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Wave Propagation at Micropolar Elastic/Fluid Saturated Porous Solid Interface

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ABSTRACT

The present paper is concerned with the reflection and transmission of longitudinal wave from a plane surface separating a micropolar elastic solid half space and a fluid saturated porous half space. Longitudinal wave impinge obliquely at the interface. Amplitude ratios of various reflected and transmitted waves are obtained and computed numerically for a specific model and results obtained are depicted graphically with angle of incidence of incident wave and material properties of half spaces. It is found that these amplitude ratios depend on angle of incidence of the incident wave. A particular case of reflection at free surface of micro polar elastic solid has been deduced and discussed with the help of graphs. A special case when fluid saturated porous half space reduced to empty porous solid has also been deduced and discussed from the present investigation.

Keywords: *Porous solid, micropolar elastic solid, reflection, refraction, longitudinal wave, amplitude ratios.*

1. INTRODUCTION

Most of natural and man-made materials, including engineering, geological and biological media, possess a microstructure. The ordinary classical theory of elasticity fails owing to the microstructure of the material. To overcome this problem Suhubi and Eringen (1964), Eringen and Suhubi (1964) developed a theory in which they considered the microstructure of the material and they showed that the motion in a granular structure material is characterized not by a displacement vector but also by a rotation vector. Gautheir (1982) found aluminum-epoxy composite to be a micropolar material and investigated the values of relevant parameters based on a specimen of an aluminum-epoxy composite. Many problems of waves and vibrations have been discussed in micropolar elastic solid by several researchers. Some of them are Parfitt and Eringen (1969), Tomar and Gogna (1992), Tomar and Kumar (1995), Singh and Kumar (2007) etc.

In the case of bodies with definite internal structure i.e. sand, fissured rocks, cemented sandstones, limestone’s and other sediments permeated by groundwater or oil (i.e. for porous materials), the existing theories needs to be upgraded. Bowen (1980) and de Boer and Ehlers (1990a, 1990b) developed an interesting theory for porous medium having all constituents to be incompressible. There are sufficient reasons for considering the fluid saturated porous constituents as incompressible. For example, consider the composition of soil in which the solid constituents as well as liquid constituents which are generally water or oils are incompressible. Therefore, the assumption of incompressible constituents meets the properties appearing the in many branches of engineering. .

Based on this theory, many researchers like de Boer and Liu (1994, 1995), de Boer and Liu (1996), Liu (1999), Yan et.al. (1999), Kumar and Hundal (2003), de Boer and Didwania (2004), Tajuddin and Hussaini (2006), Kumar et.al. (2011) etc. studied some problems of wave propagation in fluid saturated porous media.

Using the theory of de Boer and Ehlers (1990) for fluid saturated porous medium and Eringen (1968) for micro polar elastic solid, the reflection and transmission phenomenon of longitudinal wave at an interface between micropolar elastic solid half space and fluid saturated porous half space is studied. The reflection coefficient of reflected waves at the free surface of micropolar elastic solid half space have also been obtained as a particular case. A special case when fluid saturated porous half space reduced to empty porous solid has been deduced and discussed with the help of graphs.

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2. BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

For medium M_1 (micropolar elastic solid)

The equation of motion in micropolar elastic medium are given by Eringen (1968) as

$$({c_1}^2+{c_3}^2)\nabla^2\phi=\frac{\partial^2\phi}{\partial t^2}, \tag{1}$$

$$({c_2}^2+{c_3}^2)\nabla^2\vec{U}+{c_3}^2\nabla\times\vec{\Phi}=\frac{\partial^2\vec{U}}{\partial t^2}, \tag{2}$$

$$({c_4}^2\nabla^2-2{\omega_0}^2)\vec{\Phi}+{\omega_0}^2\nabla\times\vec{U}=\frac{\partial^2\vec{\Phi}}{\partial t^2}, \tag{3}$$

where

$${c_1}^2=\frac{\lambda+2\mu}{\rho},\quad {c_2}^2=\frac{\mu}{\rho},\quad {c_3}^2=\frac{\kappa}{\rho},\quad {c_4}^2=\frac{\gamma}{\rho j},\quad {\omega_0}^2=\frac{\kappa}{\rho j}, \tag{4}$$

Parfitt and Eringen (1969) have shown that eq. (1) corresponds to longitudinal wave propagating with velocityV_1, given by ${V_1}^2={c_1}^2+{c_3}^2$, and eqs. (2)- (3) are coupled equations in vector potentials $\vec{U}$ and $\vec{\Phi}$ and these correspond to coupled transverse and micro-rotation waves. If $\frac{\omega^2}{{\omega_0}^2}>2$, there exist two sets of coupled-wave propagating with velocities $1/\lambda_1$ and $1/\lambda_2$; where

$${\lambda_1}^2=\frac{1}{2}\left[B-\sqrt{B^2-4C}\right],\quad {\lambda_2}^2=\frac{1}{2}\left[B+\sqrt{B^2-4C}\right], \tag{5}$$

where

$$B=\frac{q(p-2)}{\omega^2}+\frac{1}{({c_2}^2+{c_3}^2)}+\frac{1}{{c_4}^2},\qquad C=\left(\frac{1}{{c_4}^2}-\frac{2q}{\omega^2}\right)\frac{1}{({c_2}^2+{c_3}^2)},$$

$$p=\frac{\kappa}{\mu+\kappa},\quad q=\frac{\kappa}{\gamma}. \tag{6}$$

We consider a two dimensional problem by taking the following components of displacement and micro rotation as

