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PERISTALTIC FUID FLOW THROUGH MAGNETIC FIELD AT LOW REYNOLDS NUMBER IN A FLEXIBLE CHANNEL UNDER AN OSCILATORY FLUX

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ABSRACT

The present paper investigates the peristaltic fluid flow through magnetic field at low Reynolds number in a flexible channel under an oscilatory flux. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity and transverse velocity and also Shere stress. The effects of various physical parameters on axial velocity, transverse velocity, shere stress and fluid flux have been computed numerically. It is noted axial velocity increases with increase in Reynolds number 'R' and transverse velocity remains positive for all constrictions and increases towards the boundary with increase in Reynolds number 'R' and negative and towards axis at dilation.

Key-words: *Peristaltic fluid flow, the time average flux, Reynolds number, Magnetic field.*

1. INTRODUCTION

The study of fluids transport by means of peristaltic waves in both the mechanical and physiological situations has been the subject of scientific research since the first investigation of Latham [1]. Peristaltic pumping is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of distensible duct containing liquid. Certain physiological phenomena, like urine transport from kidney to bladder through the ureter, movement of chime in the gastrointestinal tract, the movements of spermatozoa in the ductus efferentes of the male reproductive tract and ovum in the female fallopian tube, the locomotion of some warms, transport of lymph in the lymphatic vessels and vasomotion of small blood vessels such as arterioles, venules and capillaries involve the peristaltic motion. In addition, peristaltic pumping occurs in many practical applications involving biomechanical systems. Also, finger and roller pumps are frequently used for pumps corrosive or very pure materials so as to prevent direct contact of the fluid with the pumps internal surfaces. Moreover, by using the principle of peristalsis, some biomechanical instruments, e.g., heart-lung machine, have been fabricated. The magneto hydrodynamic (MHD) flow of a fluid in a channel with elastic rhythmically contracting walls (peristaltic flow) is of interest in connection with certain problems of the movement of conductive physiological fluids, e.g., the blood and blood pump machines, and with the need for theoretical research on the operation of a peristaltic MHD compressor. All important literature up to 1978 on peristaltic transport has been documented by Rath [2]. Fung and Yih [3] analyzed the role of Reynolds number and wavelength in peristaltic motion of moderate amplitude, making use of perturbation method with an amplitude ratio as the perturbation parameter. Burns and parkes [4] have discussed the peristaltic flow produced by sinusoidal peristaltic wave along flexible walls of a channel under the pressure gradient. Latham and Shapiro [5], Boyarsky *et al* [6] have investigated this phenomenon using different configurations. Also a number of recent investigations have reported the pulsatile nature of blood flow in pulmonary arteries and different portions of mesentery. The effect of moving magnetic field on blood flow was studied by Stud *et al* [7], and they observed that the effect of suitable moving magnetic field accelerates the speed of blood. Srivastava and Agarwal [8] considered the blood as an electrically conducting fluid and constitute a suspension of red cell in plasma. Also Agarwal and Anwaruddin [9] studied the effect of magnetic field on blood flow by taking a simple mathematical model for blood through an equally branched channel with flexible walls executing peristaltic waves long wavelength approximation method and observed, for the flow of blood in arteries with arterial disease like arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as a blood pump in carrying out cardiac operations. The interaction of peristaltic flow with pulsatile fluid in a circular cylindrical tube is studied by Srivastava and Srivastava [10]. Mekhemer Kh.S *et al* [11] studied by Non-linear peristaltic transport of MHD flow through porous medium. Mishra and

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Ramachandra Rao [12] have investigated the flow in an asymmetric channel generated by peristaltic waves propagating on the walls with different amplitudes and phases. Effects of a magnetic field on trapping through peristaltic motion for generalize Newtonian fluid in a channel has been studied by Abd El Hakeem *et al* [13]. Peristaltically induced transport of a MHD biviscosity fluid in a non-uniform tube has been studied by Eldabe *et al* [14], The peristaltic couple fluid flow through channels with flexible walls has been studied Ravikumar *et al* [15, 16, 17 and 18]. Peristaltic flow of a couple stress fluid through porous medium in a channel at low Reynolds number studied by Raghunatha Rao *et al* [19].

The present research aimed is to investigate the peristaltic fluid flow through magnetic field at low Reynolds number in a flexible channel under an oscillatory flux. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for axial velocity. The non-linear equations governing the flow through magnetic field are solved under long wavelength approximation. The existence of separation in the flow field is discussed for different values of the governing parameters. The behavior of the primary and secondary velocity components, the shear stresses on the wall, the time average fluxes have been analyzed in either case for different sets of parameters.

