \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \tag{7}$$

and $\frac{\partial\theta}{\partial\bar{t}} = \frac{1}{P_r} + \frac{\partial^2\theta}{\partial\bar{y}^2}$ (8)

With the following boundary conditions:

$$\bar{t} \le 0: \quad \bar{u} = 0, \quad \theta = 0, \quad \bar{C} = 0 \text{ for all values of } \bar{y}$$
$$\bar{t} > 0: \quad \bar{u} = 1, \quad \theta = \bar{t}, \quad \bar{C} = \bar{t}, \quad at\ y = 0 \tag{9}$$
$$\bar{u} \to 0, \quad \theta \to 0, \quad \bar{C} \to 0 \quad as\ y \to \infty$$

Dropping bars in the above equations, we have

$$\frac{\partial u}{\partial t} = G_r\theta + G_m C + \frac{\partial^2 u}{\partial y^2} - M_1 u \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} + \frac{\partial^2 C}{\partial y^2} \tag{11}$$

and $\frac{\partial\theta}{\partial t} = \frac{1}{P_r} + \frac{\partial^2\theta}{\partial y^2}$ (12)

With the following boundary conditions:

$$t \le 0: u = 0, \theta = 0, C = 0 \text{ for all values of } \bar{y}$$

$$t > 0: \quad u = 1, \quad \theta = t, \quad C = t, \quad \text{at } \bar{y} = 0 \tag{13}$$

$$u \to 0, \quad \theta \to 0, \quad C \to 0 \text{ as } \bar{y} \to \infty$$

The dimensionless governing equations (10) to (12), subject to the boundary conditions (13), are solved by the usual Laplace – transform technique with some help from the article “An algorithm for generating some inverse Laplace transforms of exponential from” by Hetnarski, R.B.[2]. The solutions are derived as below.

$$\bar{\theta} = t\left[\left(1 + \frac{y^2 pr}{2t}\right) erfc\left(\frac{y\sqrt{pr}}{2\sqrt{t}}\right) - \frac{y\sqrt{pr}}{\sqrt{\pi t}} \exp\left(\frac{-k^2 pr}{4t}\right)\right] \tag{14}$$

$$\bar{C} = t\left[\left(1 + \frac{y^2 s_c}{2t}\right) erfc\left(\frac{y\sqrt{s_c}}{2\sqrt{t}}\right) - \frac{y\sqrt{s_c}}{\sqrt{\pi t}} \exp\left(\frac{-y^2 s_c}{4t}\right)\right] \tag{15}$$

$$\begin{aligned} u = {} & \mathrm{G_1T_1}(y,t) + \mathrm{G_2T_2}(\mathrm{y,t}) + \mathrm{G_3T_3}(\mathrm{y,t}) - \mathrm{G_4T_4}(\mathrm{y,t}) - \mathrm{G_2T_5}(\mathrm{y,t}) \\ & + \mathrm{G_2T_6}(\mathrm{y,t}) + \mathrm{a}\ \mathrm{G_2T_7}(\mathrm{y,t}) - \mathrm{G_3T_8}(\mathrm{y,t}) + \mathrm{G_3T_9}(\mathrm{y,t}) + \mathrm{G_3T_{10}}(\mathrm{y,t}) \end{aligned} \tag{16}$$

where $T_1 = \frac{1}{2}\left[e^{-y\sqrt{M_1}} erfc(\eta - \sqrt{M_1 t}) + e^{y\sqrt{M_1}} erfc(\eta + \sqrt{M_1 t})\right]$

$$T_2 = \frac{e^{at}}{2}\left[e^{-y\sqrt{a+M_1}} erfc(\eta - \sqrt{(a+M_1)t}) + e^{y\sqrt{a+M_1}} erfc(\eta + \sqrt{(a+M_1 t)})\right]$$

$$T_3 = \frac{e^{bt}}{2}\left[e^{-y\sqrt{b+M_1}} erfc(n - \sqrt{(b+M_1)t}) + e^{y\sqrt{b+M_1}} erfc(n + \sqrt{(b+M_1 t)})\right]$$