$$\vec{u}=(u.0.w),\qquad \vec{\Phi}=(0,\Phi_2,0), \tag{7}$$

where

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z},\qquad w=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}, \tag{8}$$

and components of stresses as

$$t_{zz}=(\lambda+2\mu+\kappa)\frac{\partial^2\phi}{\partial z^2}+\lambda\frac{\partial^2\phi}{\partial x^2}+(2\mu+\kappa)\frac{\partial^2\psi}{\partial x\,\partial z}, \tag{9}$$

$$t_{zx}=(2\mu+\kappa)\frac{\partial^2\phi}{\partial x\,\partial z}-(\mu+\kappa)\frac{\partial^2\psi}{\partial z^2}+\mu\frac{\partial^2\psi}{\partial x^2}-\kappa\Phi_2, \tag{10}$$

$$m_{zy}=\gamma\frac{\partial\Phi_2}{\partial z}. \tag{11}$$

For medium M_2(fluid saturated incompressible porous half space)

Following de Boer and Ehlers (1990b), the governing equations in a fluid-saturated incompressible porous medium are

$$\operatorname{div}(\eta^S\dot{\mathbf{x}}_S+\eta^F\dot{\mathbf{x}}_F)=0. \tag{12}$$

$$\operatorname{div}\mathbf{T}_{\mathbf{E}}^{\mathbf{S}}-\eta^S\operatorname{grad}p+\rho^S(\mathbf{b}-\ddot{\mathbf{x}}_s)-\mathbf{P}_{\mathbf{E}}^{\mathbf{F}}=0, \tag{13}$$

$$\operatorname{div}\mathbf{T}_{\mathbf{E}}^{\mathbf{F}}-\eta^F\operatorname{grad}p+\rho^F(\mathbf{b}-\ddot{\mathbf{x}}_F)+\mathbf{P}_{\mathbf{E}}^{\mathbf{F}}=0, \tag{14}$$

where $\dot{\mathbf{x}}_i$ and $\ddot{\mathbf{x}}_i$ $(i = S, F)$ denote the velocities and accelerations, respectively of solid (S) and fluid (F) phases of the porous aggregate and p is the effective pore pressure of the incompressible pore fluid. ρ^S and ρ^F are the densities of the solid and fluid phases respectively and $\mathbf{b}$ is the body force per unit volume. $\mathbf{T}_\mathbf{E}^\mathbf{S}$ and $\mathbf{T}_\mathbf{E}^\mathbf{F}$ are the effective stress in the solid and fluid phases respectively, $\mathbf{P}_\mathbf{E}^\mathbf{F}$ is the effective quantity of momentum supply and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \tag{15}$$

If $\mathbf{u}_S$ and $\mathbf{u}_F$ are the displacement vectors for solid and fluid phases, then

$$\dot{x}_S = \dot{\mathbf{u}}_S, \quad \ddot{\mathbf{x}}_s = \ddot{\mathbf{u}}_s, \quad \dot{\mathbf{x}}_F = \dot{\mathbf{u}}_F, \quad \ddot{\mathbf{x}}_F = \ddot{\mathbf{u}}_F. \tag{16}$$

The constitutive equations for linear isotropic, elastic incompressible porous medium are given by de Boer, Ehlers and Liu (1993) as

$$\mathbf{T}_\mathbf{E}^\mathbf{S} = 2\mu^S \mathbf{E}_S + \lambda^S (E_S . \mathbf{I})\mathbf{I}, \tag{17}$$

$$\mathbf{T}_\mathbf{E}^\mathbf{F} = 0, \tag{18}$$

$$\mathbf{P}_\mathbf{E}^\mathbf{F} = -\mathbf{S}_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S), \tag{19}$$

where λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and $\mathbf{E}_S$ is the linearized Langrangian strain tensor defined as

$$\mathbf{E}_S = \frac{1}{2}(\text{grad}\,\mathbf{u}_S + \text{grad}^T \mathbf{u}_S), \tag{20}$$

In the case of isotropic permeability, the tensor $\mathbf{S}_v$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990b) as

$$\mathbf{S}_v = \frac{(\eta^F)^2 \gamma^{FR}}{K^F}\mathbf{I}, \tag{21}$$

where γ^{FR} is the specific weight of the fluid and K^F is the Darcy's permeability coefficient of the porous medium.

Making the use of (16) in equations (12)-(1 4), and with the help of (17)-(20), we obtain

$$\text{div}(\eta^S \dot{\mathbf{u}}_S + \eta^F \dot{\mathbf{u}}_F) = 0, \tag{22}$$

$$(\lambda^S + \mu^S)\text{grad div}\,\mathbf{u}_S + \mu^S \text{div grad}\,\mathbf{u}_S - \eta^S \text{grad p} + \rho^S(\mathbf{b} - \ddot{\mathbf{u}}_s) + S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0, \tag{23}$$

$$-\eta^F \text{grad p} + \rho^F(\mathbf{b} - \ddot{\mathbf{u}}_F) - S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0. \tag{24}$$