2. FORMULATION OF THE PROBLE

We consider a peristaltic flow of a couple stress fluids in a symmetric channel with flexible walls, which are excited by a traveling longitudinal wave resulting in a peristaltic motion. An oscillatory time dependent flux is being imposed on then peristaltic flow. Choosing the cartesian coordinate system 0(x, y), the flexible walls are represented by

$$y = \pm a_0 \, s\left[X - ct/\lambda\right]$$

Where ‘ a_0 ’ is the wave amplitude, ‘c’ is the wave velocity, ‘ λ ’ is the wave length and‘ s ’ is an arbitrary function of the normalized axial coordinate $x^* = \left[X - ct/\lambda\right]$.

In the case of incompressible fluids when the body forces the body moments are absent, the equations of motion is

$$\rho \frac{DV}{Dt} = [-\nabla p + \mu \nabla^2 \mathrm{V} - \eta \nabla^4 \mathrm{V}]$$

The equations of motion for two-dimensional flow incompressible couple stress fluid in the component form are

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} = 0 \qquad (2.1)$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] - \sigma {B_0}^2 u \qquad (2.2)$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - \sigma {B_0}^2 v \qquad (2.3)$$

(Suffices t, x, y denote differentiation with respect to the respective variable). (u, v) are the velocity components along 0(x, y) directions respectively, ‘p’ is the fluid pressure, ‘ ρ ’is the density of the fluid, ‘ μ ’ is the coefficient of the viscosity,’ η ’ is the coefficient of couple stress.

The flow being two-dimensional in view of the incompressibility of the flow using (2.1) we introduce a stream function Ψ such that

$$u = -\Psi\mathrm{y} \text{ and } v = \Psi_x \qquad (2.4)$$

Substituting (2.4) in (2.2) and (2.3) and eliminating p, the governing equations in terms of Ψ reduces to

$$\frac{\partial}{\partial \mathrm{t}}[\nabla^2\Psi] - \left[\Psi_{\mathrm{y}}\nabla^2\Psi_{\mathrm{x}}\right] + \left[\Psi_{\mathrm{x}}\nabla^2\Psi_{\mathrm{y}}\right] = \frac{\mu}{\rho}[\nabla^4\Psi] - \frac{\sigma {B_0}^2}{\rho}\nabla^2\Psi \qquad (2.5)$$

Where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$

The relevant conditions on Ψ are

$$\Psi_{\mathrm{x}} = 0\,; \quad \Psi_{\mathrm{yy}} = 0, \quad \mathrm{y} = 0 \qquad (2.6)$$

$$\Psi = \Psi_{\mathrm{f}}[1 + \mathrm{k}e^{\mathrm{i}\omega \mathrm{t}}] - \mathrm{a}_0\mathrm{cs}, \quad \mathrm{y} = \mathrm{a}_0\mathrm{s}[\mathrm{x}] \qquad (2.7)$$

$$\Psi_{yyy} = 0, \quad y = a_0 s[x] \tag{2.8}$$

(2.6) guarantees the vanishing of the transverse flow on the axis of channel in view of the symmetry. (2.7) corresponds to the no slip of the axial velocity on the channel and also guarantees the assumption of the imposed oscillatory flux across the channel. (2.8) is the boundary condition related to couple stress fluids.

We define the following non-dimensional variables

$$x^* = \left[\frac{X-ct}{\lambda}\right] \quad y^* = \left[\frac{Y}{a_0}\right] \quad t^* = [\omega t]$$

$$\Psi^* = \left[\frac{\Psi}{a_0 c}\right] \qquad \Psi_f{}^* = \left[\frac{\Psi_f}{a_0 c}\right] \quad \varepsilon = \left[\frac{a_0}{\lambda}\right]$$

Introducing these non- dimensional variables in (2.5) the governing equation in terms of Ψ reduces to (on dropping the asterisks)

$$\left(-R\varepsilon^3\Psi_{xxx} - R\varepsilon\Psi_{xyy} - R\varepsilon^3\Psi_y\Psi_{xxx} - R\varepsilon\,\Psi_y\Psi_{xyy} + R\varepsilon^3\Psi_y\Psi_{xxy} + R\varepsilon\,\Psi_x\Psi_{yyy}\right) = \left(\varepsilon^4\Psi_{xxxx} + \Psi_{yyyy} + 2\varepsilon^2\Psi_{xxyy} - RM\varepsilon^2\Psi_{xx} - RM\Psi_{yy}\right) \tag{2.9}$$

