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Čech w - Closed sets in closure spaces

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ABSTRACT

The purpose of this paper is to define and study w- closed sets in closure spaces. We also introduce the concept of w - continuous functions and investigate their properties.

Key Words: *w- closed sets, w - continuous maps, w- closed maps, T_w space.*

AMS Classification: *54A05.*

1. INTRODUCTION

Čech closure spaces were introduced by E. Čech [3] and then studied by many authors [4][5][7][8]. The concept of generalized closed sets and generalized continuous maps of topological spaces were extended to closure spaces in [2]. N.Levine [6] introduced semi- open sets and semi-continuity. P. Sundaram and M. Sheik John [9] introduced w - closed sets in topological spaces.

In this paper, we introduce and study the notion of w - closed sets in closure spaces. We define a new class of space namely T_w -space and their properties are studied. Further, we introduce a class of w- continuous maps and w- closed maps and their characterizations are obtained.

PRELIMINARIES

A map k: P(X) $\rightarrow$ P(X) defined on the power set P(X) of a set X is called a closure operator on X and the pair (X, k) is called a closure space if the following axioms are satisfied.
(i) $k(\phi) = \phi$,
(ii) $A \subseteq k(A)$ for every $A \subseteq X$
(iii) $k(A \cup B) = k(A) \cup k(B)$ for all $A,B \subseteq X$

A closure operator k on a set X is called idempotent if k (A) =k [k (A)] for all $A \subseteq X$.

Definitions: 1.1 A subset A of a čech closure space (X, k) is said to be
(i) čech closed if k(A) =A
(ii) čech open if k(X-A) =X-A
(iii) čech semi-open if $A \subseteq$ k int (A)
(iv) čech pre-open if $A \subseteq$ int [k(A)]
(v) čech pre-closed if k[int (A)] $\subseteq$ A

Definition: 1.2 A čech closure space (Y, l) is said to be a subspace of (X, k) if $Y \subseteq X$ and $k(A) = k(A) \cap Y$ for each subset $A \subseteq Y$. If Y is closed in (X, k) then the subspace (Y, l) of (X, k) is said to be closed too.

Definition: 1.3 Let (X, k) and (Y, l) be čech closure spaces. A map f: (X, k) $\rightarrow$(Y, k) is said to be continuous, if f (kA) $\subseteq$ k f(A) for every subset $A \subseteq F$.

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Definition: 1.4 Let (X, k) and (Y, l) be čech closure spaces. A map f: (X, k) $\to$(Y, l) is said to be closed (resp. open) if f(F) is a closed(resp. open) subset of (Y, l) whenever F is a closed (resp. open) subset of (X, k).

Definition: 1.5 The product of a family $\{(X_\alpha, k_\alpha) ; \alpha\in I\}$ of closure spaces denoted by $\prod_{\alpha\in I}(X_\alpha, k_\alpha)$ is the closure space $(\prod_{\alpha\in I}(X_\alpha, k)$ where $\prod_{\alpha\in I} X_\alpha$ denotes the Cartesian product of sets X_α , $\alpha\in I$ and k is čech closure operator generated by the projections $\pi_\alpha : \prod_{\alpha\in I}(X_\alpha, k_\alpha) \to (X_\alpha, k_\alpha)$, $\alpha\in I$ i.e defined by $k(A) = \prod_{\alpha\in I} k_\alpha \pi_\alpha(A)$ for each $A\subseteq \prod_{\alpha\in I} X_\alpha$.
Clearly, if $\{(X_\alpha, k_\alpha): \alpha\in I\}$ is a family of closure spaces, then the projection map $\pi_\beta : \prod_{\alpha\in I}(X_\alpha, k_\alpha)\to (X_\beta,k_\beta)$ is closed and continuous for every $\beta\in I$.

Proposition: 1.6 Let $\{(X_\alpha, k_\alpha): \alpha\in I\}$ be a family of closure spaces, let $\beta\in I$ and $F\subseteq X_\beta$. Then F is a closed subset of (X_β,k_β) if and only if $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is a closed subset of $\prod_{\alpha\in I}(X_\alpha, k_\alpha)$.