For the two dimensional problem, we assume the displacement vector $\mathbf{u}_i$ $(i = F, S)$ as

$$\mathbf{u}_i = (u^i, 0, w^i) \quad \text{where} \quad i = F, S. \tag{25}$$

Equations (22) - (24) with the help of eq. (25) in absence of body forces take the form

$$\eta^S \left[\frac{\partial^2 u^S}{\partial x\,\partial t} + \frac{\partial^2 w^S}{\partial z\,\partial t}\right] + \eta^F \left[\frac{\partial^2 u^F}{\partial x\,\partial t} + \frac{\partial^2 w^F}{\partial z\,\partial t}\right] = 0, \tag{26}$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{27}$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{28}$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{29}$$

$$\left(\lambda^S+\mu^S\right)\frac{\partial\theta^S}{\partial z}+\mu^S\nabla^2 w^S-\eta^S\frac{\partial p}{\partial z}-\rho^S\frac{\partial^2 w^S}{\partial t^2}+S_v\left[\frac{\partial w^F}{\partial t}-\frac{\partial w^S}{\partial t}\right]=0, \tag{30}$$

where

$$\theta^S=\frac{\partial(u^S)}{\partial x}+\frac{\partial(w^S)}{\partial z}, \tag{31}$$

and

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}. \tag{32}$$

Also, ${t_{zz}}^S$ and ${t_{zx}}^S$ the normal and tangential stresses in the solid phase are as under

$${t_{zz}}^S=\lambda^S\left(\frac{\partial u^S}{\partial x}+\frac{\partial w^S}{\partial z}\right)+2\mu^S\frac{\partial w^S}{\partial z}, \tag{33}$$

$${t_{zx}}^S=\mu^S\left(\frac{\partial u^S}{\partial z}+\frac{\partial w^S}{\partial x}\right). \tag{34}$$

The displacement components u^j and w^j are related to the dimensional potential ϕ^j and ψ^j as

$$u^j=\frac{\partial\phi^j}{\partial x}+\frac{\partial\psi^j}{\partial z},\quad w^j=\frac{\partial\phi^j}{\partial z}-\frac{\partial\psi^j}{\partial x},\qquad j=S,F. \tag{35}$$

Using eq. (35) in equations (26)-(30), we obtain the following equations determining ϕ^S, ϕ^F, ψ^S, ψ^F and p as:

$$\nabla^2\phi^S-\frac{1}{{C_1}^2}\frac{\partial^2\phi^S}{\partial t^2}-\frac{S_v}{\left(\lambda^S+2\mu^S\right)(\eta^F)^2}\frac{\partial\phi^S}{\partial t}=0, \tag{36}$$

$$\phi^F=-\frac{\eta^S}{\eta^F}\phi^S, \tag{37}$$

$$\mu^S\nabla^2\psi^S-\rho^S\frac{\partial^2\psi^S}{\partial t^2}+S_v\left[\frac{\partial\psi^F}{\partial t}-\frac{\partial\psi^S}{\partial t}\right]=0, \tag{38}$$

$$\rho^F\frac{\partial^2\psi^F}{\partial t^2}+S_v\left[\frac{\partial\psi^F}{\partial t}-\frac{\partial\psi^S}{\partial t}\right]=0, \tag{39}$$

$$(\eta^F)^2 p-\eta^S\rho^F\frac{\partial^2\phi^S}{\partial t^2}-S_v\frac{\partial\phi^S}{\partial t}=0, \tag{40}$$

where

$$C_1=\sqrt{\frac{(\eta^F)^2\left(\lambda^S+2\mu^S\right)}{(\eta^F)^2\rho^S+(\eta^S)^2\rho^F}}. \tag{41}$$

Assuming the solution of the system of equations (36) - (40) in the form

$$\left(\phi^S,\phi^F,\psi^S,\psi^F,p\right)=\left({\phi_1}^S,{\phi_1}^F,{\psi_1}^S,{\psi_1}^F,p_1\right)\exp(i\omega t), \tag{42}$$

where ω is the complex circular frequency.

Making the use of (42) in equations (36)-(40), we obtain

$$\left[\nabla^2+\frac{\omega^2}{{C_1}^2}-\frac{i\omega S_v}{\left(\lambda^S+2\mu^S\right)(\eta^F)^2}\right]{\phi_1}^S=0, \tag{43}$$

$$[\mu^S\nabla^2 + \rho^S\omega^2 - i\omega S_v]\psi_1{}^S = -i\omega S_v\psi_1{}^F, \tag{44}$$

$$[-\omega^2\rho^F + i\omega S_v]\psi_1{}^F - i\omega S_v\psi_1{}^S = 0, \tag{45}$$

$$(\eta^F)^2 p_1 + \eta^S\rho^F\omega^2\phi_1{}^S - i\omega S_v\phi_1{}^S = 0, \tag{46}$$

$$\phi_1{}^F = -\frac{\eta^S}{\eta^F}\phi_1{}^S. \tag{47}$$

Equation (43) corresponds to longitudinal wave propagating with velocity $\overline{V}_1$, given by

$$\overline{V}_1{}^2 = \frac{1}{G_1}, \tag{48}$$

where

$$G_1 = \left[\frac{1}{C_1{}^2} - \frac{iS_v}{\omega(\lambda^S + 2\mu^S)(\eta^F)^2}\right]. \tag{49}$$

From equation (44) and (45), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{\overline{V_2}^2}\right]\psi_1{}^S = 0, \tag{50}$$

Equation (50) corresponds to transverse wave propagating with velocity$\overline{V}_2$, given by $\overline{V}_2{}^2 = 1/G_2$

where

$$G_2 = \left\{\frac{\rho^S}{\mu^S} - \frac{iS_v}{\mu^S\omega} - \frac{S_v{}^2}{\mu^S(-\rho^S\omega^2 + i\omega S_v)}\right\}, \tag{51}$$

3. FORMULATION OF THE PROBLEM

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and the plane interface z=0 separating the uniform micropolar elastic solid half spaceM_1 [z>0] and fluid saturated porous half space M_2 [z<0]. A longitudinal wave propagating through a medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to incident longitudinal wave, we get three reflected waves in the medium M_1 and two transmitted waves in medium M_2. See fig.1

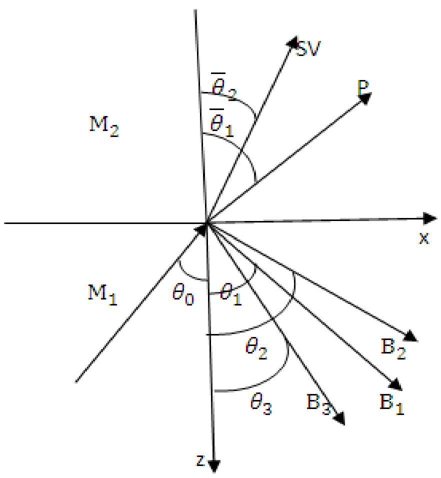

Fig.1: Geometry of the problem.