$R = \frac{\rho c a_0}{\mu}$, Reynolds number

$M = \frac{\sigma B_0{}^2 a_0}{\rho c}$, Magnetic parameter

3. METHOD OF SOLUTION

Under long wave length assumption ($\varepsilon << 1$) keeping in view of the condition (2.10) Ψ may be assumed in the form

$$\Psi = [\Psi_0 + ke^{it}\,\overline{\Psi}_0] + \varepsilon\,[\Psi_1 + ke^{it}\,\overline{\Psi}_1] \tag{3.1}$$

Substituting (3.1) in (2.9) and equating the like powers of ε , the equations corresponding to the zeroth order steady components are

$$\Psi_{0_{yyyy}} - RM\Psi_{0_{yy}} = 0 \tag{3.2}$$

$$\overline{\Psi}_{0_{yyyy}} - RM\overline{\Psi}_{0_{yy}} = 0 \tag{3.3}$$

The conditions to be satisfied by ψ_0 and $\overline{\Psi}_0$ are

$$\Psi_0 = 0 \quad y = 0 \quad ; \quad \overline{\Psi}_0 = 0 \quad y = 0 \tag{3.4}$$

$$\Psi_{0_{yy}} = 0 \quad y = 0 \quad ; \quad \overline{\Psi}_{0_{yy}} = 0 \quad y = 0 \tag{3.5}$$

$$\Psi_{0_y} = 0 \quad y = 1 \quad ; \quad \overline{\Psi}_{0_y} = 0 \quad y = 1 \tag{3.6}$$

$$\Psi_0 = 1 - S[x] \quad y = 1 \quad ; \quad \overline{\Psi}_0 = 1 \quad y = 1 \tag{3.7}$$

Solving (3.2) and (3.3) subject to the conditions (3.4-3.7), we obtain

$$\Psi_0 = \left[\left[\frac{\sqrt{RM}\cosh[\sqrt{RM}][1 - S[x]]}{\sqrt{RM}\cosh[\sqrt{RM}] - \sinh[\sqrt{RM}]}\right] * y\right] + \left[\left[\frac{[S[x] - 1]}{\sqrt{RM}\cosh[\sqrt{RM}] - \sinh[\sqrt{RM}]}\right] * [\sinh[\sqrt{RM}y]]\right]$$

$$\overline{\Psi}_0 = \left[\left[\frac{\sqrt{RM}\cosh[\sqrt{RM}]}{\sqrt{RM}\cosh[\sqrt{RM}] - \sinh[\sqrt{RM}]}\right] * y\right] - \left[\left[\frac{1}{\sqrt{RM}\cosh[\sqrt{RM}] - \sinh[\sqrt{RM}]}\right] * [\sinh[\sqrt{RM}y]]\right]$$

4. SHEAR STRESS AND FLUX

The shear stress at the upper wall y = s (x), in the dimensional form is given by

$$T = \frac{\frac{1}{2}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]\left[1 - \left(\frac{ds}{dx}\right)^2\right] + \left[\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right]\left[\frac{ds}{dx}\right]}{\left[1 + \left(\frac{ds}{dx}\right)^2\right]}$$

and it's solution is given by

$$\tau = \Big(\tfrac{1}{4}\sqrt{RM}\beta(-y(-4+\beta^2)\mathrm{Cos}[x] + \beta(-8 + 8\,\mathrm{Cos}[2x] + y\,\beta\,\mathrm{Cos}[3x]))\,\mathrm{Cosh}[\sqrt{RM}]$$
$$+ 4\sqrt{RM}\,\beta^2\,\mathrm{Cosh}[\sqrt{RM}y]\mathrm{Sin}[x]^2 + \tfrac{1}{2}(kRM\,\mathrm{Cos}[t] - (1+RM)\beta\,\mathrm{Cos}[x])$$
$$(2-\beta^2+\beta^2\,\mathrm{Cos}[2x])\,\mathrm{Sinh}[\sqrt{RM}y]\Big)/\big(2(1+\beta^2\mathrm{Sin}[x]^2)(\sqrt{RM}\,\mathrm{Cosh}[\sqrt{RM}] - \mathrm{Sinh}[\sqrt{RM}]\big)$$

The volume flux of the fluid Q is given by the formula

$Q = \int_0^S u\,dy$ and is given by

$$Q = -\,(k\,\mathrm{Cos}[t] - \beta\,\mathrm{Cos}[x])\big(\sqrt{RM}\,\mathrm{Cosh}[\sqrt{RM}]\,S(x) - \mathrm{Sinh}[\sqrt{RM}S(x))/\big(\sqrt{RM}\mathrm{Cosh}[\sqrt{RM}] - \mathrm{Sinh}[\sqrt{RM}]\big)$$

5. DISCUSSION OF THE PROBLEM AND NUMERICAL RESULTS

The interaction of flow pattern and the shear stresses has been computationally evaluated for various the governing parameters viz. 'R' the Reynolds Number, 'M' Magnetic parameter for computational purpose we choose a sinusoidal flexible boundary in the non-dimensional form y = S(x) =1+βCos[x]. For positive values of β the channel converges as 'x' varies between 0 to Π and when β is negative the channel diverges in the same limits. The points x = 0 and x = Π corresponds to points of maximum dilation and constriction respectively or vice versa according as β is positive or negative. The shear stress on the upper flexible wall is evaluated throughout the cycle of oscillations at different points within a wavelength for different sets of parameters.

