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On $b\hat{g}$ - Closed Sets in Topological Spaces

*R. Subasree & #M. Maria Singam

***Assistant Professor of Mathematics, Chandy College of Engineering, Thoothukudi, (T.N.), India**

#Associate Professor of Mathematics, V.O. Chidambaram College, Thoothukudi, (T.N.), India

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ABSTRACT

In this paper a new class of sets namely, $b\hat{g}$ – Closed Sets is introduced in Topological Spaces. We find some basic properties and applications of $b\hat{g}$ – Closed Sets. We also introduce and study a new class of space namely $Tb\hat{g}$- space.

Keywords: *$b\hat{g}$ – Closed sets, $\hat{g}$ – Open sets, $Tb\hat{g}$ – Space.*

AMS subject classification: *54A05, 54F65, 54C55.*

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1. INTRODUCTION

Levine [4], Mashhour *et al* [8], Njastad [10] and Velicko [13] introduced semi–open sets, pre-open sets, α–open sets and δ–closed sets respectively. Levine [5] introduced generalized closed (briefly g-closed) sets and studied their basic properties. Bhattacharya and Lahiri [2], Arya and Nour[1], Maki *et al* [6,7], Dontchev and Ganster[3] introduced semi-generalized closed (briefly sg–closed) sets, generalized semi-closed (briefly gs–closed) sets, generalized α-closed (briefly gα–closed) sets, α–generalized closed (briefly αg–closed) sets and δ-generalized closed (briefly δg-closed) sets respectively. Veera Kumar [12] introduced $\hat{g}$ –Closed sets in topological spaces. Andrijevic [14] introduced a new class of generalized open sets called b-open sets. Also Ahmad Al-Omari and Mohd. Salmi MD.Noorani [15] introduced generalized b-closed (briefly gb–closed) sets. M.LellisThivagar, B. Meera Devi and E. Hatir[16] introduced a new class of sets called $\delta\hat{g}$ –closed sets.

The purpose of this present paper is to define a new class of sets called $b\hat{g}$–Closed sets and also to obtain some basic properties of $b\hat{g}$–Closed sets in topological spaces. Applying these sets, we obtain a new space called $Tb\hat{g}$ – Space.

2. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset of X, cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. Let us recall the following definitions.

Definition 2.1: A subset A of a space (X, τ) is called a
i) Semi–open set if A ⊑ cl[Int (A)]
ii) α–open set if A ⊑ Int[cl(Int (A))]
iii) b–open set if A ⊑ cl[Int (A)] ∪ Int [cl(A)]

The complement of a semi–open (resp. α–open, b–open) set is called semi-closed (resp. α–closed, b–closed) set.

The intersection of all semi-closed (resp. α-closed, b-closed) sets of X containing A is called the semi-closure (resp. α-closure, b-closure) of A and is denoted by scl(A) (resp. αcl(A), bcl(A)). The family of all semi-open (resp. α-open, b-open) subsets of a space X is denoted by SO(X) (resp. αO(X), bO(X)).

Definition 2.2: A subset A of a space (X, τ) is called a
i) generalized closed (briefly g-closed) set [5] if cl(A)⊑U whenever A⊑U and U is open set in (X, τ).
ii) semi-generalized closed (briefly sg-closed) set [2] if scl(A)⊑U whenever A⊑U and U is a semi-open set in (X, τ).
iii) generalized semi-closed (briefly gs-closed) set [1] if scl(A) ⊑U whenever A⊑U and U is open set in (X, τ).
iv) α-generalized closed (briefly αg-closed) set [7] if αcl(A) ⊑U whenever A⊑U and U is open set in (X, τ).
v) generalized α-closed (briefly gα-closed) set [6] if αcl(A) ⊑U whenever A⊑U and U is α-open set in (X, τ).