In medium M_1

$$\phi = B_0 \exp\{ik_0 (x \sin\theta_0 - z \cos\theta_0) + i\omega_1 t\} + B_1 \exp\{ik_0 (x \sin\theta_1 + z \cos\theta_1) + i\omega_1 t\}, \tag{52}$$

$$\psi = B_2 \exp\{i\delta_1 (x \sin\theta_2 + z \cos\theta_2) + i\omega_2 t\} + B_3 \exp\{i\delta_2 (x \sin\theta_3 + z \cos\theta_3) + i\omega_3 t\}, \tag{53}$$

$$\Phi_2 = EB_2 \exp\{i\delta_1(x \sin\theta_2 + z\cos\theta_2) + i\omega_2 t\} + FB_3 \exp\{i\delta_2(x\sin\theta_3 + z\cos\theta_3) + i\omega_3 t\}, \tag{54}$$

where

$$E = \frac{\delta_1{}^2\left(\delta_1{}^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno .}}, \tag{55}$$

$$F = \frac{\delta_2{}^2\left(\delta_2{}^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno.}}, \tag{56}$$

and

$$\text{deno.} = p\left(2q - \frac{\omega^2}{c_4{}^2}\right), \quad \delta_1{}^2 = \lambda_1{}^2\omega^2, \quad \delta_2{}^2 = \lambda_2{}^2\omega^2 . \tag{57}$$

In medium M_2

$$\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\}\left[A_1 \exp\left\{i\bar{k}_1\left(x\sin\bar{\theta}_1 - z\cos\bar{\theta}_1\right) + i\bar{\omega}_1 t\right\}\right], \tag{58}$$

$$\{\psi^S, \psi^F\} = \{1, m_3\}\left[A_2 \exp\left\{i\bar{k}_2\left(x\sin\bar{\theta}_2 - z\cos\bar{\theta}_2\right) + i\bar{\omega}_2 t\right\}\right], \tag{59}$$

where

$$m_1 = -\frac{\eta^S}{\eta^F}, \quad m_2 = -\left[\frac{\eta^S\omega_1{}^2\rho^F - i\omega_1 S_v}{(\eta^F)^2}\right], \quad m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - \omega_2{}^2\rho^F}, \tag{60}$$

and B_0, B_1, B_2, B_3 are amplitudes of incident P-wave, reflected P-wave, reflected coupled transverse and reflected micro-rotation waves respectively. Also A_1 and A_2 are amplitudes of transverse P-wave and SV-wave, respectively and to be determined from boundary conditions.

4. BOUNDARY CONDITIONS

The appropriate boundary conditions are the continuity of displacement, micro rotation and stresses at the interface separating media M_1 and M_2. Mathematically, these boundary conditions at z=0 can be expressed as:

$$t_{zz} = t_{zz}{}^S - p, \qquad t_{zx} = t_{zx}{}^S, \quad m_{zy} = 0, \quad u = u^S, \quad w = w^S, \tag{61}$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{\lambda_1^{-1}} = \frac{\sin\theta_3}{\lambda_2^{-1}} = \frac{\sin\bar{\theta}_1}{\bar{V}_1} = \frac{\sin\bar{\theta}_2}{\bar{V}_2}, \tag{62}$$

For longitudinal wave,

$$V_0 = V_1, \quad \theta_0 = \theta_1, \tag{63}$$

Also

$$k_0 V_1 = \delta_1\lambda_1^{-1} = \delta_2\lambda_2^{-1} = \bar{k}_1\bar{V}_1 = \bar{k}_2\bar{V}_2 = \omega, \quad \text{at } z = 0 \tag{64}$$

Making the use of potentials given by equations (52)-(54) and (58)-(59) in the boundary conditions given by (61) and using (62)-(64), we get a system of five non homogeneous which can be written as

$$\sum_{j=1}^{5} a_{ij} Z_j = Y_i, \qquad (i = 1,2,3,4,5) \tag{65}$$

Where

$$Z_1 = \frac{B_1}{B_0}, \quad Z_2 = \frac{B_2}{B_0}, \quad Z_3 = \frac{B_3}{B_0}, \quad Z_4 = \frac{A_1}{B_0}, \quad Z_5 = \frac{A_2}{B_0}, \tag{66}$$

where Z_1 to Z_5 are the amplitude ratios of reflected longitudinal wave, reflected coupled-wave at an angle θ_2, reflected coupled-wave at an angle θ_3,refracted P-wave and refracted SV-wave, respectively. Also a_{ij} and Y_i in non-dimensional form are as

$$a_{11} = \left[\frac{\lambda}{\mu} + D_2\cos^2\theta_1\right],\quad a_{12} = D_2\,\frac{{\delta_1}^2}{k_0^2}\sin\theta_2\cos\theta_2,\quad a_{13} = D_2\sin\theta_3\cos\theta_3\,\frac{{\delta_2}^2}{k_0^2},$$