Fig 1 to 5 corresponds to the behavior of the shear stress in a cycle of oscillations at different points of wave length for various in governing parameters R and M. Fig 1 to 3 reveals no separation occurs in flow field for R≥2 , M=2 this is because shear stress on the wall does not vanish or become negative in a wave length throughout entire cycle of oscillations. This is true even if M≥2 , R=2 (Fig 4 and 5). Thus the phenomenon of no separation is observed in the flow field for lower of R. We now discuss the behaviour of axial velocity for various in the governing parameters R & M etc. Fig.6 reveals the axial velocity distribution (u) increases in its magnitude with an increasing the values of R. Fig.7 reveal the axial velocity distribution (u) increases by increasing the values of M. Thus the phenomenon of axial velocity increases by increasing the values of R and M. We now discuss the behaviour of transverse velocity for various in the governing parameters R & M etc. Fig.8 reveals of transverse velocity (v) remains positive for all constrictions and increases towards the boundary for all $R \geq 2,$ M=2 and transverse velocity (v) negative and towards axis at a dilation. This is true even if M ≥2, R=2(Fig 9). The stress on the upper wall and fluid flux are evaluated for various in the governing parameters and tabulated 1 and 2. The shear stress is positive at a constricted region and negative at dilation. The shear stress is increasing in magnitude in an increase in M and β. The fluid flux in negative at a constricted region and positive at dilation.