$$T_3 = \frac{e^{bt}}{2}\left[e^{-y\sqrt{b+M_1}} erfc(n - \sqrt{(b+M_1)t}) + e^{y\sqrt{b+M_1}} erfc(n + \sqrt{(b+M_1 t)})\right]$$

$$T_4 = \frac{e^{at}}{2}\left[e^{-y\sqrt{a+M_1}} erfc(\eta - \sqrt{(a+M_1)t}) + e^{y\sqrt{a+M_1}} erfc(\eta + \sqrt{(a+M_1 t)})\right]$$

$$T_5 = \frac{e^{at}}{2}\left[e^{-y\sqrt{p_r}\sqrt{a}} erfc(\eta\sqrt{p_r} - \sqrt{at}) + e^{y\sqrt{p_r}\sqrt{a}} erfc(\eta\sqrt{p_r} + \sqrt{at})\right]$$

$$T_6 = erfc(\eta\sqrt{p_r})$$

$$T_7 = t\left[(1+2\eta^2 p_r) erfc(\eta\sqrt{p_r}) - 2\eta\sqrt{\frac{p_r}{\pi}} e^{-\eta^2 p_r}\right]$$

$$T_8 = \frac{e^{bt}}{2}\left[e^{-y\sqrt{S_c}\sqrt{b}} erfc(\eta\sqrt{S_c} - \sqrt{bt}) + e^{y\sqrt{S_c}\sqrt{b}} erfc(\eta\sqrt{S_c} + \sqrt{bt})\right]$$

$$T_9 = erfc(\eta\sqrt{S_c})$$

$$T_{10} = t\left[(1+2\eta^2 S_c) erfc(\eta\sqrt{S_c}) - 2n\sqrt{\frac{S_c}{\pi}} e^{-\eta^2 S_c}\right]$$

SKIN FRICTION

We now study skin friction from velocity field. It is given in non-dimensional for as

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} \tag{17}$$

Therefore using equation (14), we get τ :

$$\begin{aligned}
\tau = {} & 2G_1\left[\sqrt{M_1} erf\left(\sqrt{M_1 t}\right) + \frac{e^{-M_1 t}}{\sqrt{\pi t}}\right] - \frac{2tG_r}{M_1}\sqrt{\left[\frac{tp_r}{\pi}\right]} - \frac{2tG_m}{M_1}\sqrt{\left[\frac{ts_c}{\pi}\right]} \\
& + \left(\frac{G_r + G_m}{M_1}\right)\left(2t\sqrt{M_1} + \frac{1}{2\sqrt{M_1}}\right) erf\left(\sqrt{M_1 t}\right) + \frac{2\sqrt{t}e^{-M_1 t}}{\sqrt{\pi}} \\
& - G_2\left[e^{at}\sqrt{(M_1+a)} erf\left(\sqrt{(M_1+a)t}\right) + \frac{e^{-M_1 t}}{\sqrt{\pi t}} - 2\sqrt{apr} erf(\sqrt{at}) - 2\sqrt{\frac{p_r}{\pi t}} e^{-at} + \sqrt{\frac{p_r}{\pi t}}\right] \\
& - G_3\left[e^{bt}\sqrt{(M_1+b)} erf(\sqrt{(M_1+b)t} + \frac{e^{-M_1 t}}{\sqrt{\pi t}} - 2\sqrt{bS_c} erf(\sqrt{bt}) - 2\sqrt{\frac{S_c}{\pi t}} e^{-bt} + \sqrt{\frac{S_c}{\pi t}}\right]
\end{aligned}$$