Corresponding author: *R. Subasree
****Assistant Professor of Mathematics, Chandy College of Engineering, Thoothukudi, (T.N.), India***

vi) δ-generalized closed (briefly δg-closed) set [3] if clδ(A)⊑U whenever A⊑U and U is open set in (X, τ).
vii) $\hat{g}$ -closed set [12] if cl(A) ⊑U whenever A⊑U and U is a semi-open set in (X, τ).
viii) α$\hat{g}$ -closed set [9] if αcl(A) ⊑U whenever A⊑U and U is $\hat{g}$-open set in (X, τ).
ix) gb-closed set [15] if bcl(A) ⊑U whenever A⊑U and U is open set in (X, τ).

The complement of a g-closed (resp. sg-closed, gs-closed, αg-closed, gα-closed, δg-closed, $\hat{g}$ –closed and α$\hat{g}$ –closed and gb - closed) set is called g-open (resp. sg-open, gs-open, αg-open, gα-open, δg-open, $\hat{g}$ –open and α$\hat{g}$ –open and gb - open) set.

Theorem 2.3: Every open set is $\hat{g}$ –open.

Proof: Let A be a open set in X. Then A^c is closed. Therefore, cl(A^c) = A^c ⊑ X and X is semi-open. This implies A^c is $\hat{g}$ –closed. Hence A is $\hat{g}$ –open.

Definition 2.4: A space (X, τ) is called a
(i) T1/2 – space [5] if every g–closed set in it is closed.
(ii) Tα$\hat{g}$ – space [9] if every α$\hat{g}$ –closed set in it is α - closed.
(iii) b – T1/2 space [15] if every singleton set is either b-open or b-closed.
(iv) Tgs – space [15] if and only if every gb–closed is b-closed.

3. b$\hat{g}$ –CLOSED SETS

We introduce the following definition.

Definition 3.1: A subset A of a space (X, τ) is said to be a b$\hat{g}$ –closed set if bcl(A) ⊑U, whenever A⊑U and U is a $\hat{g}$ –open set in (X, τ).

Example 3.2: Let X = {a, b, c} and τ = {X, Φ, {a, c}} then {X, Φ, {a}, {b}, {c}, {a, b},{b, c}}are b$\hat{g}$ –closed sets.

Proposition 3.3: Every b-closed set is b$\hat{g}$ –closed set.

Proof: Let A be a b-closed set and U be any $\hat{g}$ –open set such that A⊑U. Since A is b-closed, bcl(A) =A for every subset A of X. Therefore bcl(A) = A⊑U and hence A is a b$\hat{g}$ –closed set.

Remark 3.4: The converse of the above theorem need not be true as shown in the following example.

Example 3.5: Let X = {a, b, c} and τ = {X, Φ, {a}, {a, b}} b-closed sets of X = {X, Φ, {b}, {c}, {b, c}} b$\hat{g}$ –closed sets of X = {X, Φ, {b}, {c}, {a, c},{b, c}} Here {a, c} is b$\hat{g}$ –closed but not b-closed in (X, τ).

Proposition 3.6: Every g-closed set is b$\hat{g}$ –closed set.

Proof: Let A be any g-closed set. Let U be any open set containing A such that clA⊑U. Since every open set is $\hat{g}$ –open, we have bclA ⊑ clA ⊑ U for every subset A of X. Hence A is b$\hat{g}$ –closed set.

Remark 3.7: The converse of the above theorem need not be true as shown in the following example.

Example 3.8: Let X = {a, b, c} and τ = {X, Φ, {a}, {a, b}} g-closed sets of X = {X, Φ, {c}, {a, c}, {b, c}} b$\hat{g}$ –closed sets of X = {X, Φ, {b}, {c}, {a, c}, {b, c}}

Here {b} is b$\hat{g}$ –closed but not g-closed in (X, τ).

Proposition 3.9: Every $\hat{g}$-closed set is b$\hat{g}$ –closed set.

Proof: The Proof is straight forward. Since by theorem **3.01[12]**, "Every $\hat{g}$–closed set is g–closed" and by the Proposition **3.6**, "Every g–closed set is b$\hat{g}$ –closed". Hence We have every $\hat{g}$–closed set is b$\hat{g}$ –closed.

Remark 3.10: The converse of the above theorem need not be true as shown in the following example.