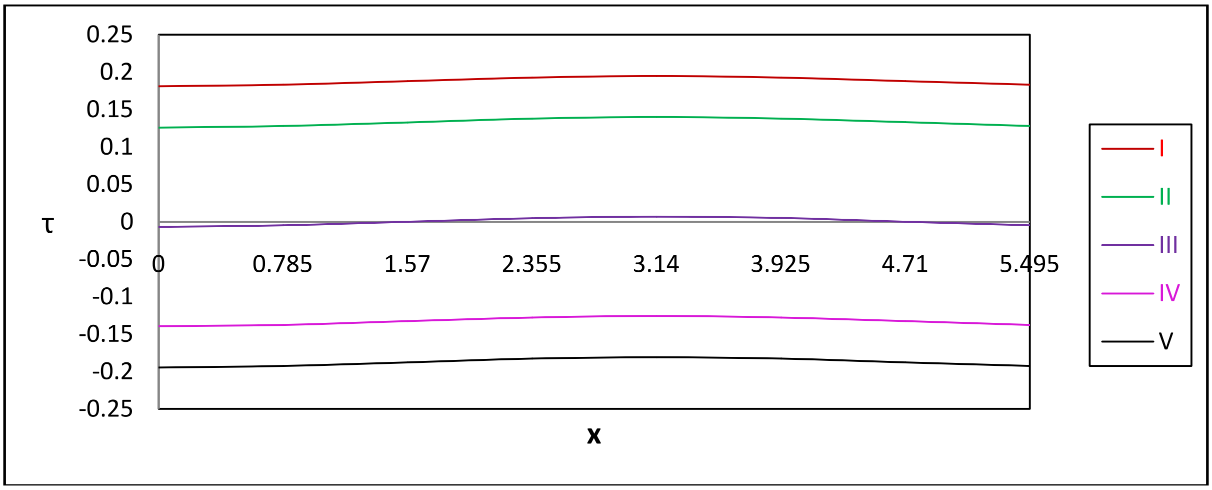

Fig.1: Shere stresses τ for R=2, M=2, β=0.005, Є=0.01, k=0.1, y =1.0043

	I	II	III	IV	V
t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π

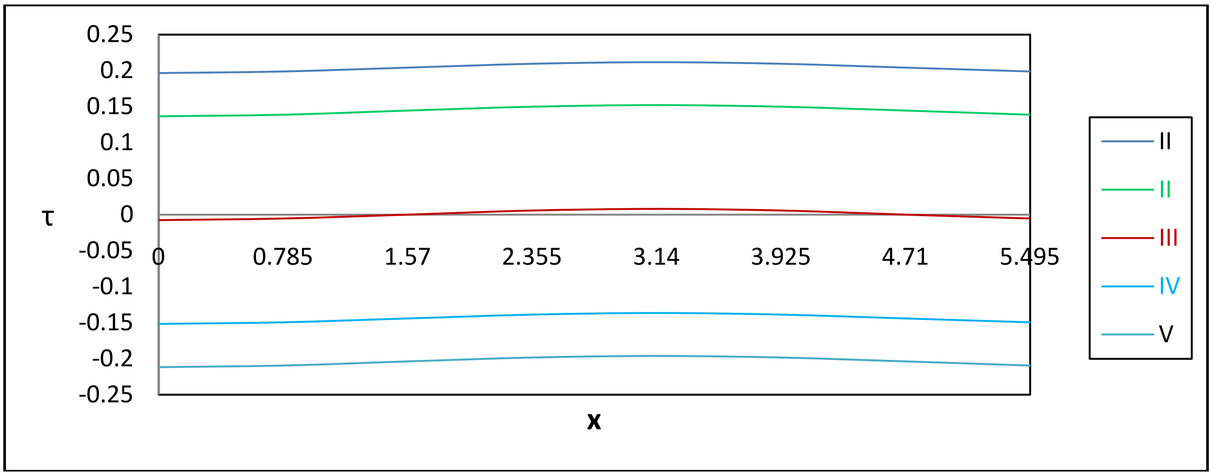

Fig.2: Shere stresses τ for R=3, M=2, β=0.005, Є=0.01, k=0.1, y =1.0043

	I	II	III	IV	V
t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π

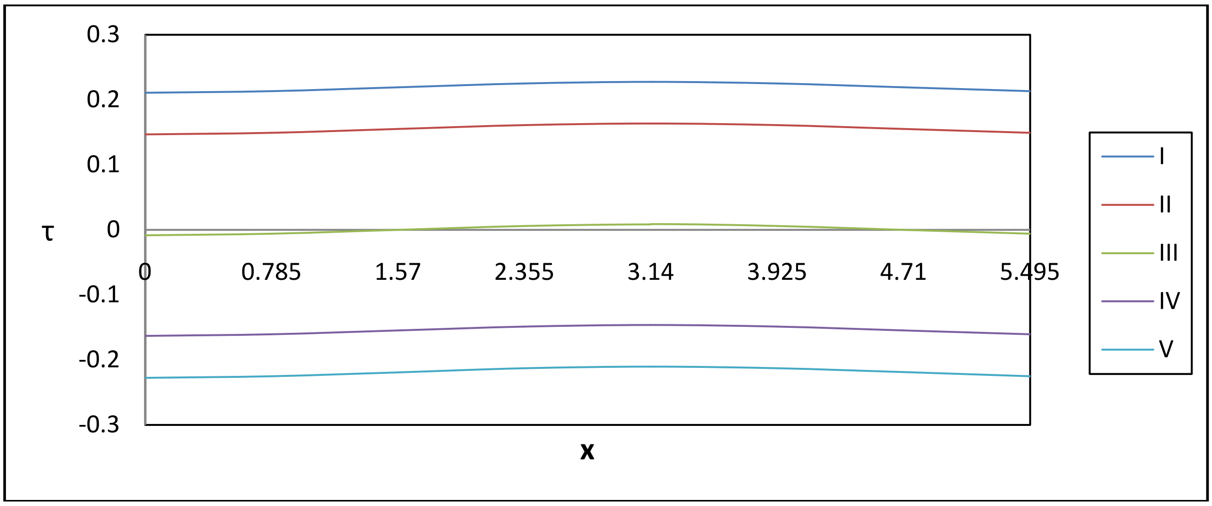

Fig.3: Shere stresses τ for R=4, M=2, β=0.005, Є=0.01, k=0.1, y =1.0043

	I	II	III	IV	V
t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π

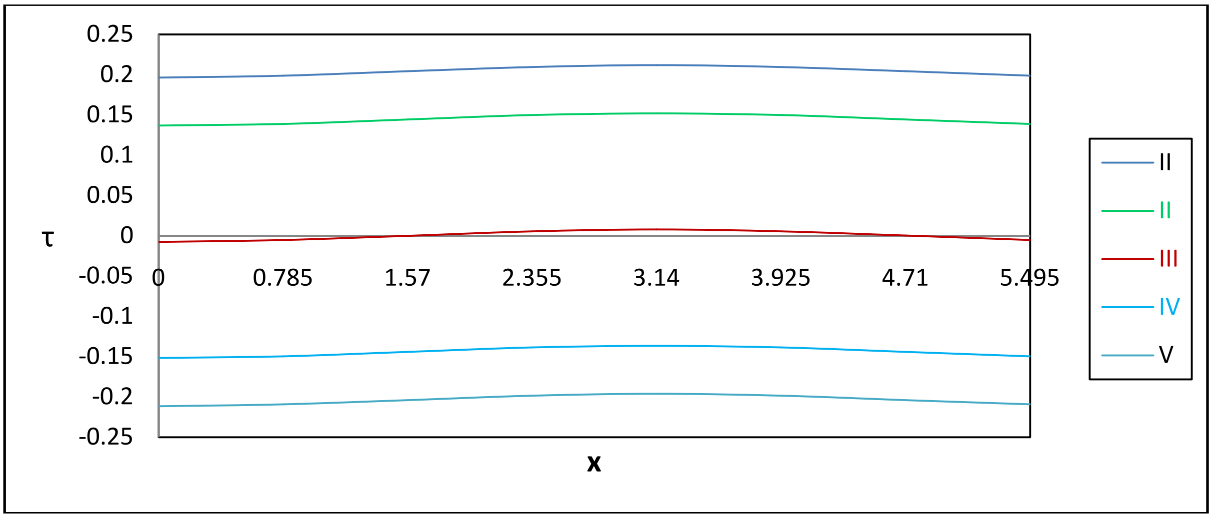

