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CONTRA REGULAR PRE-CLOSED MAPPINGS

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ABSTRACT

The aim of this paper is to introduce and study the concept of contra rp-closed mappings and the interrelationship between other contra-closed maps.

Keywords: *rp-open set, rp-open map, rp-closed map, contra-closed map, contra-pre closed map and contra rp-closed map.*

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1. INTRODUCTION

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced α-open and α-closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-clsoed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced β-open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about β-closed mappings in the year 2000. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. In the year 2010, S. Balasubramanian and P.A.S.Vyjayanthi introduced *v*-open mappings and in the year 2011 they further defined almost *v*-open mappings. In the last year S. Balasubramanian and P.A.S.Vyjayanthi introduced *v*-closed and Almost *v*-closed mappings. Inspired with these concepts and its interesting properties we in this paper tried to study a new variety of closed maps called contra rp-closed maps. Throughout the paper X, Y means topological spaces (X, τ) and (Y,σ) on which no separation axioms are assured.

§2. PRELIMINARIES

Definition 2.1: $A\subseteq X$ is said to be

a) regular open[pre-open; semi-open; α-open; β-open] if $A = int(cl(A))$ [$A \subseteq int(cl(A))$; $A\subseteq cl(int(A))$; $A\subseteq int(cl(int(A)))$; $A\subseteq cl(int(cl(A)))$] and regular closed[pre-closed; semi-closed; α-closed; β-closed] if $A = cl(int(A))$[$cl(int(A)) \subseteq A$; $int(cl(A))\subseteq A$; $cl(int(cl(A)))\subseteq A$; $int(cl(int(A))) \subseteq A$]
b) θ-pre-closed if $A = pCl_\theta(A) = \{x\in X : pcl(V)\cap A \neq \phi$; for all $V\in PO(X, x)\}$; $pCl_\theta(A)$ is θ-pre-closure of A. The complement of a θ-pre-closed set is said to be θ-pre-open.
c) *rp*-open if $A = pint(pcl(A))$ and *rp*-closed if $A = pcl(pint(A))$.
d) *rp*-dense in X if $rpcl(A) = X$.
e) θ-closed if $A = Cl_\theta(A)$. The complement of a θ-closed set is said to be θ-open.

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f) g-closed[rg-closed; pg-closed] if cl(A)⊂U[rcl(A)⊂U; pcl(A)⊂U] whenever A⊂U and U is open[r-open; pre-open] in X.
g) g-open[rg-open, pg-open] if its complement X - A is g-closed[rg-closed, pg-closed].
h) Zero[pre-zero; semi-zero] set of X if there exists a continuous [pre-continuous; semi-continuous] function *f:* X → R such that A = {x∈X : *f*(x) = 0}. Its complement is called co-zero[co-pre-zero; co-semi-zero] set of X.

Remark 1: We have the following implication diagrams for open sets.

rα-open → regularpre open
↑ ↓
r-open → open → α-open → pre-open → β-open

Definition 2.2: A function *f: X→Y* is said to be
a) continuous[resp:semi-continuous, r-continuous, rp-continuous] if the inverse image of every open set is open [resp: semi open, regular open, regular pre-open].
b) irresolute [resp: r-irresolute, rp-irresolute] if the inverse image of every semi open [resp: regular open, rp-open] set is semi open [resp: regular open, rp-open].
c) closed[resp: semi-closed, r-closed] if the image of every closed set is closed [resp: semi closed, regular closed].
d) g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed].

Definition 2.3: A function *f: X→Y* is said to be
a) contra closed if the image of every closed set in *X* is open in *Y*.
b) contra semi-closed if the image of every closed set in *X* is semi-open in *Y*.
c) contra pre-closed if the image of every closed set in *X* is pre-open in *Y*.
d) contra rα-closed if the image of every closed set in *X* is rα-open in *Y*.

Definition 2.4: *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

3. CONTRA *rp*-CLOSED MAPPINGS:

Definition 3.1: A function *f: X→Y* is said to be contra *rp*-closed if the image of every closed set in *X* is *rp*-open in *Y*.

Theorem 3.1: Every contra *rα*-closed map is contra *rp*-closed but not conversely.

