# INVENTORY SYSTEMS WITH SALES TIME DEPENDING ON PRODUCION TIME AND SIZE 

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#### Abstract

A production and sales storage system is considered. During the operation time a machine produces random number of products. After operation time, sales time starts and it has one among two distinct distributions depending on the magnitude of production is within or exceeding a random threshold magnitude. Two models are treated. Sales time depends on the number and size of the products produced. In Model A the operation times have exponential distributions and the production times are general. Model B considers the case when the operation times have general distributions and the production times are exponential. Joint transforms of the variables, means, variances, covariance and numerical results are presented


Key word: Storage system, production and sales times, Random magnitude of production, Joint transform.
Subject classification: [2000]: 30C45,30C80,90B05.

## 1. INTRODUCTION

Storage systems of $(s, S)$ type was studied by Arrow, Karlin and Scrat [1]. Such systems with random lead times and unit demand were treated by Danial and Ramanarayanan [2] .Models with bulk demands were analyzed by Ramanarayanan and Jacob [9]. Murthy and Ramanarayanan [4,5,6,7] considered several ( $s, S$ ) inventory systems. Kun- Shan Wu and Liang -Yuh Ouyang [3] have studied ( $Q, r, L$ ) inventory model with defective items. So far no models in these areas have been studied in which the sale time is depending on the number or size of the products produced. In manufacturing models to get the return on investment and to pay minimum interest, it is natural that when the production is more, the sales time is made short so as to cut cost. It has been noticed that when the units produced are more, financial supports for the customers are provided to clear products early. These are widely felt in perishable commodity sectors where many banking institutions provide required finance for the purchase. In this paper we consider two models. In model A we consider the operation times have exponential distributions and the production times have general distributions. In model B we study the case when the operation times have general distributions and the production times have exponential distributions. After the production, the sales time starts. In models A and B, the sale time varies depending on the number of products produced and operation times are within or in excess of the threshold limits. The joint transforms, the means of production times and sale times, the variances, the covariance of the variables and numerical examples are also presented.

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## 2. MODEL A : MARKOVIAN OPERATION TIME AND GENERAL RANDOM PRODUCTION

The main assumptions of the model are given below.

1. The machine operation time T is a random variable with exponential distribution with parameter $\lambda$.
2. The inter-production times of products are independent random variables with cumulative distribution function Cdf $F(x)$ and probability density function pdf $f(x)$. Let N be the number of productions completed during the operation time $T$. The size of each production is a random variable with general distribution function $\operatorname{Cdf} H(x)$ and $h(x)$. Let Z be the total size of production in operation time.
3. The sale time S has general distribution either ( $i$ ) with $\operatorname{Cdf} G_{11}(y)$ and $\operatorname{pdf} g_{11}(x)$ when the operation time is within the time threshold U and the total size of the products produced Z is less than the threshold V , or (ii) with Cdf $G_{10}(y)$ and pdf $g_{10}(x)$ when the operation time is within the time threshold U and Z is more than the threshold V, or (iii) with Cdf $G_{01}(y)$ and pdf $g_{01}(x)$ when the operation time is more than the threshold U and Z is less than the threshold V , or (iv) with $\operatorname{Cdf} G_{00}(y)$ and pdf $g_{00}(x)$ when the operation time is more than the threshold U and Z is more than the threshold V , in order to provide change in selling rate. When no unit is produced during operation time sales time S is used for other purpose.
4. The time threshold U has exponential distribution with parameter $\mu$ and the size threshold V has exponential distribution with parameter $\alpha$.
We may derive the joint distribution of T and S as follows. Noting that the operation time has exponential distribution and inter production times are general, we may derive the joint distribution of $\mathrm{T}, \mathrm{N}, \mathrm{Z}$ and S as follows. The joint probability density function of $\mathrm{T}, \mathrm{Z}$ and S and probability function of N is