Fig.4: Shere stresses τ for M=3, R=2, β=0.005, Є=0.01, k=0.1, y =1.0043

	I	II	III	IV	V
t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π

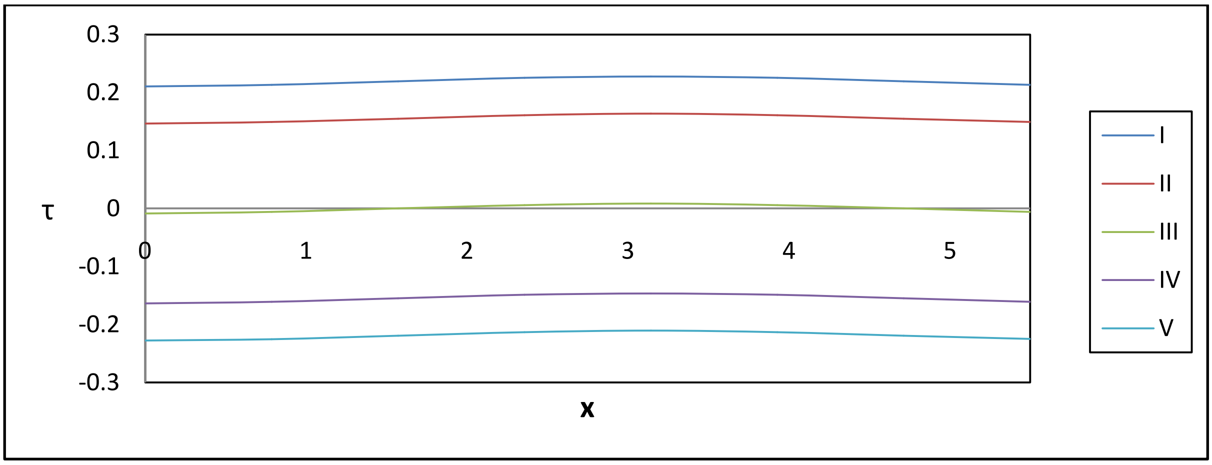

Fig.5: Shere stresses τ for M=4, R=2, β=0.005, Є=0.01, k=0.1, y =1.0043

	I	II	III	IV	V
t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π

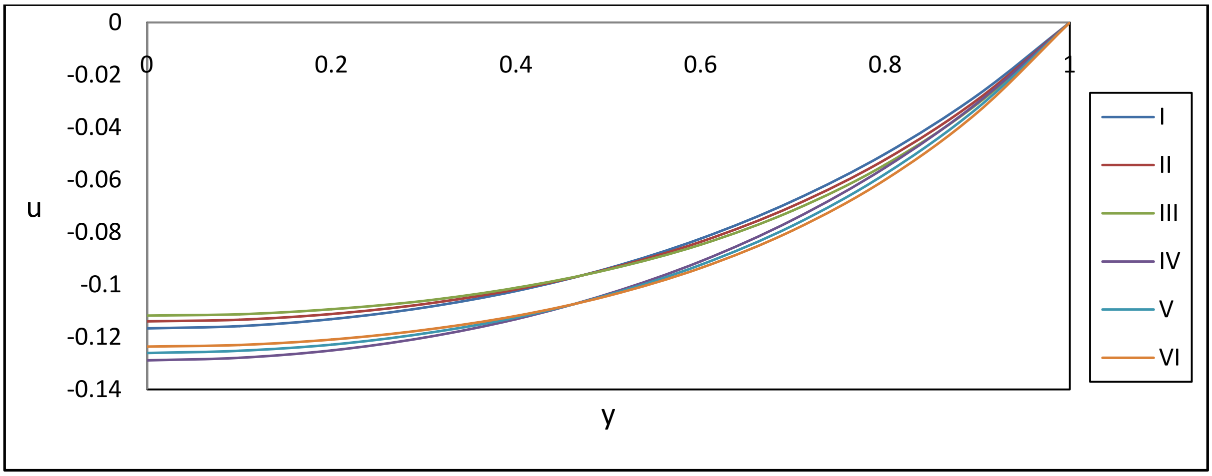

Fig.6: u with R when M=2, Є=0.01, k=0.1, $x = t = \frac{\pi}{6}$

	I	II	III	IV	V	VI
R	2	3	4	2	3	4
β	0.005	0.005	0.005	-0.005	-0.005	-0.005

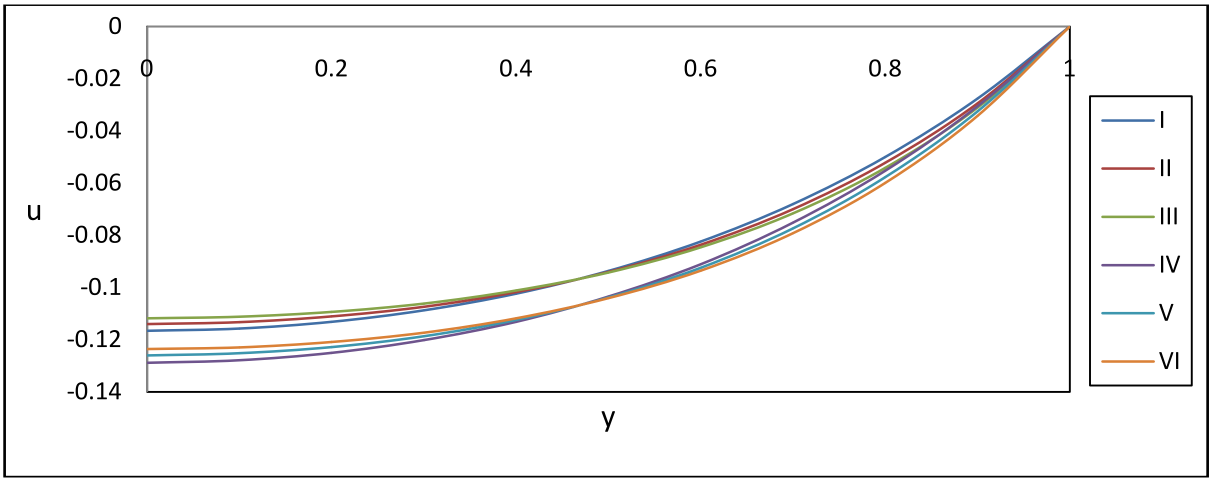