Example 3.11: Let X = {a, b, c} and τ = {X, Φ, {a}} $\hat{g}$–closed sets of X = {X, Φ,{b, c}} b$\hat{g}$ –closed sets of X = {X, Φ, {b}, {c}, {a, b},{a, c},{b, c}} Here {{b},{c},{a, b},{a, c}} are b$\hat{g}$ –closed but not $\hat{g}$-closed in (X, τ).

Proposition 3.12: Every gs–closed set is b$\hat{g}$ –closed.

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Proof: Let A be any gs-closed set. Let U be any open set containing A such that sclA $\sqsubseteq$ U. Since every open set is $\hat{g}$ –open, we have bclA $\sqsubseteq$ sclA $\sqsubseteq$ U for every subset A of X. Hence A is b$\hat{g}$ –closed set.

Remark 3.13: The converse of the above theorem need not be true as shown in the following example.

Example 3.14: Let X = {a, b, c} and τ = {X, Φ, {a, c}} gs–closed sets of X = {X, Φ,{b}, {a, b}, {b, c}} b$\hat{g}$ –closed sets of X = {X, Φ, {a}, {b}, {c}, {a, b},{b, c}} Here {a}, {c} are b$\hat{g}$ –closed but not gs-closed in (X, τ).

Proposition 3.15: Every αg-closed set is b$\hat{g}$ –closed.

Proof: Let A be any αg-closed set. Let U be any open set containing A such that αclA$\sqsubseteq$U. Since every open set is $\hat{g}$ –open, we have bclA $\sqsubseteq$ αclA $\sqsubseteq$ U for every subset A of X. Hence A is b$\hat{g}$ –closed set.

Remark 3.16: The converse of the above theorem need not be true as shown in the following example.

Example 3.17: Let X = {a, b, c} and τ = {X, Φ, {a}, {b}, {a, b}} αg–closed sets of X = {X, Φ,{c}, {a, c}, {b, c}} b$\hat{g}$ –closed sets of X = {X, Φ, {a}, {b}, {c}, {a, c},{b, c}} Here {a}, {b} are b$\hat{g}$ –closed but not αg-closed in (X, τ).

Proposition 3.18: Every gb-closed set is b$\hat{g}$ –closed.

Proof: Let A be any gb-closed set. Let U be any open set containing A such that bclA $\sqsubseteq$ U. Since every open set is $\hat{g}$ –open, we have bclA $\sqsubseteq$ U for every subset A of X. Hence A is b$\hat{g}$ –closed set.

Corollary 3.19: The converse of the above theorem is also true.

(i.e) Every b$\hat{g}$ –closed is gb-closed.

Proof: Let A be any b$\hat{g}$ –closed set. Let U be any $\hat{g}$ –open set containing A such that bclA $\sqsubseteq$ U. Also for every open set U, bclA $\sqsubseteq$ U for every subset A of X.

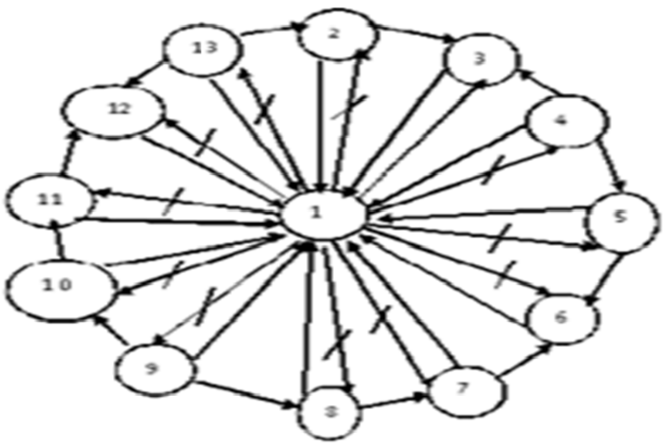
Proposition 3.20: Every α$\hat{g}$-closed set is b$\hat{g}$ –closed.