$$a_{14} = \frac{-\bar{k}_1^2\left(\lambda^S + 2\mu^S\cos^2\bar{\theta}_1\right) - m_2}{\mu\, k_0^2},\quad a_{15} = \frac{-2\mu^S\bar{k}_2^2\sin\bar{\theta}_2\cos\bar{\theta}_2}{\mu k_0^2},\quad Y_1 = -a_{11},$$

$$a_{21} = D_2\sin\theta_1\cos\theta_1,\quad a_{22} = \frac{-{\delta_1}^2}{k_0^2}\left[\left(D_1\cos^2\theta_2 - \sin^2\theta_2\right) - \frac{\kappa}{\mu}\frac{E}{{\delta_1}^2}\right],$$

$$a_{23} = \frac{-{\delta_2}^2}{k_0^2}\left[\left(D_1\cos^2\theta_3 - \sin^2\theta_3\right) - \frac{\kappa}{\mu}\frac{F}{{\delta_2}^2}\right],\quad a_{24} = \frac{\mu^S\bar{k}_1^2\sin 2\bar{\theta}_1}{\mu k_0^2},$$

$$a_{25} = \frac{\mu^S\bar{k}_2^2\left(\sin^2\bar{\theta}_2 - \cos^2\bar{\theta}_2\right)}{\mu k_0^2},\quad Y_2 = a_{21},$$

$$a_{31} = \sin\theta_1,\quad a_{32} = -\frac{\delta_1}{k_0}\cos\theta_2,\quad a_{33} = -\frac{\delta_2}{k_0}\cos\theta_3,\quad a_{34} = -\frac{\bar{k}_1}{k_0}\sin\bar{\theta}_1,$$

$$a_{35} = \frac{\bar{k}_2}{k_0}\cos\bar{\theta}_2,\quad Y_3 = -a_{31},$$

$$a_{41} = \cos\theta_1,\quad a_{42} = \frac{\delta_1}{k_0}\sin\theta_2,\quad a_{43} = \frac{\delta_2}{k_0}\sin\theta_3,\quad a_{44} = \frac{\bar{k}_1}{k_0}\cos\bar{\theta}_1,$$

$$a_{45} = \frac{\bar{k}_2}{k_0}\sin\bar{\theta}_2,\quad Y_4 = a_{41},$$

$$a_{51} = 0,\quad a_{52} = \cos\theta_2,\quad a_{53} = \frac{F\delta_2}{E\delta_1}\cos\theta_3,\quad a_{54} = 0,\quad a_{55} = 0,\quad Y_5 = 0. \tag{68}$$

5. NUMERICAL RESULTS AND DISCUSSION

The theoretical results obtained above indicate that the amplitude ratios Z_i $(i = 1,2,3,4,5)$ depend on the angle of incidence of incident wave and elastic properties of half spaces. In order to study in more detail the behaviour of various amplitude ratios, we have computed them numerically for a particular model for which the values of relevant elastic parameters are as follow

In medium M_1 , the physical constants for micropolar elastic solid are taken from Gauthier (1982) as

$$\lambda = 7.59\times10^{10}\ \mathrm{N/m^2},\qquad \mu = 1.89\times10^{10}\ \mathrm{N/m^2},\qquad \kappa = 1.49\times10^{8}\ \mathrm{N/m^2},$$
$$\rho = 2.19\times10^{3}\mathrm{kg/m^3},\qquad \gamma = 2.68\times10^{4}\ \mathrm{N},\qquad j = 1.96\times10^{-6}\mathrm{m^2},\ \frac{\omega^2}{{\omega_0}^2} = 2 \tag{69}$$

In mediumM_2, the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu (1993) as

$$\eta^S = 0.67,\quad \eta^F = 0.33,\quad \rho^S = 1.34\ \mathrm{Mg/m^3},\quad \rho^F = 0.33\ \mathrm{Mg/m^3},$$
$$\lambda^S = 5.5833\ \mathrm{MN/m^2},\qquad K^F = 0.01\frac{m}{s},\quad \gamma^{FR} = 10.00\mathrm{KN/m^3},\quad \mu^S = 8.3750\mathrm{N/m^2}, \tag{70}$$

A computer programme in MATLAB has been developed to calculate the modulus of amplitude ratios of various reflected and transmitted waves for the particular model and to depict graphically. In figures (2) - (6) solid lines show the variations of amplitude ratios when medium-I is micropolar elastic solid and medium-II is incompressible fluid saturated porous medium (FS) whereas dashed lines show the variations of amplitude ratios when medium-II becomes incompressible empty porous solid (EPS). Figures (2) - (6) indicates there is almost negligible effect of pores fluid.

Figures (7) - (9) shows the variation of the modulus of the amplitude ratios of various reflected waves at free surface of micropolar elastic solid (MES).In these figures solid lines show the variations of amplitude ratios when medium is micropolar elastic solid. In all the figures (2)-(9) the amplitude ratios first increase to their maximum values and after getting maximum values there is monotonic fall.