$$
\begin{align*}
\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial v} P(T & \leq x, V \leq v, S \leq y)=f(x, v, y) \\
= & \sum_{0}^{\infty}\left[\lambda e^{-\lambda x}\left[F_{n}(x)-F_{n+1}(x)\right] e^{-\mu x} H_{n}(v) \alpha e^{-\alpha v} g_{11}(y)\right. \\
& +\lambda e^{-\lambda x}\left[F_{n}(x)-F_{n+1}(x)\right] e^{-\mu x}\left(1-H_{n}(v)\right) \alpha e^{-\alpha v} g_{10}(y) \\
& +\lambda e^{-\lambda x}\left[F_{n}(x)-F_{n+1}(x)\right]\left(1-e^{-\mu x}\right) H_{n}(v) \alpha e^{-\alpha v} g_{01}(y) \\
& \left.+\lambda e^{-\lambda x}\left[F_{n}(x)-F_{n+1}(x)\right]\left(1-e^{-\mu x}\right)\left(1-H_{n}(v)\right) \alpha e^{-\alpha v} g_{00}(y)\right] . \tag{1}
\end{align*}
$$

Here $n=1,2,3 \ldots \ldots$. The first term of equation (1) is the part of the pdf that the operation time is $x$, the sale time is $y$, the operation time is within the threshold U , the number of productions is n and the size of production Z is within the threshold level V. The second term of equation (1) is the part of the pdf that the operation time is x which is within the threshold $U$, the sale time is $y$, the number of productions is $n$ and the size of production $Z$ is in excess of $V$, the threshold level. The third term of equation (1) is the part of the pdf that the operation time is x which is in excess of the threshold U , the sale time is y , the number of productions is n and the size of production Z is within V , the threshold level. The fourth term of equation (1) is the part of the pdf that the operation time is $x$ which is in excess of the threshold $U$, the sale time is y , the number of productions is n and the size of production Z is in excess of V , the threshold level where $F_{k}(z)$ is the k fold 'Cdf convolution' and is the probability that the size of the sum of k productions is less than z . Let us define the joint Laplace transform as follows.

$$
\begin{align*}
E\left(e^{-t T} e^{-s S} e^{-w V}\right) & =\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} f(x, v, y) e^{-t x} e^{-s y} e^{-w v} d x d v d y \\
& =\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \lambda e^{-\lambda x} \sum_{0}^{\infty}\left[F_{n}(x)-F_{n+1}(x)\right] e^{-\mu x} H_{n}(v) \alpha e^{-\alpha v} e^{-t x} e^{-s y} e^{-w v} g_{11}(y) d x d v d y \\
& +\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \lambda e^{-\lambda x} \sum_{0}^{\infty}\left[F_{n}(x)-F_{n+1}(x)\right] e^{-\mu x}\left(1-H_{n}(v)\right) \alpha e^{-\alpha v} e^{-t x} e^{-s y} e^{-w v} g_{10}(y) d x d v d y \\
& +\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \lambda e^{-\lambda x} \sum_{0}^{\infty}\left[F_{n}(x)-F_{n+1}(x)\right]\left(1-e^{-\mu x}\right) H_{n}(v) \alpha e^{-\alpha v} g_{01}(y) e^{-t x} e^{-s y} e^{-w v} d x d v d y \\
& +\int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \lambda e^{-\lambda x} \sum_{0}^{\infty}\left[F_{n}(x)-F_{n+1}(x)\right]\left(1-e^{-\mu x}\right)\left(1-H_{n}(v)\right) \alpha e^{-\alpha v} e^{-t x} e^{-s y} e^{-w v} g_{00}(y) d x d v d y \tag{2}
\end{align*}
$$