Proposition: 1.7 Let $\{(X_\alpha, k_\alpha): \alpha\in I\}$ be a family of closure spaces, let $\beta\in I$ and $G\subseteq X_\beta$. Then G is a open subset of (X_β,k_β) if and only if $G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is an open subset of $\prod_{\alpha\in I}(X_\alpha, k_\alpha)$.

2. Čech w - closed sets

Definition 2.1: Let (X, k) be a čech closure space. A subset $A \subseteq X$ is called a w - closed set if $K (A) \subseteq G$ whenever G is a semi-open subset of (X, k) with $A \subseteq G$. A subset A of X is called a w - open set if its complement is a w - closed subset of (X, k).

Proposition: 2.2 Every closed set is w - closed.

Proof: Let G be a semi-open subset of (X, k) such that $A \subseteq G$. Since A is a closed set, we have $K (A) = A \subseteq G$. Therefore A is w-closed.

The converse need not true as seen in the following example:

Example: 2.3 Let X = {a, b, c} and define a closure k on X by $k\phi = \phi$,k{a} ={a}; k{b}= {b, c}; k{c}= k{a, c} = {a, c}; k{a, b}= k{b, c}= kX = X. Then {a, b} is w - closed but it is not closed.

Proposition: 2.4 Let (X, k) be a čech closure space. If A and B are w - closed subset of (X, k), then $A \cup B$ is also w - closed.

Proof: Let G be a semi - open subset of (X, k) such that $A\cup B \subseteq G$, then $A \subseteq G$ and $B \subseteq G$. Since A and B are w - closed, we have $k (A) \subseteq G$, and $k (B) \subseteq G$, Consequently, $k (A\cup B) = k(A) \cup k(B) \subseteq G$. Therefore $A \cup B$ is w - closed.

Remark: The intersection of two w- closed sets need not be w - closed as can be seen by the following example.

Example: 2.5 Let X = {a, b, c} and define a closure k on X by $k\phi = \phi$, k{a} ={a, b}; k{b}= k{c}= k{b, c} = {b, c}; k{a, b} = k{a,c}= kX = X. If A = {a, b} and B = {a, c}, then $\{a, b\}\cap\{a, c\}=\{a\}$ which is not w - closed.

Proposition: 2.6 Let (X, k) be a čech closure space. If A is w - closed and F is semi - closed in (X, k), then $A \cap F$ is w-closed.

Proof: Let G be a semi - open subset of (X, k) such that $A \cap F \subseteq G$, Then $A \subseteq G \cup (X-F)$.and so, Since A is w - closed , $k (A) \subseteq G \cup (X-F)$, Then $k (A) \cap F \subseteq G$,Since F is semi - closed, $k (A \cap F) \subseteq G$. Therefore $A \cap F$ is w - closed.

Proposition: 2.7 Let (Y, l) be a closed subspace of (X, k). If F is a w - closed subset of (Y, l), then F is a w - closed subset of (X, k).

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Proof: Let G be semi - open set of (X, k) such that $F \subseteq G$. Since F is w - closed and $G \cap F$ is semi - open $k(F) \cap Y \subseteq G$, But Y is closed subset of (X, k) and $k(F) \subseteq G$, where G is a semi-open set. Therefore F is a w- closed set of (X, k).

The following statement is obvious

Proposition: 2.8 Let (X, k) be a čech closure space and let $A \subseteq X$. If A is both semi - open and w - closed then A is closed.

Proposition: 2.9 Let (X, k) be a čech closure space and let k be idempotent. If A is a w - closed subset of (X, k) such that $A \subseteq B \subseteq k(A)$, then B is a w - closed subset of (X, k).

Proof: Let G be a semi - open subset of (X, k) such that $B \subseteq G$. Then $A \subseteq G$, Since A is w - closed, $k(A) \subseteq G$. As G is idempotent, $k(B) \subseteq k(k(A)) = k(A) \subseteq G$, Hence B is w - closed.

Proposition: 2.10 Let (X, k) be a čech closure space and let $A \subseteq X$. If A is w - closed, then $k(A) - A$ has no non empty semi - closed subset.