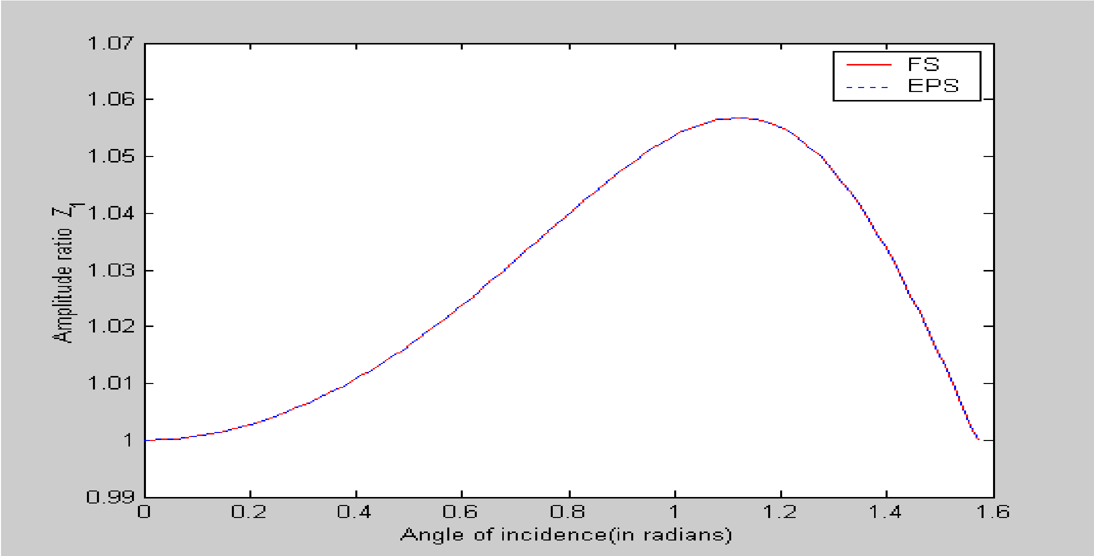

Fig.2: Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident longitudinal wave

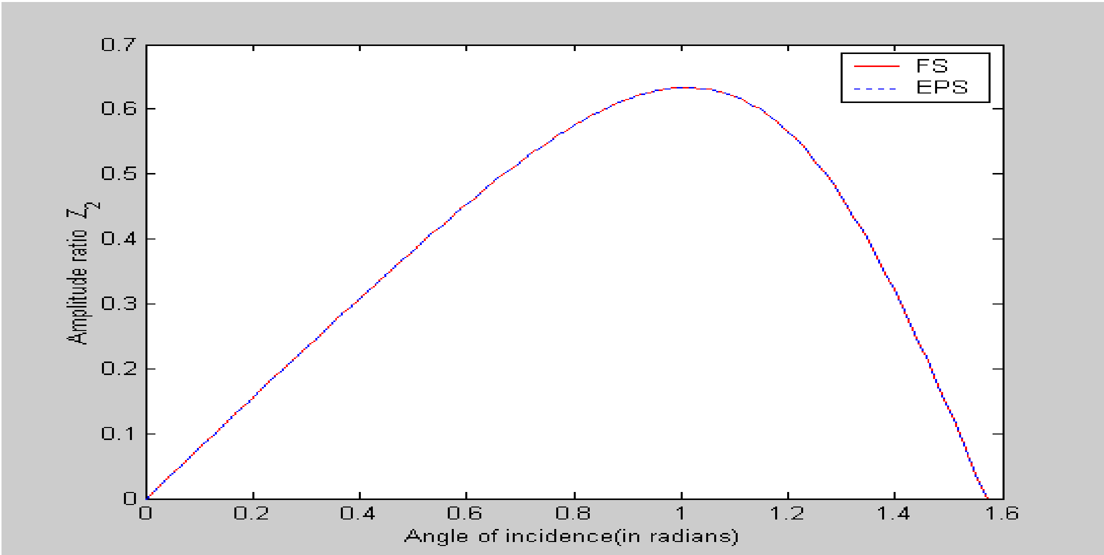

Fig.3: Variation of the amplitude ratio $|Z_2|$ with angle of incidence of the incident longitudinal wave

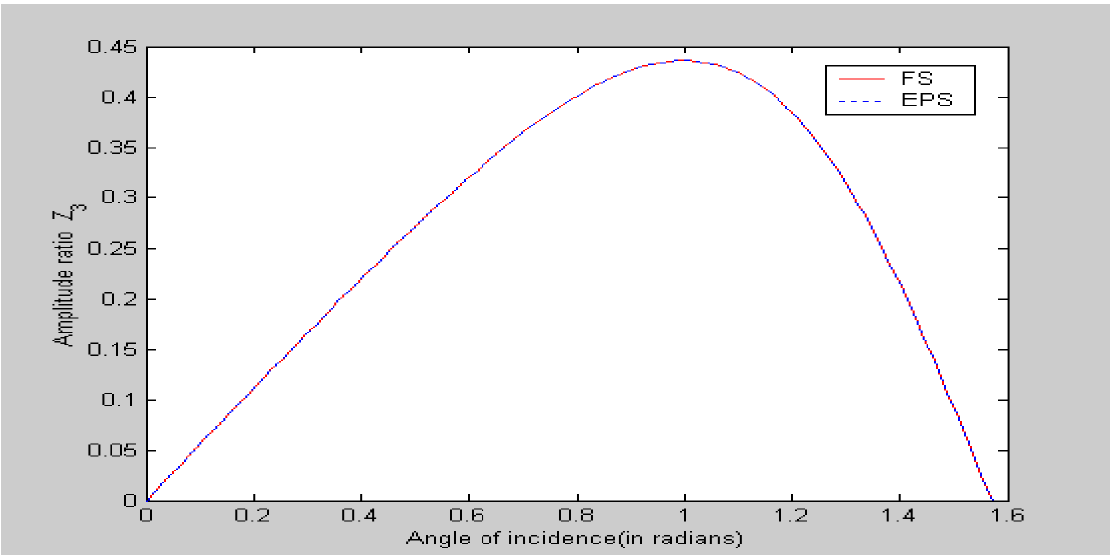

Fig.4: Variation of the amplitude ratio $|Z_3|$ with angle of incidence of the incident longitudinal wave

