International Journal of Mathematical Archive-4(10), 2013, 278-284
Available online through www.ijma.info ISSN 2229 – 5046

GENERALIZED FIXED POINT RESULTS OF COMPATIBILITY IN PROBABILISTIC METRIC SPACE

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(Received on: 07-10-13; Revised & Accepted on: 22-10-13)

ABSTRACT

A common fixed point theorem for self mapping in menger space under weak compatibility in probabilistic metric space.

Keywords: *Menger space, weakly compatible mapping, semi-compatible mapping, weakly commuting mapping, common fixed point.*

AMS subject classification: *47H10, 54H25.*

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1. INTRODUCTION

Menger in 1942[9] was first introduced the concept of probabilistic metric space. the theory of probabilistic space is of fundamental importance in probabilistic functional analysis. The most interesting reference in this direction are [1],[2],[3],[4],[5],[6] and many others have proved common fixed point theorems in probabilistic metric space and menger space

2. PRELIMINARIES

Definition 2.1: let R denote the set of real's and R^+ the non negative real's. A mapping F: R→ R^+ is called a distribution function if it non decreasing left continuous with $\inf_{t\in R} F(t) = 0$ and $\sup_{t\in R} F(t) = 1$

Definition2.2: A probabilistic metric space is an ordered pair (X, F) where X is a non empty set, L be set of all distribution function and F:$XxX \to L$. We shall denote the distribution function by F (p, q) or $F_{p,q}(x)$ will represents the values of F(p, q) at x∈R. The function F (p, q) is assumed to satisfy the following conditions:
1. $F_{p,q}(x) = 1$ for all x>0 ⇔p=q
2. $F_{p,q}(x) = 0$ for every p,q ∈X
3. $F_{p,q}(x)$ =$F_{q,p}(x)$for every p, q ∈X
4. $F_{p,q}(x) = 1$ *and* $F_{q,r,}(x) = 1$ then $F_{p,r}(x + y) = 1$ for every p,q,r∈X and in metric space (X, d). The metric d induced a mapping F:$XxX \to L$ such that $F_{p,q}(x)$ =$F_{q,p}(x)$ =H(x-d(p, q) for every p, q ∈ X where H is the distribution defined as

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases}$$

Definition2.3: A mapping *: [0, 1] X [0, 1] →[0,1] is called t-norm if
1. (a*1) = a ∀ $a \in [0,1]$
2. (a*b) = 0 ∀ $a, b \in [0,1]$
3. (a*b) = (b*a)
4. (c*d)≥ (a*b) for c≥ $a, d \geq b$
5. ((a*b)*c) = (a*(b*c))

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Example:
1. (a*b) =ab
2. (a*b) = min (a, b)
3. (a*b) = max (a+b-1; 0)

Definition 2.4: A menger space is triplet (X, F. *) where (X, F) a PM-space and Δ is a t-norm with the following condition $F_{u,w}(x+y) \geq F_{u,v}$(x)* $F_{v,w}$(y) this inequality is called menger's triangle inequality.

Example; Let X=R. (a+b) = min (a, b) ∀a, b ∈(o, 1)

$$F_{u,v}(\mathrm{x}) = \begin{cases} H(x), & for\ u \neq v \\ 1, & u = v \end{cases}$$

where

$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases}$ .Then (X, F, *) is a menger space.

Definition 2.5: let (X, F, *) is menger space with the continuous T-norms t. A sequence $\{P_n\}$ in X, if for every $\in > 0, \lambda > 0$ $there\ \ exists\ an\ intger\ N = N(\in, \lambda)$

(I). Is said to be $P_n = \cup\, p(\in, \lambda)$ for all n≥ N. or equivalently, $F_{pP_n,}(\in) > 1 - \in$ for all n≥ N. we write $P_n \to p\ as$ n→ ∞ or $\lim_{n\to\infty} P_n = p$

(II). Is said to Cauchy $F_{P_nP_m,}(\in) > 1 - \lambda$ for all n, m ≥ N.