Proof: Suppose that A is w- closed. Let F be a semi - closed set of $k(A) - A$. Then $F \subseteq k(A) \cap (X - A)$, so $A \subseteq X - F$. Consequently, since A is w – closed $F \subseteq X - k(A)$, Since $F \subseteq k(A)$, $F \subseteq (X - k(A)) \cap k(A) = \phi$, thus $F = \phi$.Therefore $k(A) - A$ contains no non empty semi - closed subset.

Proposition: 2.11 Let(X, k) be a čech closure space. A set $A \subseteq X$ is w - open if and only if $F \subseteq X - k(X-A)$ whenever F is semi - closed subset of (X, k) with $F \subseteq A$.

Proof: Suppose that A is w- open and F be a semi - closed subset of (X, k) such that $F \subseteq A$. Then $X-A \subseteq X-F$. But X-A is w- closed and X-F is semi -open. It follows that $k(X-A) \subseteq X-F$. (i.e.) $F \subseteq X - k(X-A)$.

Conversely, Let G be a semi - open subset of (X, k) such that $X-A \subseteq G$. Then $X-G \subseteq A$. Therefore $X-U \subseteq k(X-A)$. Consequently, $k(X-A) \subseteq G$. Hence X- A is w- closed and so A is w- open.

Remark: 2.12 The union of two w- open sets need not be w- open.

Proposition: 2.13 Let(X, k) be a čech closure space. If A is w- open and B is semi -open in (X, k), then $A \cup B$ is w- open.

Proof: Let F be a semi- closed subset of (X, k) such that $F \subseteq A \cup B$. Then $X - (A \cup B) \subseteq X-F$. Hence $(X-A) \cap (X-B) \subseteq X-F$. By proposition 2.6, we have, $(X-A) \cap (X-B)$ is w- closed. Therefore $k[(X-A) \cap (X-B)] \subseteq X-F$. Consequently, $F \subseteq X - k[(X-A) \cap (X-B)] = X - k[X - \cap (A \cup B)]$. Since $F \subseteq X - k[(X-A)]$, then A is w- open. Therefore $A \cup B$ is w- open.

Proposition: 2.14 Let(X, k) be a čech closure space. If A and B are w- open of (X, k), then $A \cap B$ is w – open.

Proof: Let F be a semi- closed subset of (X, k) such that $F \subseteq A \cap B$. Then $X - (A \cap B) \subseteq X - F$. Consequently, $(X-A) \cup (X-B) \subseteq X - F$. By proposition 2.4, $(X-A) \cup (X-B)$ is w – closed. Thus, $k[(X-A) \cup (X-B)] \subseteq X - F$. Hence $F \subseteq X - k[(X-A) \cup (X-B)] \subseteq X - (X - (A \cap B))$,By proposition 2.11, $A \cap B$ is w – open.

Proposition: 2.15 Let(X, k) be a čech closure space. If A is w- open subsets of (X, k), then X = G whenever G is semi open and $(X - k(X-A)) \cup (X-A) \subseteq G$.

Proof: Suppose that A is w – open. Let G be an semi open subset of (X, k) such that $(X - k(X-A)) \cup (X-A) \subseteq G$. Then $X - G \subseteq X - [(X - k(X-A)) \cup (X-A)]$. Therefore $X - G \subseteq k(X-A) \cap A$ or equivalently, $X - G \subseteq k(X-A) - (X-A)$ .But X – G is semi closed and X – A is w – closed. Then by proposition 2.10, $X - G = \phi$ .Consequently X = G.

Proposition: 2.16 Let (X, k) be a čech closure space and let $A \subseteq X$. If A is w - closed, then $k(A) - A$ is w - open.

Proof: Suppose that A is w - closed. Let F be a semi - closed set of (X, k) such that $F \subseteq k(A) - A$ .By proposition 2.10 $k(A) - A = \phi$, and hence $F \subseteq X - [(X - k(X-A))$. By proposition 2.11 $k(A) - A$ is w – open.