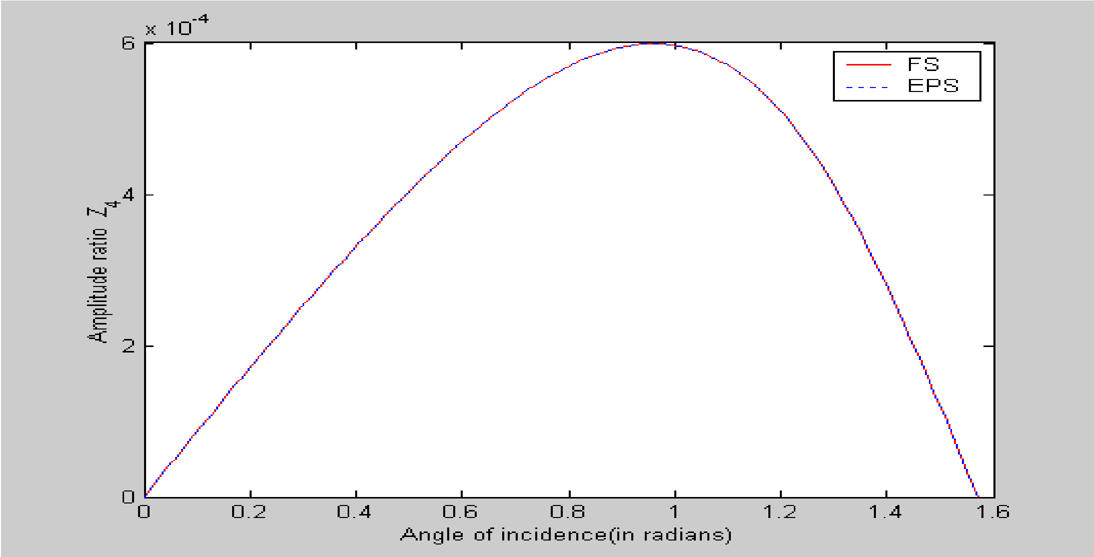

Fig.5: Variation of the amplitude ratio $|Z_4|$ with angle of incidence of the incident longitudinal wave

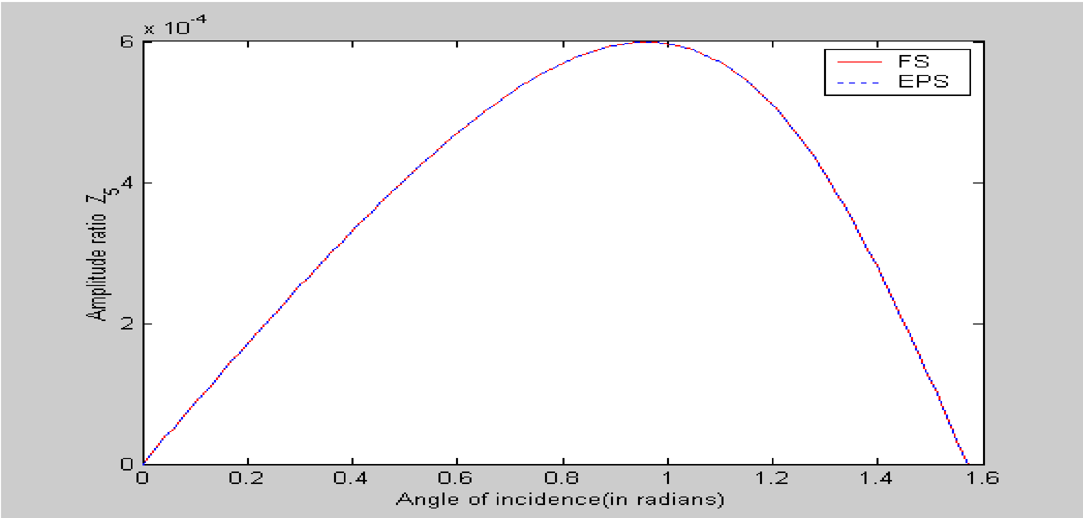

Fig.6: Variation of the amplitude ratio $|Z_5|$ with angle of incidence of the incident longitudinal wave