(III).Is said to be complete if every Cauchy sequence in X is converges to a point in X.

Definition 2.6: A coincidence point of two mappings is a point in their domain having the same image point under both mappings formally, given two mapping f, g: X→ Y we say that a point x in X is coincidence point of f and g if f(x) =g(x).

Definition 2.7: Let (X, F, *) be a menger space. two mappings f, g: X→ X are said to be weakly compatible if they commute at the coincidence point. i.e the pair {f, g} is weakly compatible pair if and only if fx=gx implies that fgx=gfx

Examples 2.9: Define athe pair A, S, [0, 1] → [0,1] by

$$A(x) = \begin{cases} x, & x \leq [0,1) \\ 1, & x > [0.1] \end{cases}, \qquad S(x) = \begin{cases} 1-x, & x \leq [0,1) \\ 1, & x > [0.1] \end{cases}$$

Then for any x∈ $[0,1]$, ASx=Sax, showing that A, S are weakly compatible maps on [0, 1]

Definition 2.8: Let (X, F,*) be a menger space. two mappings A,S: X→ X are said to be semi compatible if $F_{ASx_n,sx}$(t) → 1 for all t > 0 wherrnever $\{x_n\}$ is a sequence in X such that $Ax_n, Sx_n) \to p$ for some p in X as n→ ∞, it follow that (A, S) is semi compatible and Ay=Sy imply ASy=Say by taking $\{x_n\}$ =y and x=Ay =Sy.

Lemma 2.9: Let (X, F,*) be a menger space. If $\{P_n\}$ be a sequence in menger space, where * is continuous

I. (x*x) ≥ x ∀ x∈ $[0,1]$

II.1.∃ k ∈ (0,1) such that x> 0 and n ∈ N $F_{pn,pn+1}$(kx) ≥ $F_{pn-1,pn}$(x). Then $\{P_n\}$ is Cauchy sequence.

Lemma 2.10: ∃ k ∈ (0,1) such that $F_{x,y}$(kt) ≥ $F_{y,x}$(t) $\forall x, y \in X\ and\ k > 0\ then\ x = y$.

Lemma 2.11: If (X, d) is a metric space, then the metric d induces a mapping F:X x X→L, define by F(p, q)=H(x-d(p, q)), p, q∈X and x∈R.Further more if *:[0,1]X[0,1] →[0,1] is defined by (a*b)= min(a, b) then (X,F,*) is a menger space. It is complete if (X, d) is complete. The space (X, F,*) is obtained is called the induced menger space.

3. MAIN RESULT

Theorem 3.1: Let (X, F,*) be a complete menger space where * is continuous and (t*t) ≥ t for all t∈ $[0,1]$. Let S. T. A. and B be mapping from x into it self such that

(i).S(X)⊂B(X) AND T(X) ⊂ $A(X)$

(II).A, B, are continuous.

(iii).the pair (S,A) AND (T,B) semi compatible

(iv).there exists a number k ∈ (0,1)such that

$(F_{Sx,Ty}(K \in))^2)$≥ min{$(F_{Ax,Sx}(\in)$,$F_{By,,Ty}(\in)$,$F_{Ax,Ty}(2\in)$,$(F_{Bx,Sx}(2\in)$,}for all x, y∈ $X\ and\ \in > 0$. Then S, T, A, and B have a unique common fixed point in X.

Proof: we can find a sequence $\{y_n\}$ as follows

y_{2n} =s_{2n} =B_{2n+1} and y_{2n+1}=Tx_{2n+1}=Ax_{2n+2}.