Proof: Let A⊆*X* be closed ⇒ *f*(A) is *rα*-open in *Y* since *f:X→Y* is contra *rα*-closed ⇒*f*(A) is *rp*-open in *Y* since every rα-open set is *rp*-open. Hence *f* is contra *rp*-closed.

Example 1: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {a, b}, *X*}; σ = {ϕ, {a, c}, *Y*}. Let *f:X→Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a. Then *f* is contra *rp*-closed but not contra rα-closed.

Theorem 3.2: Every contra *r*-closed map is contra *rp*-closed but not conversely.

Proof: Let A⊆*X* be closed ⇒ *f*(A) is *r*-open in *Y* since *f:X→Y* is contra *r*-closed ⇒*f*(A) is *rp*-open in *Y* since every rα-open set is *rp*-open. Hence *f* is contra *rp*-closed.

Example 2: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {a, b}, *X*}; σ = {ϕ, {a, c}, *Y*}. Let *f:X→Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a. Then *f* is contra *rp*-closed but not contra *r*-closed.

Theorem 3.3: Every contra *rp*-closed map is contra pre-closed but not conversely.

Proof: Let A⊆*X* be closed ⇒ *f*(A) is *rp*-open in *Y* since *f:X→Y* is contra *rp*-closed ⇒*f*(A) is pre-open in *Y* since every *rp*-open set is pre-open. Hence *f* is contra pre-closed.

Theorem 3.4: Every contra *rp*-closed map is contra β-closed but not conversely.

Proof: Let A⊆*X* be closed ⇒ *f*(A) is *rp*-open in *Y* since *f:X→Y* is contra *rp*-closed ⇒*f*(A) is β-open in *Y* since every *rp*-open set is β-open. Hence *f* is contra β-closed.

Example 3: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {a, b}, *X*}; σ = {ϕ, {a, c}, *Y*}. Let *f:X→Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a. Then *f* is contra pre-closed, contra *rp*-closed, contra *β*-closed but not contra closed, contra α-closed, contra rα-closed, contra semi-closed and contra *r*-closed.

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Example 4: Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, X\}$; $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f: X\to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is contra-closed, contra-pre-closed, contra-β-closed, contra-α-closed, contra-rα-closed, contra-semi-closed but not contra-*rp*-closed and contra-r-closed.

Example 5: Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$; $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Assume $f: X\to Y$ be the identity map. Then f is not contra-closed, contra-pre-closed, contra-β-closed, contra-α-closed, contra-semi-closed, contra-rα-closed, contra-*rp*-closed and contra-r-closed.

Note 1:
a) contra closed maps and contra *rp*-closed maps are independent of each other.
b) contra α-closed map and contra *rp*-closed map are independent of each other.
c) contra semi closed map and contra *rp*-closed map are independent of each other.

Note 2: We have the following implication diagram among the closed maps.

$$\begin{tikzcd}
\text{contra-}rp\text{-closed} \arrow[rr] & & \text{contra-}p\text{-closed.} \arrow[d] \\
\text{Contra-}r\text{-closed} \arrow[u] \arrow[r] \arrow[d] & \text{contra-}r\alpha\text{-closed} \arrow[r] & \text{contra-}\beta\text{-closed} \\
\text{contra-closed} \arrow[r] & \text{contra-}\alpha\text{-closed} \arrow[r] & \text{contra-}s\text{-closed} \arrow[u]
\end{tikzcd}$$

None is reversible.

Theorem 3.5: If $R\alpha C(Y) = RPC(Y)$ then f is contra rα-closed iff f is contra *rp*-closed.

Proof: Follows from theorem 3.1

Conversely Let $A\subseteq X$ be closed $\Rightarrow f(A)$ is *rp*-open in Y since $f:X\to Y$ is contra *rp*-closed $\Rightarrow f(A)$ is $r\alpha$-open in Y since every r*p*-open set is $r\alpha$-open. Hence f is contra $r\alpha$-closed.

Theorem 3.6: If $RPC(Y) = RC(Y)$ then f is contra *r*-closed iff f is contra *rp*-closed.

Proof: Follows from theorem 3.2

Conversely Let $A\subseteq X$ be closed $\Rightarrow f(A)$ is *rp*-open in Y since $f:X\to Y$ is contra *rp*-closed $\Rightarrow f(A)$ is *r*-open in Y since every r*p*-open set is *r*-open. Hence f is contra *r*-closed.