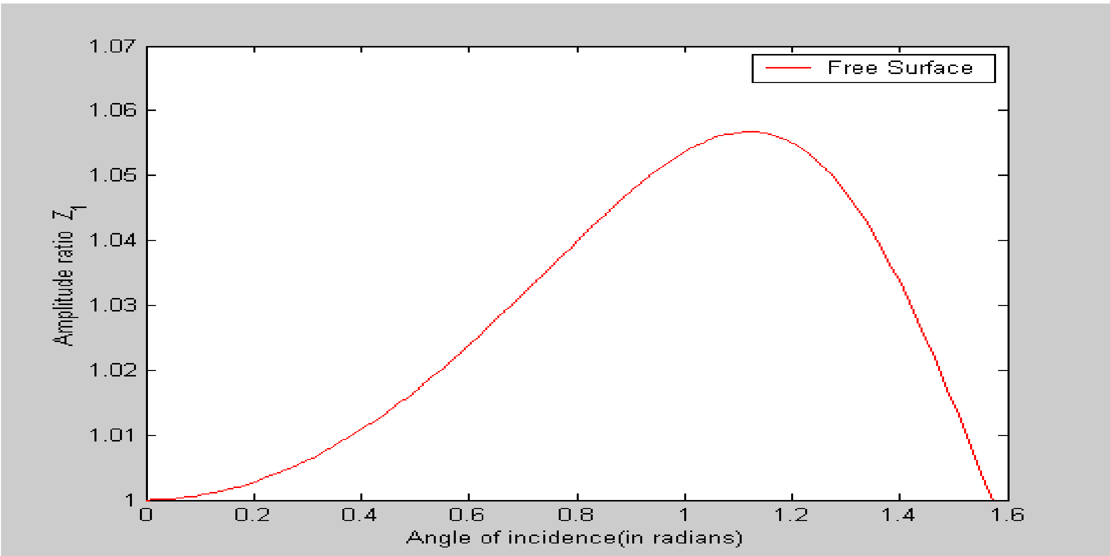

Fig.7: Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident longitudinal wave (free surface)

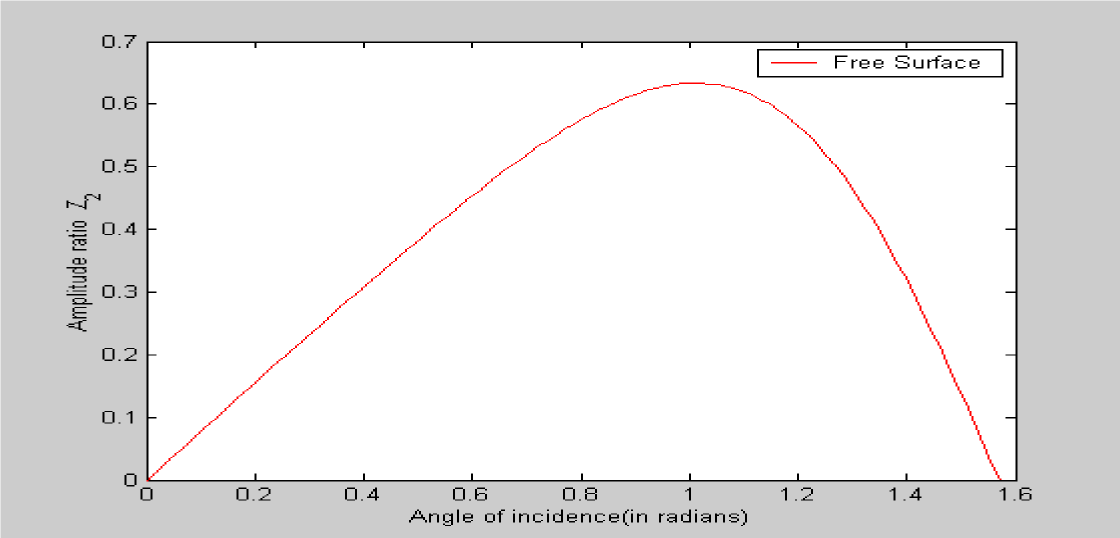

Fig.8: Variation of the amplitude ratio $|Z_1|$ with angle of incidence of the incident longitudinal wave (free surface)

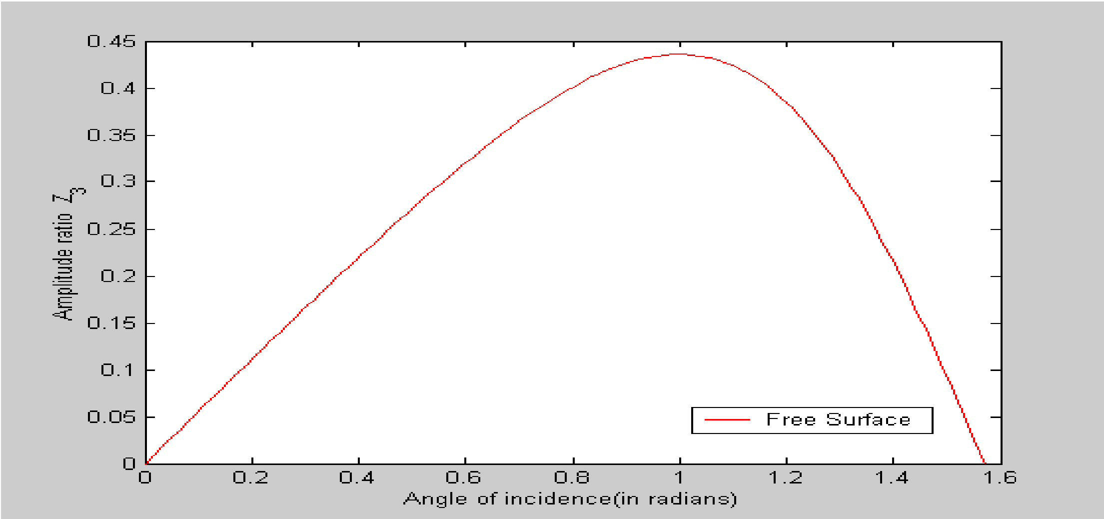

Fig.9: Variation of the amplitude ratio $|Z_3|$ with angle of incidence of the incident longitudinal wave (free surface)

7. CONCLUSION

In conclusion, a mathematical study of reflection and refraction coefficients at an interface separating micropolar elastic solid half space and fluid saturated incompressible porous half space is made when longitudinal wave is incident. It is observed that the amplitudes ratios of various reflected and refracted waves depend on the angle of incidence of the incident wave and material properties of half spaces. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is not significant on the amplitudes ratios. The model presented in this paper is one of the more realistic forms of the earth models. It may be of some use in engineering, seismology and geophysics etc.

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