$we\ shall\ prove\ that\ for\ any\ n \in N\ and\ \in > 0$

$$F_{y_{2n+1,y_{2n+2}}}(k\in) \geq F_{y_{2n,y_{2n+1}}}(\in) \tag{1}$$

$suppose$ (1) is not true. Then there exist n∈N and ∈> o such that

$$F_{y_{2n+1,y_{2n+2}}}(k\in) < F_{y_{2n,y_{2n+1}}}(\in) \tag{2}$$

$it\ follow\ from\ (iv) and\ (2)$

$(F_{Sx,Ty}(K \in))^2$

$$
\begin{aligned}
(F_{y_{2n+1,y_{2n+2}}}(k \in))^2 &= (F_{Sx_{2n+2,Tx_{2n+1}}}(k \in))^2 \\
&\geq \min\{(F_{Ax_{2n+2},sx_{2n+2}}(\in),)(F_{Bx_{2n+1},Tx_{2n+1}}(\in))(F_{Ax_{2n+2},sx_{2n+2}}(2 \in),)(F_{Bx_{2n+1},Tx_{2n+1}}(2\in) \\
&= \min\{(F_{Ax_{2n+2},sx_{2n+2}}(\in),)(F_{Bx_{2n+1},Tx_{2n+1}}(\in)(F_{Ax_{2n+2},sx_{2n+2}}(2 \in)(F_{Bx_{2n+1},Tx_{2n+1}}(2\in) \\
&= \min\{(F_{y_{2n+1},y_{2n+2}}(\in),)(F_{y_{2n},y_{2n+1}}(\in)(F_{y_{2n+1},y_{2n+1}}(2 \in)(F_{y_{2n},y_{2n+2}}(2\in) \\
&\geq \min\{(F_{y_{2n+1},y_{2n+2}}(K \in),)(F_{y_{2n},y_{2n+1}}(\in)(F_{y_{2n},y_{2n+2}}(2\in) \quad \text{since } K\in < \in
\end{aligned}
$$

And F is non decreasing.

$$\geq \min\{(F_{y_{2n+1},y_{2n+2}}(k \in),)(F_{y_{2n},y_{2n+1}}(\in)\}\min\{(F_{y_{2n},y_{2n+1}}(\in)(F_{y_{2n+1},y_{2n+2}}(\in) \tag{3}$$

Now $note, that$

$$
\begin{cases}
(a)\min\{(F_{y_{2n},y_{2n+1}}(\in)(F_{y_{2n+1},y_{2n+2}}(\in)\} >. (F_{y_{2n+1,y_{2n+2}}}(k \in))^2 \\
(b)\min\{(F_{y_{2n},y_{2n+1}}(\in)(F_{y_{2n+1},y_{2n+2}}(\in)\} \geq (F_{y_{2n+1},y_{2n+2}}(k \in) \\
(c)\min\{(F_{y_{2n},y_{2n+1}}(\in)(F_{y_{2n+1},y_{2n+2}}(\in)\}(F_{y_{2n},y_{2n+1}}(\in)
\end{cases}
$$

$$\geq \{(F_{y_{2n+1},y_{2n+2}}(k \in),)(F_{y_{2n},y_{2n+1}}(\in)$$

$$> (F_{y_{2n+1,y_{2n+2}}}(k \in))^2$$

So we get from (3) that

$(F_{y_{2n+1,y_{2n+2}}}(k \in))^2 > (F_{y_{2n+1,y_{2n+2}}}(k \in))^2$, a contradiction.

$There$ for, (1) holds, for any n∈ N and ∈>∈

0 $using\ a\ similar\ argument\ we\ obatain\ that\ for\ any$ n∈ $N$and ∈> 0

$$(F_{y_{2n},y_{2n+1}}(\in)) \geq (F_{y_{2n-1},y_{2n}}(\in)\} \tag{4}$$

Thus putting (1) and (4) to gather, we see that $(F_{y_n,y_{n+1}}(\in)) \geq (F_{y_{n-1},y_n}(\in)$for any n∈ N and ∈> 0, and hence by lemma2.9 $\{y_n\}$is a Cauchy sequence in X. Since X is complete. There exists z in X such that

$$
\begin{cases}
s_{x_{2n}} \longrightarrow z \\
B_{x_{2n+1}} \longrightarrow z \\
T_{x_{2n+1}} \longrightarrow z \\
A_{x_{2n+2}} \longrightarrow z
\end{cases}
\text{as } n \longrightarrow \infty
$$