Fig.7: u with M when R=2, Є=0.01, k=0.1, $x = t = \frac{\pi}{6}$

	I	II	III	IV	V	VI
M	2	3	4	2	3	4
β	0.005	0.005	0.005	-0.005	-0.005	-0.005

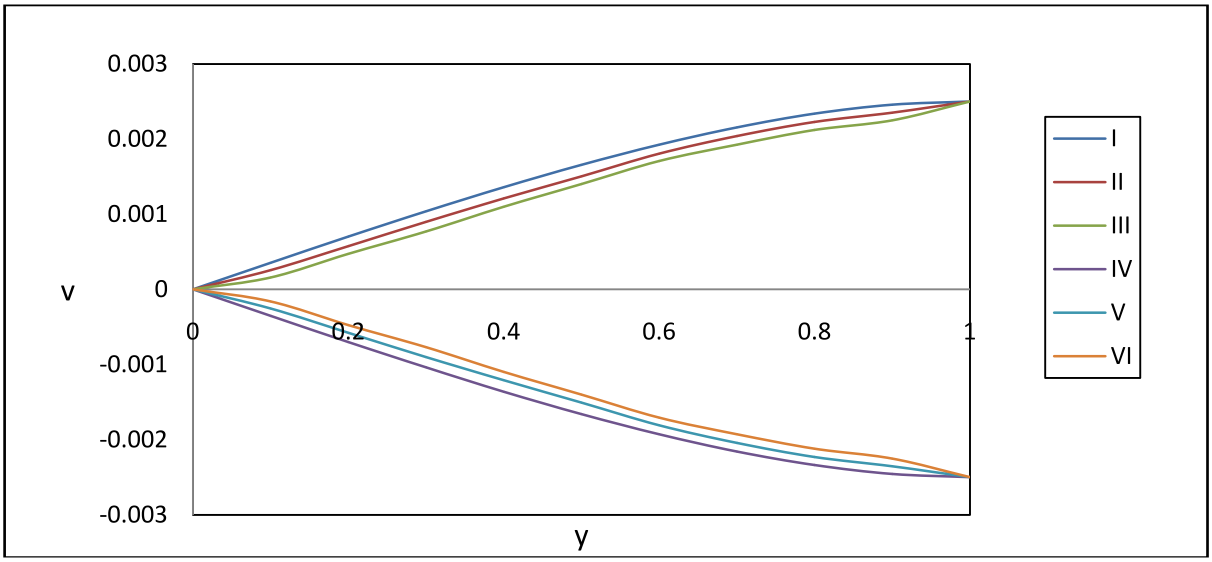

Fig.8: v with R when M=2, Є=0.01, k=0.1, $x = t = \dfrac{\pi}{6}$

	I	II	III	IV	V	VI
R	2	3	4	2	3	4
β	0.005	0.005	0.005	-0.005	-0.005	-0.005

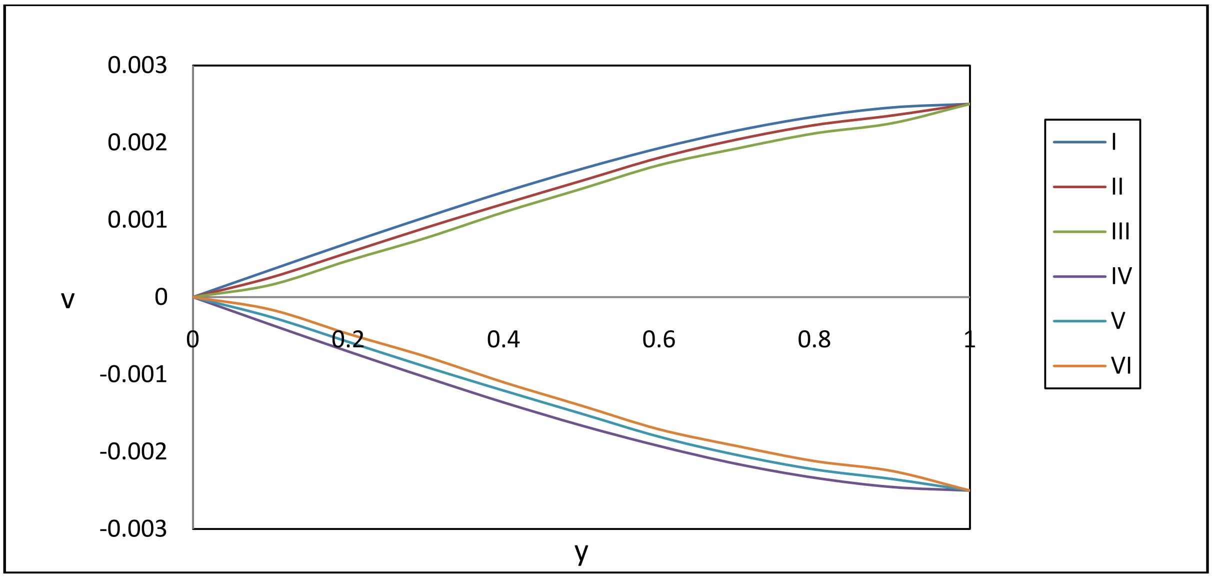

Fig.9: v with M when R=2, Є=0.01, k=0.1, $x = t = \dfrac{\pi}{6}$

	I	II	III	IV	V	VI
M	2	3	4	2	3	4
β	0.005	0.005	0.005	-0.005	-0.005	-0.005

Table - 1

SHEAR STRESS AT THE UPPER WALL (τ)

y =1.0043301, t = Π\2, x =2.355, k=0.1

R	I	II	III	IV	V	VI
2	0.0048657	0.00543912	0.00597136	-0.0048649	-0.00543824	-0.00597042
3	0.00543912	0.0062243	0.00693868	-0.0054382	-0.00622333	-0.00693763
4	0.00597136	0.00693868	0.00780655	-0.0059704	-0.00693763	-0.00780539

Table - 2
FLUID FLUX (Q)
t = Π\2, x =2.355, β = 0.005, S[x] = 1.00433

R	I	II	III	IV	V	VI
2	-0.00353118	-0.00353117	-0.00353116	0.00353118	0.00353117	0.00353116
3	-0.00353117	-0.00353116	-0.00353115	0.00353117	0.00353116	0.00353115
4	-0.00353116	-0.00353115	-0.00353113	0.00353116	0.00353115	0.00353113

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