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$0 \leq \theta \leq 1$ and $t, s, w \geq 0$ which on simplification gives

$$
\begin{align*}
E\left(e^{-t T} e^{-w V} e^{-s s}\right)= & \left(\frac{\lambda \alpha}{(\alpha+w)(\lambda+t+\mu)}\right)\left(\frac{1-f^{*}(t+\lambda+\mu)}{1-h^{*}(\alpha+w) f^{*}(t+\lambda+\mu)}\right) \\
& \left(g_{11}^{*}(s)-g_{10}^{*}(s)-g_{01}^{*}(s)+g_{00}^{*}(s)\right)+\left(\frac{\lambda \alpha}{(\alpha+w)(\lambda+t+\mu)}\right)\left(g_{10}^{*}(s)-g_{00}^{*}(s)\right) \\
& +\left(\frac{\lambda \alpha}{(\alpha+w)(\lambda+t)}\right)\left(\frac{1-f^{*}(t+\lambda)}{1-h^{*}(\alpha+w) f^{*}(t+\lambda)}\right)\left(g_{01}^{*}(s)-g_{00}^{*}(s)\right) \\
& +\left(\frac{\lambda}{t+\lambda}\right)\left(\frac{\alpha}{\alpha+w}\right) g_{00}^{*}(s) . \tag{3}
\end{align*}
$$

We may note $E\left(e^{-t T}\right)=\frac{\lambda}{t+\lambda}$ and $E\left(e^{-w V}\right)=\frac{\alpha}{\alpha+w}$ by putting zero for other variables in equation (3).

$$
\begin{gather*}
E\left(e^{-s s}\right)=\left(\frac{\lambda}{\lambda+\mu}\right)\left(\frac{1-f^{*}(\lambda+\mu)}{1-h^{*}(\alpha) f^{*}(\lambda+\mu)}\right)\left(g_{11}^{*}(s)-g_{10}^{*}(s)-g_{01}^{*}(s)+g_{00}^{*}(s)\right)+\left(\frac{\lambda}{\lambda+\mu}\right)\left(g_{10}^{*}(s)-g_{00}^{*}(s)\right) \\
+\left(\frac{1-f^{*}(\lambda)}{1-h^{*}(\alpha) f^{*}(\lambda)}\right)\left(g_{01}^{*}(s)-g_{00}^{*}(s)\right)+g_{00}^{*}(s) \tag{4}
\end{gather*}
$$

This on differentiation gives

$$
\begin{align*}
E(S)=\left(\frac{\lambda}{\lambda+\mu}\right) & \left(\frac{1-f^{*}(\lambda+\mu)}{1-h^{*}(\alpha) f^{*}(\lambda+\mu)}\right)\left(E\left(S_{11}\right)-E\left(S_{10}\right)-E\left(S_{01}\right)+E\left(S_{00}\right)\right) \\
& +\left(\frac{\lambda}{\lambda+\mu}\right) E\left(S_{10}\right)+\frac{\mu}{\lambda+\mu} E\left(S_{00}\right)+\left(\frac{1-f^{*}(\lambda)}{1-h^{*}(\alpha) f^{*}(\lambda)}\right)\left[E\left(S_{01}\right)-E\left(S_{00}\right)\right] . \tag{5}
\end{align*}
$$

We note that the joint transform of T and S is

$$
\begin{align*}
E\left(e^{-t T} e^{-s s}\right)= & \left(\frac{\lambda}{\lambda+t+\mu}\right)\left(\frac{1-f^{*}(t+\lambda+\mu)}{1-h^{*}(\alpha) f^{*}(t+\lambda+\mu)}\right)\left(g_{11}^{*}(s)-g_{10}^{*}(s)-g_{01}^{*}(s)+g_{00}^{*}(s)\right) \\
& +\left(\frac{\lambda}{\lambda+t+\mu}\right)\left(g_{10}^{*}(s)-g_{00}^{*}(s)\right) \\
& +\left(\frac{\lambda}{\lambda+t}\right)\left(\frac{1-f^{*}(t+\lambda)}{1-h^{*}(\alpha) f^{*}(t+\lambda)}\right)\left(g_{01}^{*}(s)-g_{00}^{*}(s)\right)+\left(\frac{\lambda}{t+\lambda}\right)\left(\frac{\lambda}{t+\lambda}\right) g_{00}^{*}(s) . \tag{6}
\end{align*}
$$