3. RESULTS AND DISCUSSIONS

In order to get the physical insight of the problem, numerical computations have been carried out for velocity, temperature, concentration, skin friction and the effects of various physical parameters on flow quantities are studied though graphs. The values of Pr are chosen as 0.71 and 7.0 which corresponds to air and water respectively. The values of Schmidt number are chosen as 0.60, 0.78, 0.96 which correspond to water vapour, NH_3 and CO_2respectively. The values of Reynolds number Re, Grashof number Gr, Grashof number for mass transfer Gm, Hartmann number M, Permeability parameter K, and Schmidt number Sc are chosen arbitrarily. Velocity profiles are displayed from figures 1 to 7. From these figures it is noticed that Velocity increases with increase in t, Pr, K but it shows the reveres effect in case of M, Re, Gr, Gm and Sc. It is due to the application of transvers magnetic field which acts as Lorentz's force that retards the flow. The variations of temperature under the influence of Pr are presented in figure 8. From thisfigures it is observed that temperature decreases with an increase in Pr. Concentration profiles are displayed in figure9. From this figures it is observed that concentration decreases with increase in Sc. That is the low viscosity or low mass diffusivity raises the concentration level of the fluid.

4. CONCLUSION

In this paper mass transfer effects on MHD free convection flow through porous medium past an impulsively started vertical plate with variable temperature and mass diffusion in presence of transverse magnetic field is investigated. The non-dimensional governing equations are solved by the usual Laplace transform technique. In the analysis of the flow the following conclusions are made.

i. Velocity increases with increase in t, Pr, K but it shows the reveres effect in case of M, Re, N, Gr, Gm and Sc.
ii. Temperature decreases with an increase in Pr and N
iii. Concentration decreases with increase in Sc.