Theorem 3.7: If $RPC(Y) = \alpha C(Y)$ then f is contra α-closed iff f is contra *rp*-closed.

Proof: Let $A\subseteq X$ be closed $\Rightarrow f(A)$ is α-open in Y since $f:X\to Y$ is contra α-closed $\Rightarrow f(A)$ is *rp*-open in Y since every α-open set is *rp*-open. Hence f is contra *rp*-closed.

Conversely Let $A\subseteq X$ be closed $\Rightarrow f(A)$ is *rp*-open in Y since $f:X\to Y$ is contra *rp*-closed $\Rightarrow f(A)$ is α-open in Y since every r*p*-open set is α-open. Hence f is contra α-closed.

Theorem 3.8: If f is closed and g is contra *rp*-closed then $g\ o\ f$ is contra *rp*-closed.

Proof: Let $A\subseteq X$ be closed $\Rightarrow f(A)$ is closed in $Y \Rightarrow g(f(A))$ is *rp*-open in $Z \Rightarrow g\ o\ f(A)$ is *rp*-open in Z. Hence $g\ o\ f$ is contra *rp*-closed.

Theorem 3.9: If f is closed and g is contra *r*-closed then $g\ o\ f$ is contra *rp*-closed.

Proof: Let $A\subseteq X$ be closed $\Rightarrow f(A)$ is closed in $Y \Rightarrow g(f(A))$ is *r*-open in $Z \Rightarrow g\ o\ f(A)$ is *rp*-open in Z. Hence $g\ o\ f$ is contra *rp*-closed.

Theorem 3.10: If f is closed and g is contra $r\alpha$-closed then $g\ o\ f$ is contra *rp*-closed.

Proof: Let $A\subseteq X$ be closed in X $\Rightarrow f(A)$ is closed in Y $\Rightarrow g(f(A))$ is $r\alpha$-open in Z $\Rightarrow g(f(A))$ is *rp*-open in Z $\Rightarrow g\bullet f(A)$ is *rp*-open in Z Hence $g\bullet f$ is almost contra *rp*-closed.

Theorem 3.11: If f is r-closed and g is contra *rp*-closed then $g\ o\ f$ is contra *rp*-closed.

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Proof: Let $A\subseteq X$ be closed $\Rightarrow$ $f(A)$ is r-closed in $Y \Rightarrow g(f(A))$ is rp-open in $Z \Rightarrow gof(A)$ is rp-open in Z. Hence $g\ o\ f$ is contra rp-closed.

Theorem 3.12: If f is r-closed and g is contra r-closed then $g\ o\ f$ is contra rp-closed.

Proof: Let $A\subseteq X$ be closed $\Rightarrow$ $f(A)$ is r-closed in $Y \Rightarrow g(f(A))$ is r-open in $Z \Rightarrow go\ f(A)$ is rp-open in Z. Hence $g\ o\ f$ is contra rp-closed.

Theorem 3.13: If f is r-closed and g is contra $r\alpha$-closed then $g\ o\ f$ is contra rp-closed.

Proof: Let $A\subseteq X$ be closed in X $\Rightarrow f(A)$ is r-closed in Y $\Rightarrow g(f(A))$ is $r\alpha$-open in Z $\Rightarrow g(f(A))$ is rp-open in Z $\Rightarrow g\bullet f(A)$ is rp-open in Z Hence $g\bullet f$ is contra rp-closed.

Corollary 3.1:

a) If f is closed[r-closed] and g is contra rp-closed then $g\ o\ f$ is contra pre-closed and hence contra β-closed.
b) If f is closed[r-closed] and g is contra r-closed then $g\ o\ f$ is contra pre-closed and hence contra β-closed.
c) If f is closed[r-closed] and g is contra $r\alpha$-closed then $g\ o\ f$ is contra pre-closed and hence contra β-closed.

Theorem 3.14: If f is contra closed and g is rp-open then $g\ o\ f$ is contra-rp-closed.

Proof: Let $A\subseteq X$ be closed in X $\Rightarrow f(A)$ is open in Y $\Rightarrow g(f(A))$ is rp-open in Z $\Rightarrow g\bullet f(A)$ is rp-open in Z. Hence $g\bullet f$ is contra rp-closed.

Theorem 3.15: If f is contra closed and g is r-open then $g\ o\ f$ is contra-rp-closed.