Now, suppose A is continuous. Then

$A^2_{x_{2n}} \longrightarrow Az$ and $As_{x_{2n}} \longrightarrow Az$ as n⟶ ∞ (5)

$since\ both\ of$ { Ax_{2n}} and { sx_{2n}} are convergent to z, the semi compatibility of A and S implies that

$lim_{n\to\infty}F_{ASx_{2n},ASx_{2n},}(\in) = 1.$

This in conjunction with (5) and the inequality

$F_{SAx_{2n},AZ}(\in) \geq \min\{F_{SAx_{2n},Asx_{2n}}(\frac{\in}{2}), F_{Asx_{2n},Az_{2n}}(\frac{\in}{2})$

Show that $SAx_{2n} \longrightarrow Az\ as$ n⟶ ∞

Let E= {∈ >0: $F_{Az,z}$ is continuous at ∈} since $F_{Az,z}$ is non decreasing, it can be discontinuous at only denumerable many points.

We show that $F_{Az,z}(\in) \geq F_{Az,z}(k^{-1}\in)$for any $\in belongs\ to\ E.$ by IV

$$(F_{SAx_{2n},Tx_{2n+1}}(k\in))^2 \geq \min\{(F_{A2x_{2n},sx_{2n+2}}(k^{-1}\in),)(F_{Bx_{2n+1},Tx_{2n+1}}(k^{-1}\in)(F_{A2x_{2n+2},sx_{2n+2}}(2k^{-1}\in),) \\ (F_{Bx_{2n+1},Tx_{2n}}(2k^{-1}\in) \tag{6}$$

$it\ is\ easy$ to see that we can choose a subsequence $\{n_j\}$ of natural numbers such that all the limits in (6) exists as j⟶ ∞ and satisfy

$$Lim_{j\to\infty}(F_{SAx_{2nj},Tx_{2nj+1}}(k\in))^2 \geq \min\{Lim_{j\to\infty}(F_{A2x_{2nj},sAx_{2nj}}(k^{-1}\in),)(F_{Bx_{2nj+1},Tx_{2nj+1}}(k^{-1}\in)\}$$

$$Lim_{j\to\infty}\{(F_{A2x_{2nj+1},Tx_{2nj+1}}(2k^{-1}\in),)(F_{Bx_{2nj+1},SAx_{2nj}}(2k^{-1}\in)\}$$

$$\geq \min\{Lim_{j\to\infty}(F_{A2x_{2nj},sAx_{2nj}}(k^{-1}\in),)(F_{Bx_{2nj+1},Tx_{2nj+1}}(k^{-1}\in)\} \\ Lim_{j\to\infty}(F_{A2x_{2nj+1},Tx_{2nj+1}}(2k^{-1}\in),)(F_{Bx_{2nj+1},SAx_{2nj}}(2k^{-1}\in)$$

$$\geq \min\{\text{lowerlim}_{n\to\infty}(F_{A2x_{2n},sAx_{2n}}(k^{-1}\in),)(F_{Bx_{2n+1},Tx_{2n+1}}(k^{-1}\in) \\ \underset{n\to\infty}{\text{lowerlim}}(F_{A2x_{2n+1},Tx_{2n+1}}(2k^{-1}\in),)(F_{Bx_{2n+1},SAx_{2n}}(2k^{-1}\in)$$

$$\geq \min\{F_{Az,Az}(k^{-1}\in),)(F_{z,z},(k^{-1}\in)(F_{Az,z},(2k^{-1}\in),)(F_{z,Az}(2k^{-1}\in)$$

$$\geq (F_{AZ,Z}(k^{-1}\in))^2 \tag{7}$$

Also since ∈ belong to E, it follow from lemma 2.9 that $lim_{n\to\infty}F_{SAx_{2n},,Tx_{2n+1},}(\in) = (F_{AZ,Z}(\in)$ which in conjunction with (7) shows that

