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SOLUTION OF MULTI-OBJECTIVE FUZZY INVENTORY MODEL WITH LIMITED STORAGE SPACE AND INVESTMENT THROUGH KARUSH KUHN TUCKER CONDITIONS

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ABSTRACT

A multi item multi objective model with shortages and demand dependent unit cost has been formulated along with three constraints. This paper presents a mathematical model of inventory control problem for determining the minimum total cost of multi-item multi- objective fuzzy inventory model. The three constraints are warehouse space constraints, investment amount constraint and the percentage of utilization of volume of the warehouse space. Warehouse maintenance is one of the essential parts of service operation. In this model, the warehouse space in the selling store is considered in volume. A demand dependent unit cost has been considered and this unit cost is taken in fuzzy environment. This model is solved by Kuhn-Tucker conditions method. The model is illustrated with numerical example.

Keywords: *Inventory, Membership Function, Karush -Kuhn-Tucker Condition, volume of the warehouse.*

1. INTRODUCTION

The meaning of inventory is the stock of goods for future use (production/sales). The control of inventories of physical goods is a problem common to all enterprises in any sector of an economy. The basic objective of inventory control is to reduce investment in inventories and ensuring that production process does not suffer at the same time.

In general the classical inventory problems are designed by considering that the demand rate of an item is constant and deterministic and that the unit price of an item is considered to be constant and independent in nature. But in practical situation, unit price and demand rate of an item may be related to each other. When the demand of an item is high, an item is produced in large numbers and fixed costs of production are spread over a large number of items. Hence the unit cost of the item decreases. .i.e., the unit price of an item inversely relates to the demand of that item. So demand rate of an item may be considered as a function of unit price. i.e $D_i = A_i p_i^{-\beta i}$, $A_i>0$ & $0< \beta_i <1$ Warehouse space available in the selling store plays an important role in inventory model. Warehouse space can be considered in terms of area and/or volume, but most of the researchers consider only the area of the warehouse space. Here the warehouse space in the selling store is considered in volume.

Zadeh [2] first gave the concept of fuzzy set theory. Later on, Bellman and Zadeh [5] used the fuzzy set theory to the decision-making problem. The objective are introduced as fuzzy goals over the α-cut of a fuzzy constraint set by Tanaka[3]. Zimmerman [11] gave the concept to solve multi objective linear programming problem. Sommer [4] applied the fuzzy concept to an inventory and production scheduling problem. Park [1] examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Hence we may impose warehouse space, cost parameters, number of orders, production cost etc, in fuzzy environment. Nirmal Kumar Mandal [6] formulated a multi objective fuzzy inventory model with three constraints and solved by using Geomentric programming method. In all the above articles, the warehouse space available in the selling store is taken in terms of area. If the warehouse space is taken in terms of volume then less percentage of volume of the warehouse space will be consumed. Consequently the maximum of the volume of the warehouse can be utilized effectively.

In this paper, multi objective fuzzy inventory model with shortages is considered under three constraints such as warehouse space constraint, investment amount constraint and the third one is the percentage of utilization of volume of the warehouse space. Here the volumes of the unit items are taken for calculations. The demand is dependent on unit cost and the unit cost is taken in fuzzy environment. The unit cost, lot size and shortage level are decision variables. The problem is solved using Karush -Kuhn-Tucker Conditions method.

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The Karush -Kuhn-Tucker Condition [7] are necessary conditions for identifying stationary points of a non linear constrained problem subject to inequality constraints.

The development of this method is based on the Lagrangean method. These conditions are also sufficient if the objective function and the solution space satisfy the conditions in the following table 1.1

Table: 1.1

Sense of optimization	Required conditions	
	Objective function	Solution space
Maximization	Concave	Convex Set
Minimization	Convex	Convex Set

The conditions for establishing the sufficiency of the Kuhn-Tucker Conditions [8] are summarized in the following table 1.2

Table: 1.2

Problem	Kuhn-Tucker conditions
1.Max z = f(X) subject to $h^i(X) \le 0$ $X \ge 0$, i = 1,2,…..m	$\frac{\partial}{\partial x_j} f(X) - \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial x_j} h^i(X) = 0$ $\lambda_i h^i(X) = 0, h^i(X) \le 0, i = 1,2,....m$ $\lambda_i \ge 0, i = 1,2,......m$
2.Min z = f(X) subject to $h^i(X) \ge 0$ $X \ge 0$, i= 1,2,…..m	$\frac{\partial}{\partial x_j} f(X) - \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial x_j} h^i(X) = 0$ $\lambda_i h^i(X) = 0, h^i(X) \ge 0, i = 1,2,....m$ $\lambda_i \ge 0, i = 1,2,......m$

Kuhn-Tucker Conditions also known as Karush Kuhn-Tucker (KKT) Conditions was first developed by W. Karush in 1939 as part of his M.S thesis at the University of Chicago. The same conditions were developed independently in 1951 by W. Kuhn and A. Tucker.

2. ASSUMPTIONS AND NOTATIONS

A multi-item, multi-objective inventory model with shortages is developed under the following notations and assumptions.