Proof: Let $A\subseteq X$ be closed in X $\Rightarrow f(A)$ is open in Y $\Rightarrow g(f(A))$ is r-open in Z $\Rightarrow g(f(A))$ is rp-open in Z $\Rightarrow g\bullet f(A)$ is rp-open in Z. Hence $g\bullet f$ is contra rp-closed.

Theorem 3.16: If f is contra closed and g is $r\alpha$-open then $g\ o\ f$ is contra-rp-closed.

Proof: Let $A\subseteq X$ be closed in X $\Rightarrow f(A)$ is open in Y $\Rightarrow g(f(A))$ is $r\alpha$-open in Z $\Rightarrow g(f(A))$ is rp-open in Z $\Rightarrow g\bullet f(A)$ is rp-open in Z. Hence $g\bullet f$ is contra rp-closed.

Theorem 3.17: If f is contra-r-closed and g is rp-open then $g\ o\ f$ is contra-rp-closed.

Proof: Let $A\subseteq X$ be closed in X $\Rightarrow f(A)$ is r-open in Y $\Rightarrow g(f(A))$ is rp-open in Z $\Rightarrow g\bullet f(A)$ is rp-open in Z. Hence $g\bullet f$ is contra rp-closed.

Theorem 3.18: If f is contra-r-closed and g is r-open then $g\ o\ f$ is contra-rp-closed.

Proof: Let $A\subseteq X$ be closed in X $\Rightarrow f(A)$ is r-open in Y $\Rightarrow g(f(A))$ is r-open in Z $\Rightarrow g(f(A))$ is rp-open in Z $\Rightarrow g\bullet f(A)$ is rp-open in Z. Hence $g\bullet f$ is contra rp-closed.

Theorem 3.19: If f is contra-r-closed and g is $r\alpha$-open then $g\ o\ f$ is contra-rp-closed.

Proof: Let $A\subseteq X$ be closed in X $\Rightarrow f(A)$ is r-open in Y $\Rightarrow g(f(A))$ is $r\alpha$-open in Z $\Rightarrow g(f(A))$ is rp-open in Z $\Rightarrow g\bullet f(A)$ is rp-open in Z. Hence $g\bullet f$ is contra rp-closed.

Corollary 3.2:

a) If f is contra closed [contra-r-closed] and g is rp-open then $g\ o\ f$ is contra-pre-closed and hence contra β-closed.
b) If f is contra closed [contra-r-closed] and g is r-open then $g\ o\ f$ is contra-pre-closed and hence contra β-closed.
c) If f is contra closed [contra-r-closed] and g is $r\alpha$-clsoed then $g\ o\ f$ is contra-pre-closed and hence contra β-closed.

Theorem 3.20: If $f: X\rightarrow Y$ is contra rp-closed, then $f(A^o)\subset rp(f(A))^o$

Proof: Let $A\subseteq$ X be closed and f: X→ Y is contra rp-closed gives $f(A^o)$ is rp-open in Y and $f(A^o)\subset f(A)$ which in turn gives $rp(f(A^o))^o\subset rp(f(A))^o$ (1)

Since $f(A^o)$ is rp-open in Y, $rp(f(A^o))^o = f(A^o)$ (2)

Combining above (1) and (2) we have $f(A^o)\subset rp(f(A))^o$ for every subset A of X.

Remark 2: Converse is not true in general as shown by the following example.

Corollary 3.3: If $f{:}X\to Y$ is contra r-closed, then $f(A^o)\subset rp(f(A))^o$

Proof: Let $A\subseteq X$ be closed and $f{:}\ X\to Y$ is contra r-closed gives $f(A^o)$ is r-open in Y and $f(A^o)\subset f(A)$ which in turn gives $rp(f(A^o))^o\subset rp(f(A))^o$ (1)

Since $f(A^o)$ is rp-open in Y, $rp(f(A^o))^o = f(A^o)$ (2)

Combining above (1) and (2) we have $f(A^o)\subset rp(f(A))^o$ for every subset A of X.

Theorem 3.21: If $f{:}\ X\to Y$ is contra rp-closed and $A\subseteq X$ is closed, $f(A)$ is τ_{rp}-open in Y.