$(F_{AZ,Z}(\in) \geq (F_{AZ,Z}(k^{-1}\in)$ for ∈ belong to E. (8)

$to\ conclude\ that\ Az = z\ we\ must\ show\ that\ F_{AZ,Z}(\in)$=1 for any$\in > 0. for\ this\ let$ ∈ be any member in E and put $\in_1$=∈. Then we have

$\in_1 = k^{-1}(\in_1) <. k^{-2}(\in_1) <$…….$k^{-n}(\in_1) <$………$lim_{n\to\infty}k^{-n}(\in_1) = \infty$ (9)

let ή >0 be any given positive number. Since $F_{AZ,Z}$ is left continuous at $k^{-2}(\in_1).\ there\ is\ \delta > 0.$ Such that

$F_{AZ,Z}(k^{-2}(\in_1).) \leq F_{AZ,Z}(\omega) + \frac{ή}{2}$ (10)

For all $\omega \in (k^{-2}(\in_1) - \delta, k^{-2}(\in_1))$

By the continuity of k^{-1}at $k^{-1}(\in_1)$, we choose $\in_2 \in (\in_1, k^{-1}(\in_1)) \cap$ E so that $k^{-1}(\in_1) \in (k^{-2}(\in_1) - \delta, (k^{-2}(\in_1))$, and hence with the aid of (10)

$(F_{AZ,Z}(k^{-1}(\epsilon_2))) \geq (F_{AZ,Z}(k^{-2}(\epsilon_1)))$ - $-\frac{ή}{2}$ (11)

$by\ induction, for\ any$ n$\in N$ we can choose $\epsilon_{n+1}\in$ E so that

$k^{-n+1}(\epsilon_1))) < \epsilon_{n+1} < k^{-n}(\epsilon_1)$ and $(F_{AZ,Z}(k^{-1}(\epsilon_{n+1}))) \geq (F_{AZ,Z}(k^{-(n+1)}(\epsilon_1)))$ $-\frac{ή}{2^n}$ (12)

So we have

$$\begin{aligned}
F_{AZ,Z}(\epsilon) &= F_{AZ,Z}(\epsilon_1) \\
&\geq (F_{AZ,Z}(k^{-1}(\epsilon_1))) \\
&\geq (F_{AZ,Z}(k^{-2}(\epsilon_2))) \text{ since } \epsilon_2\ \ belong\ to\ E \\
&\geq (F_{AZ,Z}(\epsilon_3) - -\tfrac{ή}{2} \\
&\geq (F_{AZ,Z}K^{-1}(\epsilon_2)) - -\tfrac{ή}{2} \\
&\geq (F_{AZ,Z}(K^{-1}(\epsilon_1)) - -\tfrac{ή}{2^2}\ \ -\tfrac{ή}{2} \\
&\quad \ldots\ldots\ldots\ldots\ldots\ldots \\
&\geq (F_{AZ,Z}(K^{-n}(\epsilon_1)) - -(\tfrac{ή}{2^{n-1}} + \tfrac{ή}{2^{n-2}}\ + \ldots\ldots -\tfrac{ή}{2}) \\
&= (F_{AZ,Z}(K^{-n}(\epsilon_1)) - ή(1 - \tfrac{1}{2^{n-1}}) \forall n \in N
\end{aligned}$$

Letting $n \to \infty$ in (13) and holding $lim_{n\to\infty}k^{-n}$ $(\epsilon_1) = \infty$, we obtain that

$F_{Az,z}(\epsilon) \geq$ 1-η for $\eta > 0$.