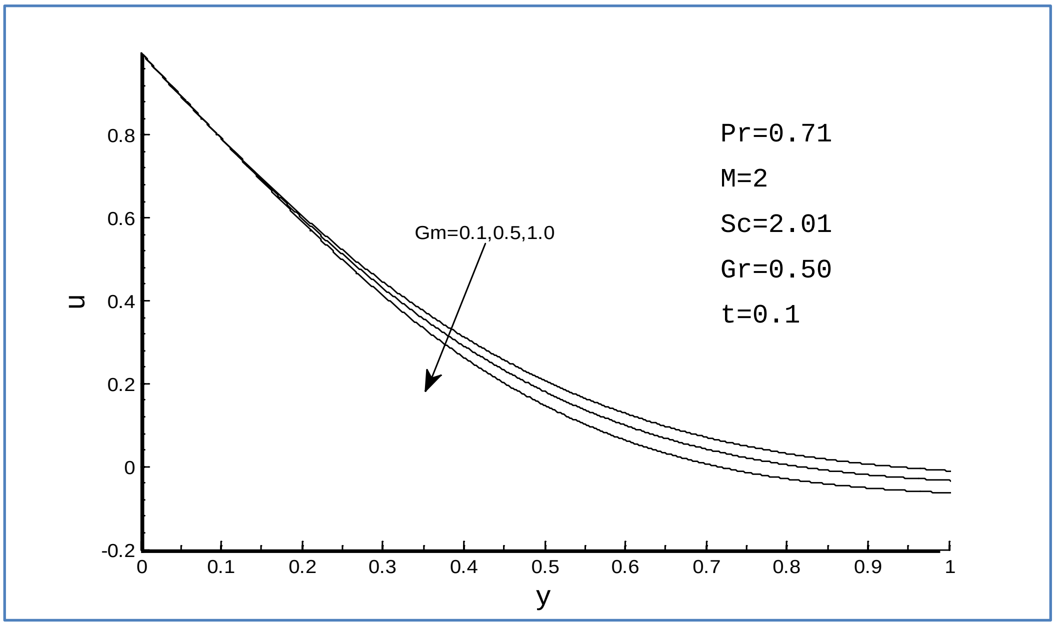

Fig.1: Effects of Gm on Velocity profiles

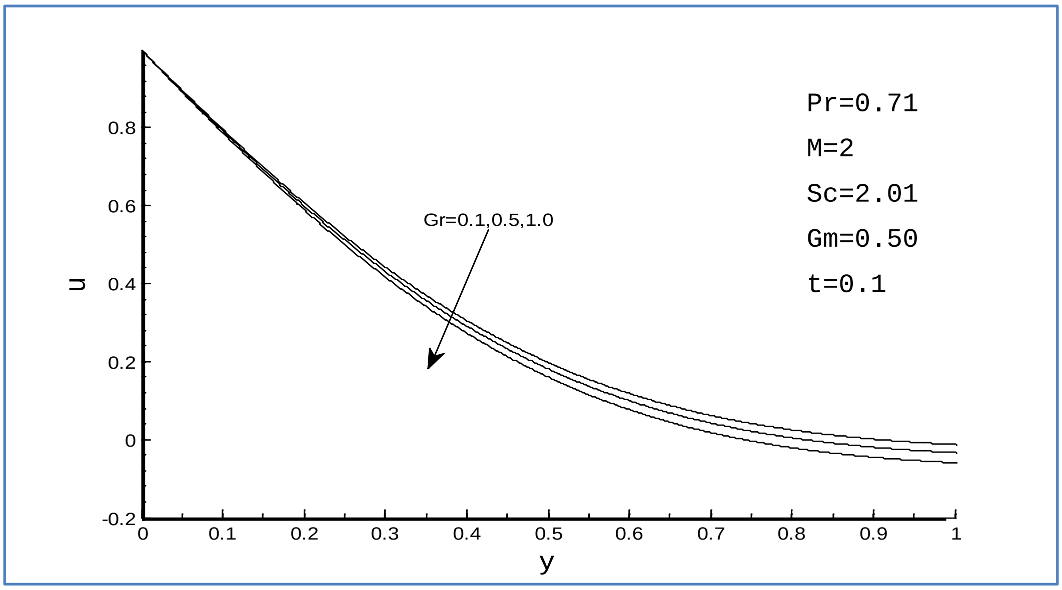

Fig.2: Effects of Gr on Velocity profiles

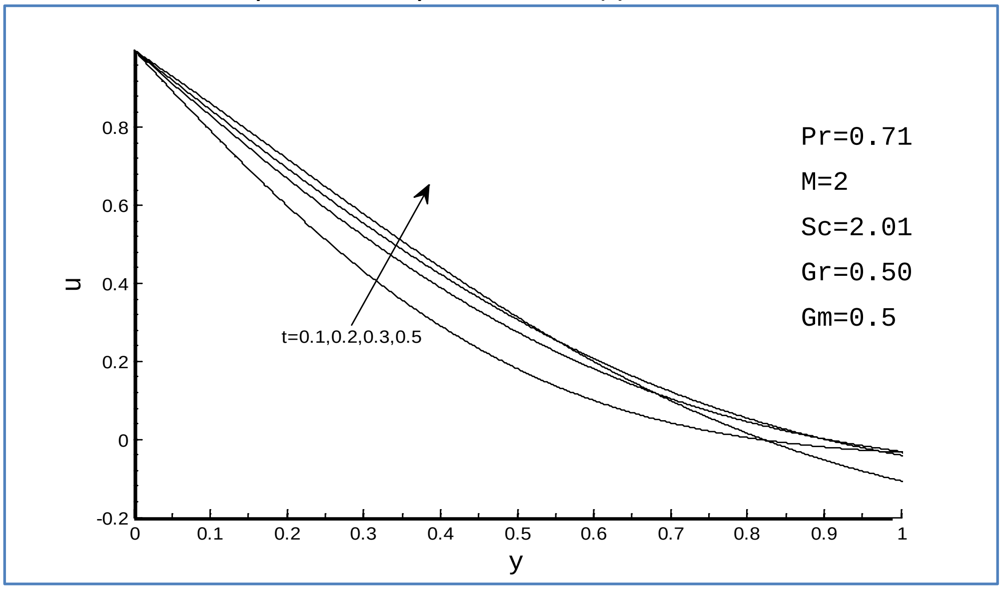