Proof: Let $A\subset X$ be closed and $f{:}\ X\to Y$ is contra rp-closed $\Rightarrow f(A^o)\subset rp(f(A))^o \Rightarrow f(A)\subset rp(f(A))^o$, since $f(A) = f(A^o)$. But $rp(f(A))^o\subset f(A)$. Combining we get $f(A) = rp(f(A))^o$. Therefore $f(A)$ is τ_{rp}-open in Y.

Corollary 3.4: If $f{:}\ X\to Y$ is contra r-closed, then $f(A)$ is τ_{rp}-open in Y if A is r-closed set in X.

Proof: Let $A\subset X$ be r-closed and $f{:}\ X\to Y$ is contra r-closed $\Rightarrow f(A^o)\subset r(f(A))^o \Rightarrow f(A^o)\subset rp(f(A))^o$ (by theorem 3.20) $\Rightarrow f(A)\subset rp(f(A))^o$, since $f(A) = f(A^o)$. But $rp(f(A))^o\subset f(A)$. Combining we get $f(A) = rp(f(A))^o$. Hence $f(A)$ is τ_{rp}-open in Y.

Theorem 3.22: If $rp(A)^o = r(A)^o$ for every $A\subset Y$, then the following are equivalent:
a) $f{:}\ X\to Y$ is contra rp-closed map
b) $f(A^o)\subset rp(f(A))^o$

Proof:
(a) $\Rightarrow$(b) follows from theorem 3.20.

(b) $\Rightarrow$ (a) Let A be any r-closed set in X, then $f(A) = f(A^o)\subset rp(f(A))^o$ by hypothesis. We have $f(A)\subset rp(f(A))^o$. Combining we get $f(A) = rp(f(A))^o = r(f(A))^o$[by given condition] which implies $f(A)$ is r-open and hence rp-open. Thus f is contra rp-closed.

Theorem 3.23: $f{:}X\to Y$ is contra rp-closed iff for each subset S of Y and each open set U containing $f^{-1}(S)$, there is an rp-open set V of Y such that $S\subseteq V$ and $f^{-1}(V)\subseteq U$.

Remark 3: Composition of two contra rp-closed maps is not contra rp-closed in general.

Theorem 3.24: Let X, Y, Z be topological spaces and every rp-open set is closed[r-closed] in Y. Then the composition of two contra rp-closed[contra r-closed] maps is contra rp-closed.

Proof: (a) Let $f{:}\ X\to Y$ and $g{:}Y\to Z$ be contra rp-closed maps. Let A be any closed set in $X \Rightarrow f(A)$ is rp-open in $Y\Rightarrow f(A)$ is closed in Y (by assumption) $\Rightarrow g(f(A))$ is rp-open in $Z \Rightarrow g\ o\ f(A)$ is rp-open in Z. Therefore $g\ o\ f$ is contra rp-closed.

(b) Let $f{:}\ X\to Y$ and $g{:}Y\to Z$ be contra rp-closed maps. Let A be any closed set in $X \Rightarrow f(A)$ is r-open in $Y \Rightarrow f(A)$ is rp-open in $Y \Rightarrow f(A)$ is r-closed in Y (by assumption) $\Rightarrow f(A)$ is closed in Y (by assumption) $\Rightarrow g(f(A))$ is r-open in $Z \Rightarrow gof(A)$ is rp-open in Z. Therefore gof is contra rp-closed.

Theorem 3.25: Let X, Y, Z be topological spaces and Y is discrete topological space in Y. Then the composition of two contra rp-closed[contra r-closed] maps is contra rp-closed.

Theorem 3.26: If $f{:}\ X\to Y$ is g-closed, $g{:}Y\to Z$ is contra rp-closed [contra r-closed] and Y is $T_{1/2}$ [r-$T_{1/2}$] then $g\ o\ f$ is contra rp-closed.

Proof: (a) Let A be a closed set in X. Then $f(A)$ is g-closed set in $Y\Rightarrow f(A)$ is closed in Y as Y is $T_{1/2}\Rightarrow g(f(A))$ is rp-open in Z since g is contra rp-closed $\Rightarrow g\ o\ f(A)$ is rp-open in Z. Hence $g\ o\ f$ is contra rp-closed.

(b) Let A be a closed set in X. Then $f(A)$ is g-closed set in $Y\Rightarrow f(A)$ is closed in Y as Y is $T_{1/2}\Rightarrow g(f(A))$ is r-open in Z since g is contra r-closed $\Rightarrow g\ o\ f(A)$ is rp-open in Z. Hence gof is contra rp-closed.