Since $\eta > 0$ is arbitrary, we conclude that $F_{Az,z}(\epsilon)$ =1 for any $\in$ belong to E. Since E is dense in $[0, \infty)$and $F_{Az,z}$ is left continuous on $(0, \infty)$. We see that $F_{Az,z}(\epsilon)$ =1 for all $\epsilon > 0$ and so Az=z. As for Sz=z, using $(F_{Az,z}(\epsilon))^2$ $=\lim_{n\to\infty}(F_{Sz,Tx_{2n+1}}(\epsilon))^2$ and (iv), we can just follow as before to obtain $F_{Sz,z}(\epsilon) \geq (F_{Sz,z}(\epsilon))^2$ for $\epsilon>$Where $F_{Sz,z}$ is continuous. Then in a similar argument as before, we conclude $F_{Sz,z}(\epsilon)$ =1$\forall\epsilon > 0$. Since S(X)$\subseteq B(X)$, there exist y in X such that By=Sz=Z. so for any $\epsilon > 0$

$$\begin{aligned}
(F_{z,Ty}\ (k\ \epsilon))^2 &= (F_{Sz,Ty}\ (k\ \epsilon))^2 \\
&\geq \min\{(F_{Az,Sz}\ (\epsilon), F_{By,Ty}\ (\epsilon)(F_{Az,Ty}\ (2\ \epsilon)(F_{By,Sz}\ (2\ \epsilon)\} \\
&\geq \min\{\ (F_{z,Ty}\ (\epsilon)(F_{z,Ty}\ (2\ \epsilon)\} \\
&\geq (F_{z,Ty}\ (\epsilon))^2
\end{aligned}$$

$(F_{z,Ty}\ (K\ \epsilon) \geq (F_{z,Ty}\ (\epsilon))$, and ty=z. up to now we have shown that SZ = Az = z = By = Ty. We are now going to show that z is a common fixed point of S, T, A, and B. Since T and B are semi compatible, we have BTy=TBy, that is Bz =Tz. Therefore for $\epsilon > 0$, we have following inequalities.

$$\begin{aligned}
(F_{z,Tz}\ (k\ \epsilon))^2 &= (F_{Sz,Tz}\ (k\ \epsilon))^2 \\
&\geq \min\{(F_{Az,Sz}\ (\epsilon), F_{Bz,Tz}\ (\epsilon)(F_{Az,Tz}\ (2\ \epsilon)(F_{Bz,Sz}\ (2\ \epsilon) \\
&\geq\ \min\{\ (F_{z,Tz}\ (\epsilon)(F_{z,Tz}\ (2\ \epsilon)\} \\
&= \min\{\ (F_{z,Tz}\ (2\ \epsilon))^2, (F_{z,Tz}\ (2\ \epsilon) \\
&\geq (F_{z,Tz}\ (\epsilon))^2
\end{aligned}$$

So Tz=z ny Lemma 2.10. This completes the proof for z being the common fixed point of S, T, A, A and B provided that A is continuous. By symmetric, if B is continuous we can prove that S, T, A and B have common fixed point in a similar way.

Next, assume that S is continuous. Then SA$x_{2n} \to Sz$. and SB$x_{2n+1} \to Sz$. as n$\to \infty$, and since S and A are semi compatible and both {Ax_{2n}}and {Sx_{2n}}are convergent to z, $\lim_{n\to\infty}(F_{ASx_{2n},SAx_{2n}}\ \frac{\epsilon}{2}$, $F_{ASx_{2n},Sz}\ \frac{\epsilon}{2}$}and both $F_{ASx_{2n},SAx_{2n}}\ \frac{\epsilon}{2}$} and {$F_{SAx_{2n},Sz}\ \frac{\epsilon}{2}$}are convergent to1. We see that $\lim_{n\to\infty}(F_{ASx_{2n},Sz}\ (\epsilon)$ =1 for $\epsilon > 0$ and so $\lim_{n\to\infty}$A$Sx_{2n} = Sz$. In inequality

$$\begin{aligned}
(F_{SBx_{2n+1},Tx_{2n+1},}\ (\epsilon))^2 \geq \min\{&(F_{ABx_{2n+1},ABx_{2n+1}}\ (k^{-1}\ \epsilon), (F_{Bx_{2n+1},Tx_{2n+1}}\ (k^{-1}\ \epsilon), \\
&(F_{ABx_{2n+1},Tx_{2n+1}}\ (2k^{-1}\ \epsilon), (F_{Bx_{2n+1},SBx_{2n+1}}\ (2k^{-1}\ \epsilon),
\end{aligned}$$