Fig.3: Effects of t on Velocity profiles

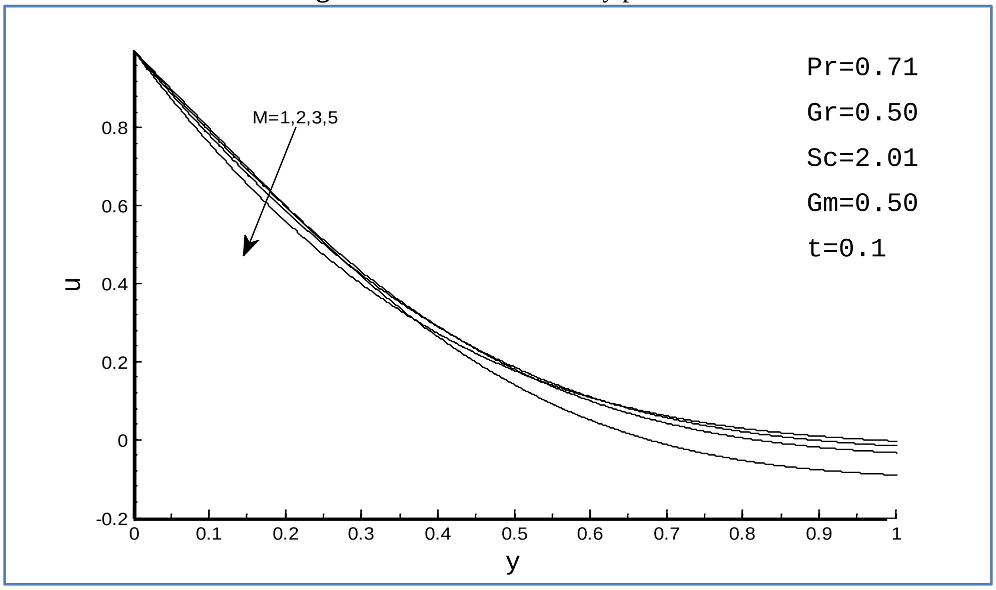

Fig.4: Effects of M on Velocity profiles

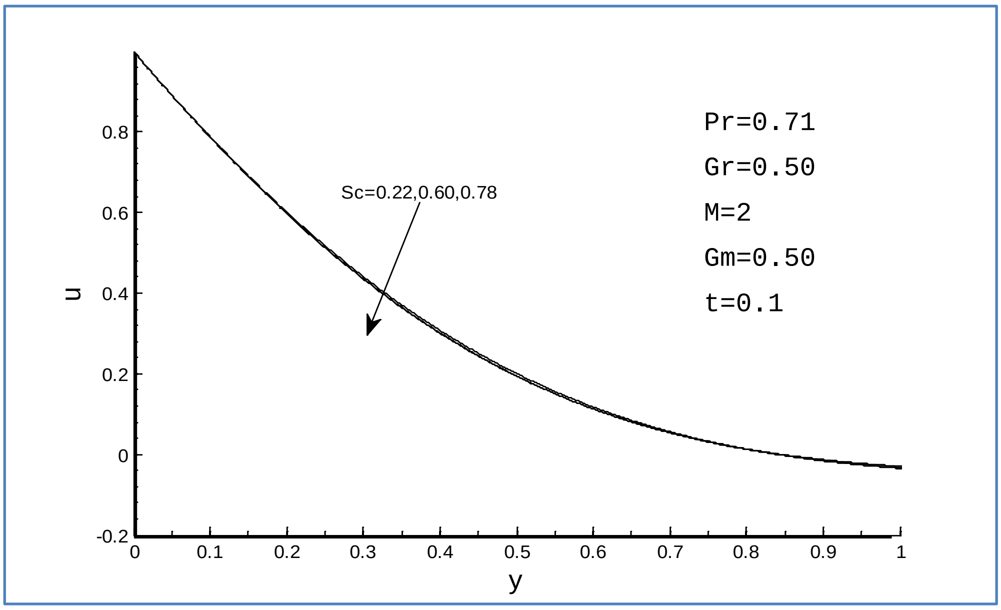

Fig.5: Effects of Sc on Velocity profiles