Proposition: 2.17 Let $\{(X_\alpha, k_\alpha): \alpha \in I\}$ be a family of closure spaces, let $\beta \in I$ and $G \subseteq X_\beta$. Then G is a w - open subset of (X_β, k_β) if and only if $G \times \prod_{\substack{\alpha \neq \beta \\ \alpha \in I}} X_\alpha$ is a w - open subset of $\prod_{\alpha \in I} (X_\alpha, k_\alpha)$.

Proof: Let F be a w - closed subset of (X_α, k_α) such that $F \subseteq G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$, Then $\pi_\beta(F) \subseteq G$. Since $\pi_\beta(F)$ is semi - closed and G is w - open in (X_β, k_β), $\pi_\beta(F) \subseteq X_\beta - k_\beta(X_\beta - G)$. Therefore $F \subseteq \pi_\beta^{-1}(X_\beta - k_\beta(X_\beta - G)) = \prod_{\alpha\in I} X_\alpha - \prod_{\alpha\in I} k_\alpha \pi_\alpha (\prod_{\alpha\in I} X_\alpha - G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha)$, hence $G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is a w- closed subset of $\prod_{\alpha\in I} (X_\alpha, k_\alpha)$.

Conversely, Let F be a semi - closed subset of (X_β, k_β) such that $F \subseteq G$. Then $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha \subseteq G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$). Since $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is semi - closed and $G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is w- open in $\prod_{\alpha\in I} (X_\alpha, k_\alpha)$.

$F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha \subseteq \prod_{\alpha\in I} X_\alpha - \prod_{\alpha\in I} k_\alpha\pi_\alpha(\prod_{\alpha\in I} X_\alpha - G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha)$.By proposition,2.11.Therefore

$\prod_{\alpha\in I} k_\alpha\pi_\alpha (X_\beta - G) \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha) \subseteq \prod_{\alpha\in I} X_\alpha - F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha = (X_\beta - F) \prod_{\alpha\in I} X_\alpha$

Consequently, $k_\beta(X_\beta - G) \subseteq X_\beta - F$ implies $F \subseteq X_\beta - k_\beta(X_\beta - G)$. Hence G is a w - open subset of (X_β, k_β).

Proposition: 2.18 Let $\{(X_\alpha, k_\alpha): \alpha\in I\}$ be a family of closure spaces, let $\beta\in I$ and $F \subseteq X_\beta$.Then F is a w- closed subset of (X_β,k_β) if and only if $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is a w - closed subset of $\prod_{\alpha\in I} (X_\alpha, k_\alpha)$.

Proof: Let F be a w - closed subset of (X_β,k_β).Then X_β -F is an w - open subset of (X_β, k_β). By proposition 2.17, $(X_\beta - F) \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha = \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha - F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is a w - open subset of $\prod_{\alpha\in I} (X_\alpha, k_\alpha)$.Hence $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is a w - closed of $\prod_{\alpha\in I} (X_\alpha, k_\alpha)$.

Conversely, let G be a w - open subset of (X_β,k_β) such that $F \subseteq G$, Then $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha \subseteq G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ .Since $F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is w - closed and $G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is semi-open in $\prod_{\alpha\in I} (X_\alpha,k_\alpha)$, $\prod_{\alpha\in I} k_\alpha\pi_\alpha(F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\beta) \subseteq G \times \prod_{\alpha\in I} X_\alpha$ .Consequently, $k_\beta(F) \subseteq G$. Therefore, F is a w - closed subset of (X_β,k_β).

Proposition: 2.19 Let $\{(X_\alpha,k_\alpha): \alpha\in I\}$ be a family of closure spaces, For each $\beta\in I$ and let $\pi_\beta : \prod_{\alpha\in I} X_\alpha \rightarrow X_\beta$ be a projection map. Then

(i) If F is a w - closed subset of $\prod_{\alpha\in I} (X_\alpha, k_\alpha)$, then $\pi_\beta(F)$ is a w- closed subset of (X_β, k_β).