Using differentiation of (6) we get

$$
\begin{aligned}
& E(T S)=\frac{1}{\lambda} E\left(S_{00}\right)+\left[E\left(S_{01}\right)-E\left(S_{00}\right)\right]\left\{\frac{1-f^{*}(\lambda)}{\lambda\left(1-h^{*}(\alpha) f^{*}(\lambda)\right)}+\frac{\left(f^{*}\right)^{\prime}(\lambda)\left(1-h^{*}(\alpha)\right.}{\left(1-h^{*}(\alpha) f^{*}(\lambda)\right)^{2}}\right\} \\
&+\left[E\left(S_{11}\right)-E\left(S_{10}\right)-E\left(S_{01}\right)+E\left(S_{00}\right)\right]\left(\frac{\lambda}{\lambda+\mu}\right) \frac{1-f^{*}(\lambda+\mu)}{(\lambda+\mu)\left(1-h^{*}(\alpha) f^{*}(\lambda+\mu)\right)} \\
&+\frac{\left(f^{*}\right)(\lambda+\mu)}{\left(1-h^{*}(\alpha) f^{*}(\lambda+\mu)\right)^{2}}+\left(\frac{\lambda}{(\lambda+\mu)^{2}}\right)\left(E\left(S_{10}\right)-E\left(S_{00}\right)\right) .
\end{aligned}
$$

Using $\operatorname{Cov}(T, S)=E(T S)-E(T) E(S)$ we get

$$
\begin{align*}
\operatorname{Cov}(T, S)=[ & \left(E\left(S_{01}\right)-E\left(S_{00}\right)\right] \frac{f^{*^{\prime}}\left(\lambda\left[1-h^{*}(\alpha)\right]\right)}{\left(1-h^{*}(\alpha) f^{*}(\lambda)\right)^{2}}+\left[E\left(S_{11}\right)-E\left(S_{10}\right)-E\left(S_{01}\right)+E\left(S_{00}\right)\right] \\
& \cdot\left\{\frac{-\mu}{(\lambda+\mu)^{2}}\left(\frac{1-f^{*}(\lambda+\mu)}{\left(1-h^{*}(\alpha) f^{*}(\lambda+\mu)\right)}\right)+\left(\frac{\lambda}{\lambda+\mu}\right) \frac{\left(f^{*}\right)^{\prime}(\lambda+\mu)\left[1-h^{*}(\alpha)\right]}{\left(1-h^{*}(\alpha) f^{*}(\lambda+\mu)\right)^{2}}\right\} \\
& +\left(\frac{-\mu}{(\lambda+\mu)^{2}}\right)\left[E\left(S_{10}\right)-E\left(S_{00}\right)\right] . \tag{7}
\end{align*}
$$

We may find that

$$
\begin{align*}
E\left(e^{-w V} e^{-s s}\right)= & \left(\frac{\lambda \alpha}{(\alpha+w)(\lambda+\mu)}\right)\left(\frac{1-f^{*}(\lambda+\mu)}{1-h^{*}(\alpha+w) f^{*}(\lambda+\mu)}\right)\left(g_{11}^{*}(s)-g_{10}^{*}(s)-g_{01}^{*}(s)+g_{00}^{*}(s)\right) \\
& +\left(\frac{\lambda \alpha}{(\alpha+w)(\lambda+\mu)}\right)\left(g_{10}^{*}(s)-g_{00}^{*}(s)\right) \\
& +\left(\frac{\alpha}{\alpha+w}\right)\left(\frac{1-f^{*}(\lambda)}{1-h^{*}(\alpha+w) f^{*}(\lambda)}\right)\left(g_{01}^{*}(s)-g_{00}^{*}(s)\right)+\left(\frac{\alpha}{\alpha+w}\right) g_{00}^{*}(s) . \tag{8}
\end{align*}
$$