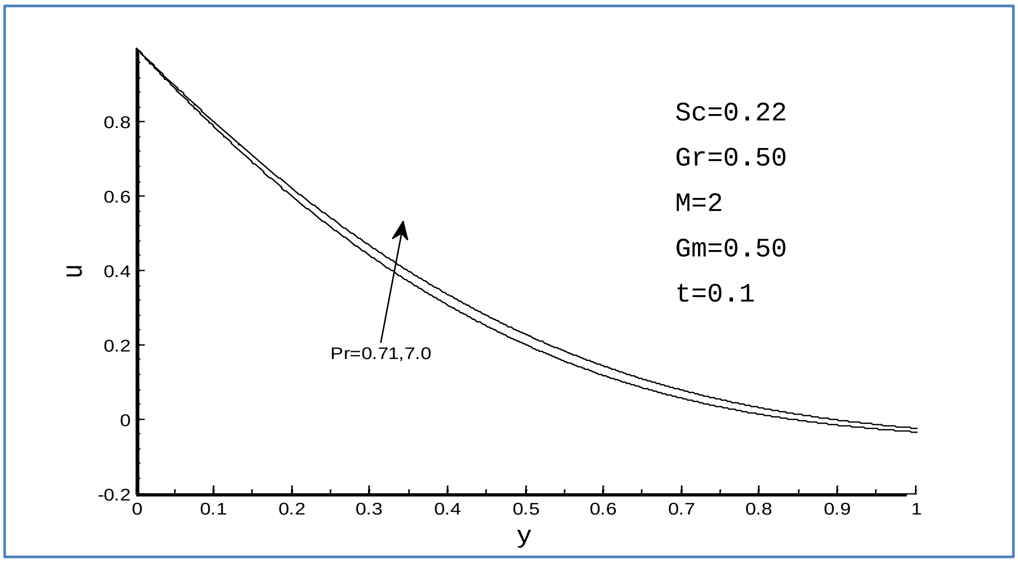

Fig.6: Effects of Pr on Velocity profiles

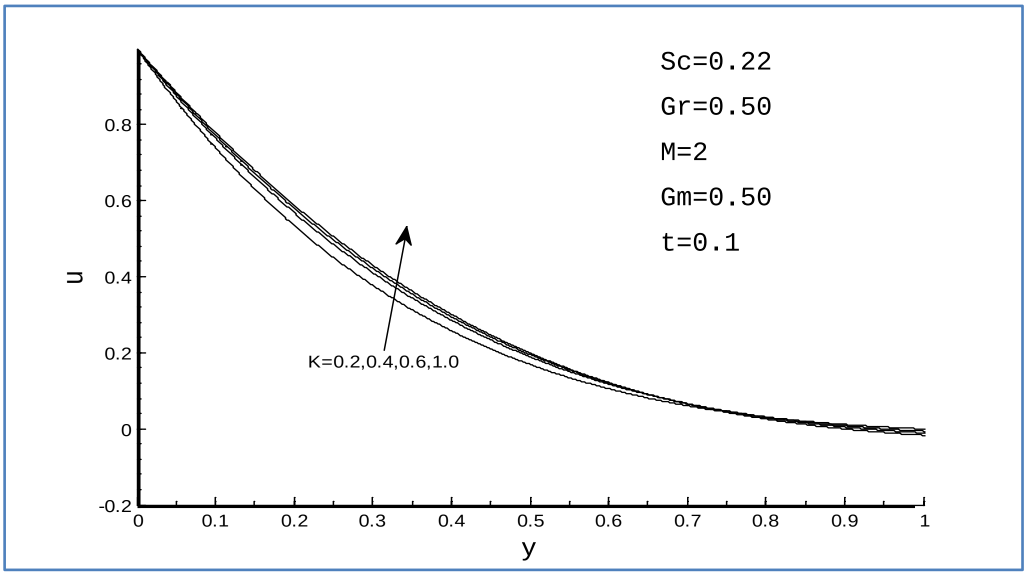

Fig.7: Effects of K on Velocity profiles