(ii) If F is a w-closed subset of (X_β, k_β), then $\pi_\beta^{-1}(F)$ is a w- closed subset of $\prod_{\alpha\in I} (X_\alpha, k_\alpha)$,

Proof: Let F be a w-closed subset of $\prod_{\alpha\in I} (X_\alpha, k_\alpha)$ and let G be a semi-open subset of (X_β,k_β) such that $\pi_\beta(F) \subseteq G$. Then $F \subseteq \pi_\beta^{-1}(G) = G \times \prod_{\alpha\in I} X_\alpha$. Since F is a w - closed and $G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\beta$ is semi - open, $\prod_{\alpha\in I} k_\alpha\pi_\alpha(F) \subseteq G \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ .Consequently $k_\beta\pi_\beta(F) \subseteq G$.

Hence π_β (F) is a w - closed subset of (X_β, k_β).

(ii) Let F be a w - closed subset of (X_β, k_β),Then π_β^{-1}(F) = F × $\prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$

.Therefore, we have, F × $\prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is a w - closed subset of $\prod_{\alpha\in I}$ (X_α, k_α).Therefore, π_β^{-1}(F) is a w - closed subset of $\prod_{\alpha\in I}$ (X_α, k_α).

Definition: 2.20 A closure space (X, k) is said to be a Tw -space if every w-closed subset of (X, k) is closed.

Proposition: 2.21 Let (X, k) be closure space. Then
(i) If (X, k) is a T_w -space then every singleton subset of X is either semi - closed or open.
(ii)If every singleton subset of X is a semi - closed subset of (X, k), then (X, k) is a T_w -space.

Proof:

i. Suppose that (X. k) is a T_w -space. Let x∈X and assume that{x} is not semi - closed. Then X-{x} is not semi - open. This implies X-{x} is w - closed since X is the only semi - open set which contains X-{x}. Since (X, k) is a T_w - space, X-{x} is closed or equivalently {x} is open.

ii. Let A be a w - closed subset of (X, k).Suppose that x∉A. Then {x} ⊆ X-{x}.

Since A is w - closed and X-{x} is semi – open, k (A) ⊆ X-{x}, (i.e) {x} ⊆ X- k (A).

Hence x∉ k (A) and thus k (A) ⊆ A. Therefore A is closed subset of (X, k). Hence (X, k) is a T_w -space.

3. Čech w - continuous maps

Definition: 3.1 Let (X, k) and (Y, l) be a čech closure space. A mapping f: (X, k) →(Y, l) is said to be w - continuous, if f^{-1}(F) is w - closed set of (X, k) for every closed set F in (Y, l).

Proposition: 3.2 Every continuous map is w - continuous.

Proof: Let f : (X, k) → (Y, l) be continuous, Let F be a closed set of (Y, l).Since f is continuous, then f^{-1}(F) is θ - closed set of (X, k).Since every closed set is w - closed of (X, k),we have f^{-1}(F) is a w - closed set of (X, k).Therefore f is a w – continuous map.

Proposition: 3.3 Let (X, k) be a T_W space and let (Y, l) be a čech closure space. If f: (X, k) → (Y, l) is said to be semi - continuous, then f is w - continuous,

Proof: Let F be a closed subset of (Y, l).Since F is semi - continuous, then f^{-1}(F) is semi - closed set of (X, k).Since (X, k) is a T_W space, f^{-1}(F) is a w- closed set of (X, k). Hence, f is w - continuous,

The following statement is obvious.
Proposition: 3.4 Let (X, k), (Y, l) and (Z, m) be closure spaces. If f: (X, k) →(Y, l) is w - continuous and g: (Y, l) →(Z, m) is continuous then g ∘ f: (X, k) →(Z, m) is w - continuous.

Proposition: 3.5 Let, (Z, m) be closure spaces and let (Y, l) be a T_{w} - space .If f: (X, k) →(Y, l) and g: (Y, l) →(Z, m) are w - continuous, then g ∘ f: (X, k) → (Z, m) is w - continuous.