We can imitate the procedure for the case that A is continuous to show that $F_{Sz,z}(\in) \geq (F_{Sz,,z}\ (k^{-1} \in)$, for any $\in > 0$ where $F_{Sz,z}$ is continuous, and then show that $F_{Sz,z}(\in) = 1$ for any $\in > 0$. So Sz=z. Since S(x)⊆ B(X). We can choose y∈ X. we can choose Y∈X such that By=Sz=z. then for any $\in > 0$ and n∈ N. So Tz=z by Lemma 2.10 This completes the proof for z being the common fixed point of S, T, A, A and B provided that A is continuous .By symmetric, if B is continuous we ca prove that S, T, A and B have common fixed point in a similar way.

Next, assume that S is continuous. Then SA$x_{2n} \to Sz$. and SB$x_{2n+1} \to Sz$. as n→ ∞, and since S and A are semi compatible and both {Ax_{2n}} and {Sx_{2n}}are convergent to z, $\lim_{n\to\infty}(F_{ASx_{2n},SAx_{2n}}\frac{\in}{2}, F_{ASx_{2n},Sz}\frac{\in}{2}\}$and both $F_{ASx_{2n},SAx_{2n}}\frac{\in}{2}\}$ and $\{F_{SAx_{2n},Sz}\frac{\in}{2}\}$are convergent to 1. We see that $\lim_{n\to\infty}(F_{ASx_{2n},Sz}(\in) = 1$ for $\in > 0$and so $\lim_{n\to\infty} ASx_{2n} = Sz$. In inequality

$$(F_{SBx_{2n+1},Tx_{2n+1},}(\in))^2 \geq \min\{(F_{ABx_{2n+1},ABx_{2n+1}}(k^{-1}\in), (F_{Bx_{2n+1},Tx_{2n+1}}(k^{-1}\in), (F_{ABx_{2n+1},Tx_{2n+1}}(2k^{-1}\in), (F_{Bx_{2n+1},SBx_{2n+1}}(2k^{-1}\in),$$

We can imitate the procedure for the case that A is continuous to show that $F_{Sz,z}(\in) \geq (F_{Sz,,z}\ (k^{-1} \in)$,for any $\in > 0$ where $F_{Sz,z}$ is continuous , and then show that $F_{Sz,z}(\in) = 1$for any $\in > 0$. So Sz=z. Since S(x)⊆ B(X). We can choose y∈ X. We can choose Y∈X such that By=Sz=z. then for any $\in > 0$ and n∈ N *we have*

$$(F_{SBx_{2n+1},y}(\in))^2 \geq \min\{(F_{ABx_{2n+1},SBx_{2n+1}}(k^{-1}\in), (F_{By,Ty}(k^{-1}\in), (F_{ABx_{2n+1},Ty}(2k^{-1}\in), (F_{By,SBx_{2n+1}}(2k^{-1}\in),$$

$$= \min\{(F_{ASx_{2n+1},S^2x_{2n}}(k^{-1}\in), (F_{z,Ty}(k^{-1}\in), (F_{ASx_{2n}S^2x_{2n},}(2k^{-1}\in), (F_{z,S^2x_{2n}}(2k^{-1}\in),$$