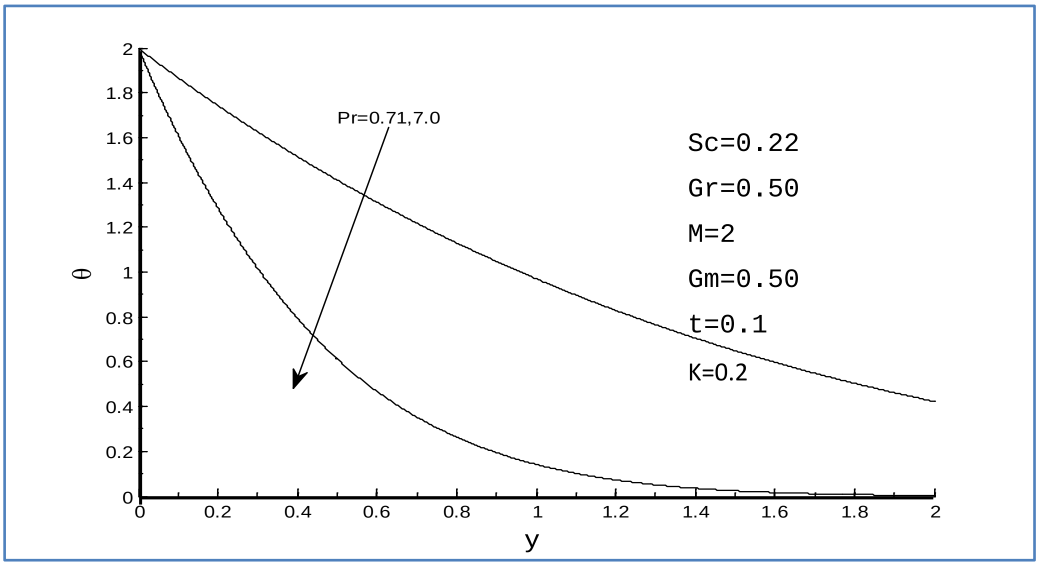

Fig.8. Effects of Pr on Temperature profiles

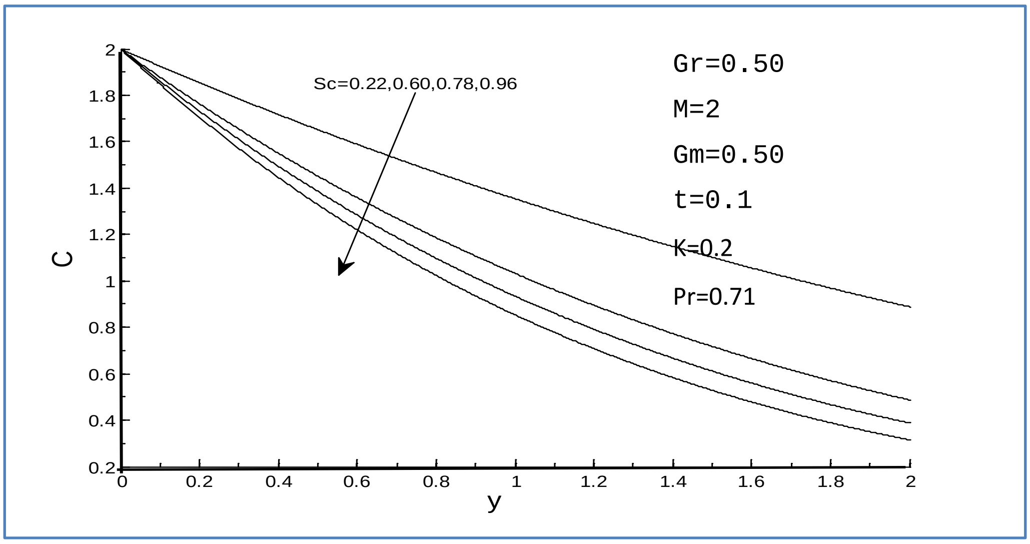

Fig.9. Effects of Sc on Concentration profiles

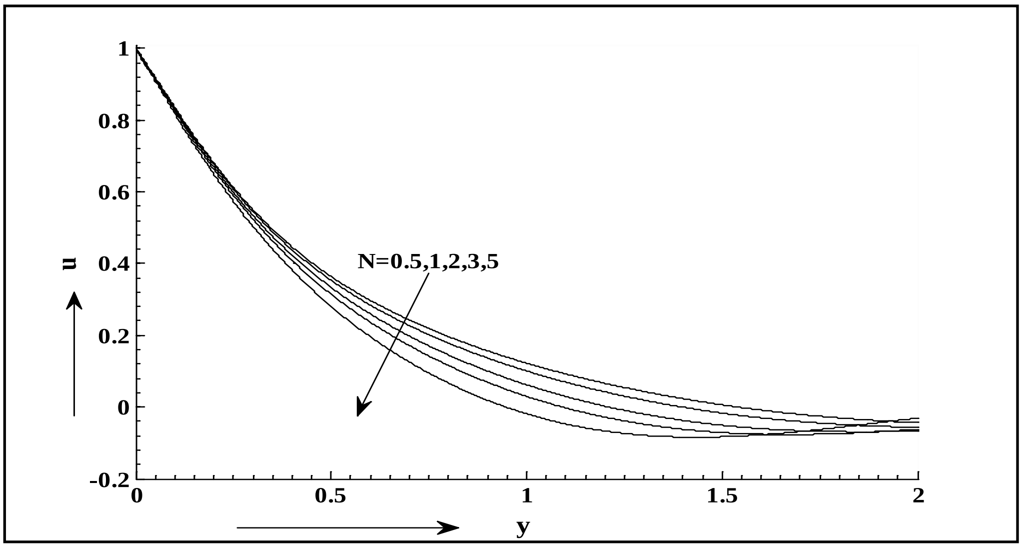