Proof: Let F be a closed subset of (Z, w).Since g is w - continuous, then g^{-1}(F) is w - closed set of (Y, l).Since (Y, l) is a T_W space, g^{-1}(F) is a closed set of (Y, l) which implies that $(g \circ f)^{-1}$ (F) is a w – closed subset of (X, k). Hence, g ∘ f is w - continuous,

Proposition: 3.6 Let {(X_α, k_α): α∈I} and {(Y_α, l_α): α∈I }be families of closure spaces. For each α∈I, let f_α: $X_\alpha \to Y_\alpha$ be a map and f: $\prod_{\alpha\in I} X_\alpha \to \prod_{\alpha\in I} Y_\alpha$ be a map defined by $f((x_\alpha)_{\alpha\in I}) = (f_\alpha(x_\alpha)_{\alpha\in I})$.If f: $\prod_{\alpha\in I}$ (X_α , k_α)→$\prod_{\alpha\in I}$ (Y_α, l_α) is w - continuous, then f_α: (X_α, k_α)→ (Y_α, l_α) is w - continuous for each α∈I.

Proof: Let $\beta \in I$ and F be a closed subset of (Y_β, l_β). Then F x $\prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} Y_\alpha$ is a closed subset of $\prod_{\alpha\in I}(Y_\alpha, l_\alpha)$, Since f is w - continuous, $f^{-1}(F \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} Y_\alpha) = f_\beta^{-1}(F) \times \prod_{\substack{\alpha\neq\beta \\ \alpha\in I}} X_\alpha$ is a w - closed subset of $\prod_{\alpha\in I}(X_\alpha, k_\alpha)$.By proposition 2.18, $f_\beta^{-1}(F)$ is a w-closed subset of (X_β, k_β), Hence f_β is w - continuous.

Definition: 3.7 Let (X, k) and (Y, l) be closure spaces. A map f: (X, k) $\rightarrow$(Y, l) is called w – irresolute if $f^{-1}(F)$ is w - closed set in (X, k) for every w - closed set F in (Y, l).

Definition: 3.8 Let (X, k) and (Y, l) be closure spaces. A map f: (X, k) $\rightarrow$(Y, l) is called w–closed if f(F) is a w - closed subset of (Y, l)for every closed set F of (X, k).

Proposition: 3.9 Let (X, k), (Y, l) and (Z, m) be closure spaces. If f: (X, k) $\rightarrow$(Y, l) and g : (Y, l) $\rightarrow$(Z, m) be map. Then

(i) If f is closed and g is w - closed, then $g \circ f$ is w - closed

(ii) If $g \circ f$ is w - closed and f is continuous and surjective, then g is w - closed.

(iii) If $g \circ f$ is closed and g is w - continuous and injective, then f is w - closed.

Proposition: 3.10 A map f: (X, k) $\rightarrow$(Y, l) is w – closed if and only if, for each subset B of Y and each open subset G with $f^{-1}(B) \subseteq G$, there is a w - open subset V of (Y, l) such that $B \subseteq V$ and $f^{-1}(V) \subseteq G$.

Proof: Suppose f is w - closed. Let B be a subset of (Y, l) and G be an open subset of (X, k) such that $f^{-1}(B) \subseteq G$, Then f(X-G) is a w - closed subset of (Y, l). Let V= Y-f(X-G).Since V is w - open and $f^{-1}(V) = f^{-1}(Y\text{-}f(X\text{-}G)) = X\text{-} f^{-1}(f(X\text{-}G)) \subseteq X\text{-}(X\text{-}G) = G$. Therefore, V is w - open, $B \subseteq V$ and $f^{-1}(V) \subseteq G$.

Conversely, Suppose F is a closed subset of (X, k), Then $f^{-1}(Y\text{-} f(F)) \subseteq X\text{-}F$ and X-F is open. By hypothesis there is a w - open subset V of (Y, l) such that $Y\text{-}f(F) \subseteq V$ and $f^{-1}(V) \subseteq X\text{-}F$.

Therefore $F \subseteq X\text{-} f^{-1}(V)$.Hence $Y\text{-}V \subseteq f(F) \subseteq f(X\text{-} f^{-1}(V)) \subseteq Y\text{-}V \Rightarrow f(F) = Y\text{-}V$. Thus f(F) is w - closed. Therefore f is w - closed.

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