As the case that A is continuous. We can take limit via a suitable subsequence $\{n_j\}$ of natural numbers to get

$$(F_{z.Ty,}(k\in))^2 \geq \min\{(F_{z,Ty,}(k^{-1}\in), (F_{z,Ty,}(2k^{-1}\in),\}$$
$$\geq (F_{z.Ty,}(k^{-1}\in))^2 \text{ for } \in > 0 \text{ where } F_{z.Ty,} \text{ is continuous.}$$

Thus ty=z. in summary we have shown that By=Ty=z. Now Since T(X)⊆ $A(X)$ there exists x∈ X such that z=Sz=By=Ty=Ax. Then we get Ax=Sx from the following inequalities.

$$(F_{Sx.Ax,}(k\in))^2 = (F_{Sx.Ty,}(k\in))^2$$
$$\geq \min\{(F_{Ax,Sx}(\in), F_{By,Ty}(\in)(F_{Ax,Ty}(2\in)(F_{By,Sx}(2\in)$$
$$= \min\{(F_{Ax,Sx}(\in)(F_{Ax,Sx}(2\in)\}$$
$$\geq (F_{Ax.Sx,}(\in))^2$$

Let ξ Ax=Sx=Ty=By. since S and A are semi compatible and hence Ax=Sx, we get ASx=Sax, that is A$\xi = S\xi$. *Then for any* $\in > 0$

$$(F_{S\xi,\xi,}(k\in))^2 = (F_{S\xi,T\xi,}(k\in))^2$$
$$\geq \min\{(F_{A\xi,S\xi}(\in), F_{By,Ty}(\in)(F_{A\xi,Ty}(2\in)(F_{By,S\xi}(2\in)\}$$
$$= \min\{(F_{S\xi,\xi,}(k\in))^2(F_{S\xi,\xi}(2\in\}$$
$$\geq (F_{S\xi,\xi,}(\in))^2$$

Which implies that S $\xi = \xi = A\xi$.Next $\nu \in X$ such that B $\nu = S\ \xi = \xi$. Then

$$(F_{\xi,T\nu,}(k\in))^2 = (F_{S\nu,T,\nu}(k\in))^2$$
$$\geq \min\{(F_{A\xi,S\xi}(\in), F_{B\nu,T\nu}(\in)(F_{A\xi,T\nu}(2\in)(F_{B,\nu S\xi}(2\in)\}$$
$$= \min\{(F_{\xi,S\nu}(\in), (F_{\xi,T\nu}(2\in)\}$$
$$\geq (F_{\xi,T\nu,}(\in))^2 \forall \in > 0$$

Hence T $\nu = \xi$. Since T and B are semi compatible and T ν = B ν. We have T$B\ \nu$ = BT ν that is T $\xi = B\xi$. Then we conclude that T $\xi = \xi$ from the following inequalities.

$$(F_{y,z,}(k\in))^2 = (F_{Sy,T,z}(k\in))^2$$
$$\geq \min\{(F_{Ay,Sy}(\in), F_{Bz,Tz}(\in)(F_{Ay,Tz}(2\in)(F_{Bz,Sy}(2\in)\}$$
$$= \min\{(F_{y,z,}(2\in))^2, (F_{y,z}(2\in)\}$$
$$\geq (F_{y,z,}(\in))^2\ \forall \in > 0$$

We conclude that y=z by virtue of lemma 2.10

Corollary 3.2: Let (X, F, *) be a complete menger space where * is continuous and (t*t) ≥ t for all t∈ [0,1]. Let S. T. A. and B be mapping from x into it self such that
(i).S(X)⊂B(X)
(II).B is continuous.
(iii).the pair (S, B) is semi compatible
(iv).there exists a number $k \in (0,1)$ such that

$(F_{Sx,Ty}(K \in))^2) \geq \min\{(F_{Ax,Sx}(\in), F_{By,,Ty}(\in), F_{Ax,Ty}(2\in), (F_{Bx,Sx}(2\in),\}$ for all x, y∈ X and $\in > 0$. Then S, and B have a unique common fixed point in X.

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Source of support: Nil, Conflict of interest: None Declared