Fig.5: Effects of N on Velocity profiles

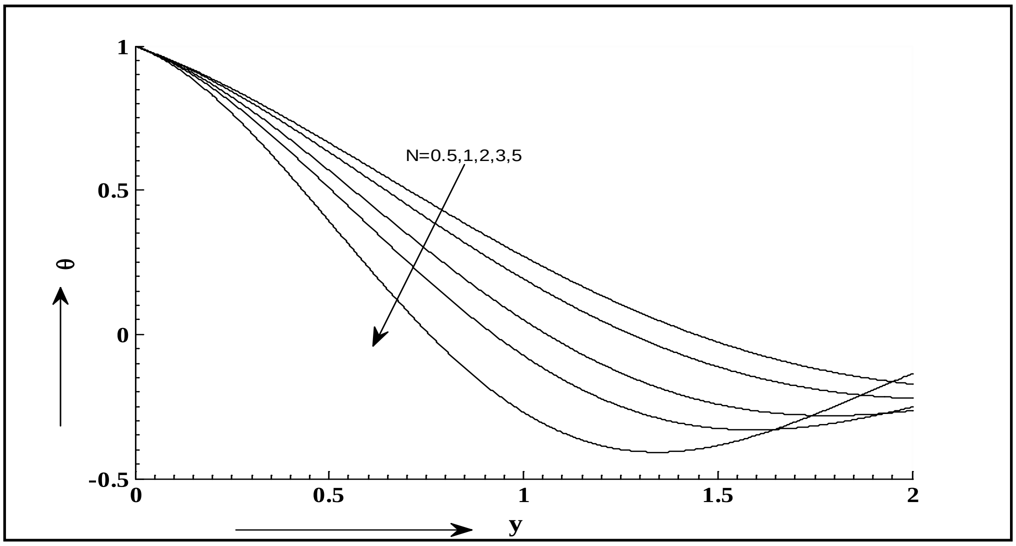

Fig.6: Effect of radiation parameter N on temperature profiles

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