Proof: Let A be any α$\hat{g}$-closed set. Let U be any $\hat{g}$-open set containing A such that αclA $\sqsubseteq$ U. Now for every subset A of X, we have bclA $\sqsubseteq$ αclA $\sqsubseteq$ U. Hence A is b$\hat{g}$ –closed set.

Remark 3.21: The converse of the above theorem need not be true as shown in the following example.

Example 3.22: Let X = {a, b, c} and τ = {X, Φ, {a}, {b, c}} α$\hat{g}$–closed sets of X = {X, Φ,{a}, {b, c}} b$\hat{g}$ –closed sets of X = {X, Φ, {a}, {b}, {c}, {a, b}, {a, c},{b, c}}Here {b}, {c},{a, b},{a, c} are b$\hat{g}$ –closed but not α$\hat{g}$-closed in (X, τ).

Remark 3.23: The following diagram shows the relationships of b$\hat{g}$ –closed sets with other known existing sets.
A $\longrightarrow$ B represents A implies B but not conversely.



1. b$\hat{g}$ –closed	**2.** b-closed	**3.** gb-closed	**4.** $\hat{g}$ –closed
5. g-closed	**6.** αg-closed	**7.** gα-closed	**8.** α$\hat{g}$ –closed
9. α-closed	**10.** semi-closed	**11.** sg-closed	**12.**gs-closed
13. Closed			

4. CHARACTERIZATION

Remark 4.1: The finite union of bĝ –closed set need not be bĝ –closed set.

Example 4.2: Let X= {a, b, c} and τ = {X, Φ, {a}, {b}, {a, b}} bĝ –closed sets of X = {X, Φ, {a}, {b}, {c}, {a, c}, {b, c}} Here {a} ∪ {b} = {a, b} is not bĝ –closed.

Remark 4.3: Intersection of any two bĝ –closed sets need not be bĝ –closed.

Example 4.4: Let X= {a, b, c} and τ = {X, Φ, {a}} bĝ –closed sets of X = {X, Φ, {b}, {c}, {a, b}, {a, c}, {b, c}} Here {a, b} ∩ {a, c} = {a} is not bĝ –closed.

Proposition 4.5: Let A be a bĝ –closed set of (X, τ). Then bclA-A does not contain a nonempty ĝ –closed set.

Proof: Suppose A is a bĝ –closed set. Let F be aĝ –closed set contained in bclA-A. Now F^c is a ĝ –open set of (X, τ) such that A ⊑ F^c. Since A is bĝ –closed, we have bclA ⊑ F^c. Hence F ⊑ $(bclA)^c$.

Also F ⊑ bclA-A.

Therefore F ⊑ bclA ∩ $(bcl\ A)^c$ = Φ.

Hence, F must be Φ.

Proposition 4.6: If A is ĝ –open and bĝ –closed set of (X, τ), then A is b-closed.

Proof: Since A is ĝ –open and bĝ –closed, we have bclA ⊑ A. Hence, A is b-closed.

Proposition 4.7: The intersection of a bĝ –closed and a b-closed set of (X, τ) is always bĝ –closed.

Proof: Let A be a bĝ –closed set and F be a b-closed set. If U is a ĝ –open set with A∩F ⊑ U.

To Prove: bcl(A∩F) ⊑ U.

Since A∩F ⊑ U $\Longrightarrow$ A ⊑ (U ∪ F^c)

Since A is bĝ –closed, bclA ⊑ (U ∪ F^c) $\Longrightarrow$ bclA ∩ F ⊑ U (1)

Now, bcl(A∩F) ⊑ bclA ∩ bcl F
⊑ bclA ∩ F[since F is b-closed, bclF ⊑ F]
⊑ U [from (1)]

Hence, bcl(A∩F) ⊑ U.

Therefore, intersection of any bĝ –closed set and a b-closed set of (X, τ) is always bĝ –closed.

5. APPLICATIONS

Definition 5.1: A Space (X, τ) is called a Tbĝ–space, if every bĝ–closed set in it is b-closed.

Proposition 5.2: Every T1/2 space is Tbĝ–space**.**

Proof: Since in a T1/2 space, every g-closed set is closed. Also we know that [15] every closed set is b-closed and from Proposition **3.6**, "Every g-closed set is bĝ –closed". Hence every bĝ –closed is b-closed.