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Corollary 3.5: If f: $X \to Y$ is g-open, g: $Y \to Z$ is contra rp-closed [contra r-closed] and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is contra p-closed and hence contra β-closed.

Theorem 3.27: If f: $X \to Y$ is rg-open, g:$Y \to Z$ is contra rp-closed [contra r-closed] and Y is r-$T_{1/2}$, then $g\ o\ f$ is contra rp-closed.

Proof: Let A be a closed set in X. Then f (A) is rg-closed in $Y \Rightarrow f$ (A) is r-closed in Y since Y is r-$T_{1/2} \Rightarrow f$ (A) is closed in Y since every r-closed set is closed $\Rightarrow$ g(f(A)) is rp-open in $Z \Rightarrow$ g o f(A) is rp-open in Z. Hence gof is contra rp-closed.

Corollary 3.6: If f: $X \to Y$ is rg-open, g:$Y \to Z$ is contra rp-closed [contra r-closed] and Y is r-$T_{1/2}$, then $g\ o\ f$ is contra pre-closed and hence contra β-closed.

Theorem 3.28: If f: $X \to Y$, g:$Y \to Z$ be two mappings such that gof is contra rp-closed [contra r-closed] then the following statements are true.

a) If f is continuous [r-continuous] and surjective then g is contra rp-closed.
b) If f is g-continuous, surjective and X is $T_{1/2}$ then g is contra rp-closed.
c) If f is rg-continuous, surjective and X is r-$T_{1/2}$ then g is contra rp-closed.

Proof: (a) Let A be a closed set in $Y \Rightarrow f^{-1}$(A) is closed in $X \Rightarrow$ ($g\ o\ f$) (f^{-1}(A)) is rp-open in $Z \Rightarrow g$(A) is rp-open in Z. Hence g is contra rp-closed.

(b) Let A be a closed set in Y. $\Rightarrow f^{-1}$(A) is g-closed in $X \Rightarrow f^{-1}$(A) is closed in X [since X is $T_{1/2}$] $\Rightarrow$ ($g\ o\ f$)(f^{-1}(A)) is rp-open in $Z \Rightarrow g$(A) is rp-open in Z. Hence g is contra rp-closed.

(c) Let A be a closed set in $Y \Rightarrow f^{-1}$(A) is g-closed in $X \Rightarrow f^{-1}$(A) is closed in X [since X is r-$T_{1/2}$] $\Rightarrow$ ($g\ o\ f$)(f^{-1}(A)) is rp-open in $Z \Rightarrow g$(A) is rp-open in Z. Hence g is contra rp-closed.

Corollary 3.7: If f: $X \to Y$, g: $Y \to Z$ be two mappings such that gof is contra rp-closed [contra r-closed] then the following statements are true.

a) If f is continuous [r-continuous] and surjective then g is contra pre-closed and hence contra β-closed.
b) If f is g continuous, surjective and X is $T_{1/2}$ then g is contra pre-closed and hence contra β-closed.
c) If f is rg-continuous, surjective and X is r-$T_{1/2}$ then g is contra pre-closed and hence contra β-closed.

Theorem 3.29: If X is rp-regular, f: $X \to Y$ is r-open, r-continuous, rp-closed surjective and $\bar{A} = A$ for every rp-closed set in Y then Y is rp-regular.

Proof: Let $p \in U \in RPO(Y)$. Then there exists a point x∈X such that f(x) = p as f is surjective. Since X is rp-regular and f is r-continuous there exists $V \in RO(X)$ such that $x \in V \subseteq \overline{V} \subseteq f^{-1}(U)$ which implies $p \in f(V) \subseteq f(\overline{V}) \subseteq f(f^{-1}(U)) = U \to$ (1)

Since f is rp-closed, $f(\overline{V}) \subseteq$ U, By hypothesis $\overline{f(\overline{V})} = f(\overline{V})$ *and* $\overline{f(\overline{V})} = \overline{f(V)} \to$ (2)

By (1) & (2) we have $p \in f(V) \subseteq f(\overline{V}) \subseteq U$ and $f(V)$ is rp-open. Hence Y is rp-regular.