NOTATIONS

n = number of items
B = Total investment cost for replenishment
l = inside length of warehouse
b = inside breadth of warehouse
h = maximum height of shelf
V = Volume of warehouse
For i^{th} item: (i= 1,2,………..n)
$D_i = D_i(p_i)$ demand rate[function of unit cost price]
Q_i = lot size (decision variable)
M_i = Shortage level (decision variable)
S_i = Set-up cost per cycle
H_i = Inventory holding cost per unit item
m_i = Shortage cost per unit item

p_i = price per unit item (fuzzy decision variable)
l_i = Length of unit item i
b_i = breadth of unit item i
h_i = height of unit item i
v_i = Volume of unit item i
V_w = Percentage of utilization of volume of warehouse.
TC (p,Q,M) = expected annual total cost

ASSUMPTIONS
(i) Replenishment is instantaneous
(ii) lead time is zero
(iii) Demand is related to the unit price as

$$D_i = \frac{A_i}{p_i^{\beta_i}} = A_i p_i^{-\beta_i}$$

where A_i (>0) and β_i($0 < \beta_i < 1$) are constants and real numbers selected to provide the best fit of the estimated price function. A_i>0 is an obvious condition since both D_i and p_i must be non-negative. Volume of the unit item is defined by

$v_i = l_i \times b_i \times h_i$

To calculate the volume of the warehouse space, multiply the length of the dimensions of the inside of the warehouse, that is, multiply the inside length, inside breadth and maximum shelf height. i.e., Volume = l x b x h

3. FORMULATION OF INVENTORY MODEL WITH SHORTAGES

The Total cost = Production cost + Set up cost + Holding cost + Shortage cost

$$TC(p_i, Q_i, M_i) = p_i D_i + \frac{S_i D_i}{Q_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{m_i M_i^2}{2Q_i}$$

$$= A_i p_i^{1-\beta_i} + \frac{A_i S_i}{Q_i} p_i^{-\beta_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{m_i M_i^2}{2Q_i}$$

for i=1,2,3,………n

$$\text{Min}\, TC(p_i, Q_i, M_i) = \sum_{i=1}^{n} \left[A_i p_i^{1-\beta_i} + \frac{A_i S_i}{Q_i} p_i^{-\beta_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{m_i M_i^2}{2Q_i} \right]$$

There are some restrictions on available resources in inventory problems that cannot be ignored to derive the optimal total cost.

The limitation on the available warehouse space in the store $\sum_{i=1}^{n} v_i Q_i \le V$

The upper limit of the total amount investment $\sum_{i=1}^{n} p_i Q_i \le B$

where p_i, Q_i, M_i >0 (i=1,2….n)

Percentage of utilization of volume of the warehouse

$$\frac{VXV_W}{\left(\sum_{i=1}^{n} v_i Q_i\right) 100} = 1,\ 0 \le V_W \le 100$$

4. INVENTORY MODEL IN FUZZY ENVIRONMENT

When p_i's are fuzzy decision variables, the above crisp model under fuzzy environment reduces to

$$MinTC\left(\tilde{p}, Q, M\right) = \sum_{i=1}^{n} \left[A_i \tilde{p}_i^{1-\beta_i} + \frac{A_i S_i}{Q_i} \tilde{p}_i^{-\beta_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{m_i M_i^2}{2Q_i} \right]$$

Subject to the constraints

$$\sum_{i=1}^{n} v_i Q_i \leq V$$

$$\sum_{i=1}^{n} \widetilde{p_i} Q_i \leq B$$

$$\frac{VXV_W}{\left(\sum_{i=1}^{n} v_i Q_i\right)100} = 1$$

and $\tilde{p}$, Q_i, M_i >0 (i=1,2….n), $0 \leq V_W \leq 100$

[Here cap '~ 'denotes the fuzzification of the parameters]

5. MEMBERSHIP FUNCTION

The membership function for the fuzzy variable p_i is defined as follows

$$\mu_{p_i}(X) = \begin{cases} 1, p_i \leq L_{L_i} \\ \dfrac{U_{L_i} - p_i}{U_{L_i} - L_{L_i}}, L_{L_i} \leq p_i \leq U_{L_i} \\ 0, p_i \geq U_{L_i} \end{cases}$$

Here U_{Li} and L_{Li} are upper limit and lower limit of p_i respectively.

6. NUMERICAL EXAMPLE

The model is illustrated for one item (i=1) and also the common parametric values assumed for the given model are i = 1, A_1 = 113, S_1 = \$100, H_1 = \$1, B = \$1400, m_1=\$1, l_1 = 2m, b_1 = 3m, h_1 = 4m, l = 10m, b = 12m, h = 30m and \$3≤ p_1 ≤\$13.

From the given values v_1 = 24 cubic m and V = 3600 cubic m.

The proposed model is solved by Karush-Kuhn-Tucker conditions and the optimal results are presented in the table 6.1

Table: 6.1

β_1	p_1	μ_{P1} value	Q_1	$\mathbf{M_1}$	$\mathbf{V_W}$	Expected Total cost
0.85	5.49	0.7507	103.15	51.58	**68.77**	197.44
0.86	6.42	0.6580	95.64	47.82	**63.76**	194.39
0.87	7.60	0.5400	88.05	44.03	**58.70**	191.09
0.88	9.11	0.3890	80.48	40.24	**53.65**	187.52
0.89	11.10	0.1900	72.90	36.45	48.60	183.68

7. CONCLUSION

In this paper we have proposed a concept of the optimal solution of the inventory problem with fuzzy cost price per unit item. Fuzzy set theoretic approach of solving an inventory control problem is realistic as there is nothing like fully rigid in the world.Here the fuzzy inventory model is taken with three constraints, particularly volumes of the unit items are taken in the warehouse space constraint. By solving the above fuzzy inventory model using Kuhn-Tucker condition method we have the values of imprecise variable for decision making.The result reveals the minimum expected annual total cost of the inventory model and also the optimal percentage of utilization of the volume of the warehouse. In the result, percentage of utilization of the volume of the warehouse space is less. It can be increased by changing the values like volume of the warehouse space. Investment cost etc.The proposed model can be developed with many limitations like number of orders, total cost etc.

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