Remark 5.3: The converse of the above theorem need not be true.

(i.e) Every Tbĝ–space need not be a T1/2 space as shown in the following example.

Example 5.4: Let Let X={a, b, c} and τ = {X, Φ, {a, b}, {c}} bĝ –closed sets of X = {X, Φ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} b–closed sets of X = {X, Φ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} g–closed sets of X = { X, Φ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}closed sets of X = {X, Φ, {a, b}, {c}}Here (X, τ) is a Tbĝ-space but not a T1/2 space.

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Proposition 5.5: Every Tαĝ–space is Tbĝ–space**.**

Proof: The Proof is straight forward. Since, from Proposition **3.20,** every αĝ–closed set is bĝ –closed and also since every α-closed set is b-closed, we have every bĝ-closed set is b-closed. Hence the Proof.

Remark 5.6: The converse of the above theorem need not be true.

(i.e) Every Tbĝ–space need not be a Tαĝ–space as shown in the following example.

Example 5.7: Let Let X={a, b, c} and τ = {X, Φ, {a, c}} bĝ –closed sets of X = {X, Φ, {a}, {b}, {c}, {a, b}, {b, c}}b–closed sets of X = {X, Φ, {a}, {b}, {c}, {a, b}, {b, c}}αĝ–closed sets of X = {X, Φ, {b}, {a, b}, {b, c}} α–closed sets of X = {X, Φ, {b}}Here (X, τ) is a Tbĝ-space but not Tαĝ-space.

Proposition 5.8: Every Tbĝ–space is b-T1/2 – space**.**

Proof: Let xεX and (X, τ) is a Tbĝ-space. Let {x}εX is not a b-closed set. Then X – {x} is not a b-open set.

To Prove: {x} is b-open.

Thus, A= X – {x} is a bĝ –closed set, since the only open set containing A is X.

Since X is Tbĝ–space, A is b-closed.

Therefore X – {x} is b-closed $\Longrightarrow$ {x} is b-open. Hence the Proof.

Remark 5.9: The converse of the above theorem need not be true.

(i.e) Every b-T1/2–space need not be a Tbĝ–space as shown in the following example.

Example 5.10: Let Let X={a, b, c, d} and τ = {X, Φ, {a, b}, {b}, {b, c, d}} bĝ –closed sets of X = {X, Φ, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, b, c}, {a, c, d}, {a, b, d}} b–closed sets of X = {X, Φ, {a}, {c}, {d}, {a, c}, {a, d}, {c, d}, {a, c, d}}b–open sets of X = {X, Φ, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {b, c, d}, {a, b, d}} Here (X, τ) is a b-T1/2–space but not Tbĝ–space.

Proposition 5.11: Every Tbĝ–space is Tgs–space**.**

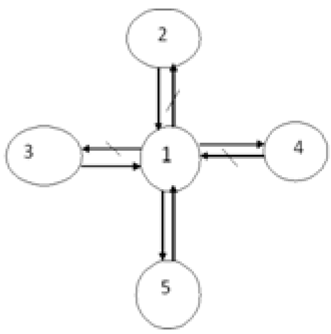
Proof: Let (X, τ) be a Tbĝ–space. Let A⊑ X be a gb–closed set, since by Proposition **3.18**, A is also a bĝ –closed set. Also since X is a Tbĝ–space, then A is b-closed.

Remark 5.12: The converse of the above theorem is also true.

Corollary 5.13: Every Tgs–space is also a Tbĝ–space

Proof: Let (X, τ) be a Tgs-space. Let A ⊑ X be a bĝ–closed set, since by Corollary **3.19**, A is also a gb–closed set. Also since X is a Tgs–space, then A is b-closed.

Remark 5.14: The following diagram shows the relationships about Tbĝ–space with other known existing spaces.



1. Tbĝ–space **2**. T1/2–space **3.** Tαĝ–space
4. b-T1/2–space **5**. Tgs–space

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