Corollary 3.8: If X is rp-regular, f: $X \to Y$ is r-open, r-continuous, rp-closed, surjective and $\bar{A} = A$ for every r-closed set in Y then Y is rp-regular.

Theorem 3.30: If f: $X \to Y$ is contra rp-closed and A is an closed set of X then f_A:(X, τ(A)) → (Y, σ) is contra rp-closed.

Proof: Let F be a closed set in A. Then F = A∩E for some closed set E of X and so F is closed in $X \Rightarrow f$(A) is rp-open in Y. But f(F) = f_A(F). Therefore f_A is contra rp-closed.

Theorem 3.31: If f: $X \to Y$ is contra r-closed and A is an closed set of X then f_A:(X, τ(A)) → (Y, σ) is contra rp-closed.

Proof: Let F be a closed set in A. Then F = A∩E for some closed set E of X and so F is closed in $X \Rightarrow f$(A) is r-open in Y. $\Rightarrow f$(A) is rp-open in Y. But f(F) = f_A(F). Therefore f_A is contra rp-closed.

Corollary 3.9: If f: $X \to Y$ is contra rp-closed [contra r-closed] and A is an closed set of X then f_A:(X, τ(A)) → (Y, σ) is contra pre-closed and hence contra β-closed.

Theorem 3.32: If $f: X \to Y$ is contra *rp*-closed, X is $T_{1/2}$ and A is g-closed set of X then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is contra *rp*-closed.

Proof: Let F be a closed set in A. Then $F = A \cap E$ for some closed set E of X and so F is closed in $X \Rightarrow f(A)$ is *rp*-open in Y. But $f(F) = f_A(F)$. Therefore f_A is contra *rp*-closed.

Theorem 3.33: If $f: X \to Y$ is contra-*r*-closed, X is $T_{1/2}$ and A is g-closed set of X then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is contra *rp*-closed.

Proof: Let F be a closed set in A. Then $F = A \cap E$ for some closed set E of X and so F is closed in $X \Rightarrow f(A)$ is *r*-open in $Y \Rightarrow f(A)$ is *rp*-open in Y. But $f(F) = f_A(F)$. Therefore f_A is contra *rp*-closed.

Corollary 3.10: If $f: X \to Y$ is contra *rp*-closed [contra *r*-closed], X is $T_{1/2}$, A is g-closed set of X then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is contra pre-closed and hence contra β-closed.

Theorem 3.34: If $f_i: X_i \to Y_i$ be contra *rp*-closed [contra *r*-closed] for i =1, 2. Let $f: X_1 x X_2 \to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f: X1xX2 \to Y_1 x Y_2$ is contra *rp*-closed.

Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1) \text{ x } f_2(U_2)$ is *rp*-open set in $Y_1 x Y_2$. Then $f(U_1 x U_2)$ is *rp*-open set in $Y_1 x Y_2$. Hence f is contra *rp*-closed.

Corollary 3.11: If $f_i: X_i \to Y_i$ be contra *rp*-closed [contra *r*-closed] for i =1, 2. Let $f: X1xX2 \to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f: X_1 x X_2 \to Y_1 x Y_2$ is contra pre-closed and hence contra β-closed.

Theorem 3.35: Let $h: X \to X_1 x X_2$ be contra *rp*-closed. Let $f_i: X \to X_i$ be defined as $h(x)=(x_1,x_2)$ and $f_i(x) = x_i$. Then $f_i: X \to X_i$ is contra *rp*-closed for i =1, 2.

Proof: Let U_1 be closed in X_1, then $U_1 x\ X_2$ is closed in $X_1 x\ X_2$, and $h(U_1 x\ X_2)$ is *rp*-open in X. But $f_1(U_1) = h(U_1 x\ X_2)$, therefore f_1 is contra *rp*-closed. Similarly we can show that f_2 is also contra *rp*-closed and thus $f_i: X \to X_i$ is contra *rp*-closed for i = 1, 2.

Corollary 3.12: Let $h: X \to X_1 x X_2$ be contra *rp*-closed. Let $f_i: X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then $f_i: X \to X_i$ is contra pre-closed and hence contra β-closed for i =1, 2.

CONCLUSION

In this Paper we introduced the concept of *rp*-closed mappings, almost *rp*-closed mappings, studied their basic properties and the interrelationship between other closed maps.

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