$$
E(V S)=\left(\frac{-\lambda}{\alpha(\lambda+\mu)}\right)\left(\frac{1-f^{*}(\lambda+\mu)}{1-h^{*}(\alpha) f^{*}(\lambda+\mu)}\right)
$$

$$
+\left(\frac{\lambda}{\lambda+\mu}\right) \frac{1-f^{*}(\lambda+\mu)}{\left(1-f^{*}(\lambda+\mu) h^{*}(\alpha)\right)^{2}}\left(h^{*}\right)^{\prime}(\alpha) f^{*}(\lambda+\mu)\left(\left(g_{11}^{*}\right)^{\prime}(s)\right.
$$

$$
\left.-\left(g_{10}^{*}\right)^{\prime}(s)-\left(g_{01}^{*}\right)^{\prime}(s)+\left(g_{00}^{*}\right)^{\prime}(s)\right)
$$

$$
+\left[\frac{-\lambda}{\alpha(\lambda+\mu)}\right]\left(\left(g_{10}^{*}\right)^{\prime}(s)-\left(g_{00}^{*}\right)^{\prime}(s)\right)
$$

$$
\left.+\left[\left(\frac{-1}{\alpha}\right)\left(\frac{1-f^{*}(\lambda)}{1-h^{*}(\alpha) f^{*}(\lambda)}\right)+\frac{1-f^{*}(\lambda)}{\left(1-f^{*}(\lambda) h^{*}(\alpha)\right)^{2}}\left(h^{*}\right)^{\prime}(\alpha) f^{*}(\lambda)\right]\left[\left(g_{01}^{*}\right)^{\prime}(s)-\left(g_{00}^{*}\right)^{\prime}(s)\right)\right]
$$

$$
\begin{equation*}
+\left(\frac{-1}{\alpha}\right)\left(g_{00}^{*}\right)^{\prime}(s) \tag{9}
\end{equation*}
$$

$$
\begin{align*}
\operatorname{Cov}(V, S)= & -\left[\left(\frac{\lambda}{\lambda+\mu}\right) \frac{1-f^{*}(\lambda+\mu)}{\left(1-f^{*}(\lambda+\mu) h^{*}(\alpha)\right)^{2}} h^{* \prime}(\alpha) f^{*}(\lambda+\mu)\right]\left[E\left(S_{11}\right)-E\left(S_{10}\right)-E\left(S_{01}\right)+E\left(S_{00}\right)\right] \\
& -\left[\frac{1-f^{*}(\lambda)}{\left(1-f^{*}(\lambda) h^{*}(\alpha)\right)^{2}} h^{* \prime}(\alpha) f^{*}(\lambda)\right]\left[E\left(S_{01}\right)-E\left(S_{00}\right)\right] \tag{10}
\end{align*}
$$

## 3. MODEL B : GENERAL OPERATION AND RANDOM MAKOVIAN PRODUCTION TIME

The main assumptions of the model are given below.

1. The machine operation time T is a random variable with cumulative distribution function $\mathrm{Cdf} F(x)$ and probability density function pdf $f(x)$.
2. The inter-production times of products are independent random variables with exponential distribution with rate $\lambda$. Let N be the number of products produced during the operation time T . The size of each production is a random variable with general distribution function Cdf $H(x)$ and $h(x)$. Let Z be the total size of production in operation time.
3. The sale time $S$ has general distribution either (i) with $\operatorname{Cdf} G_{11}(y)$ and $\operatorname{pdf} g_{11}(x)$ when the operation time is within the time threshold U and the total size of the products produced Z is less than the threshold V , or (ii) with Cdf $G_{10}(y)$ and pdf $g_{10}(x)$ when the operation time is within the time threshold U and Z is more than the threshold V, or (iii) with Cdf $G_{01}(y)$ and pdf $g_{01}(x)$ when the operation time is more than the threshold U and Z is less than the threshold V , or (iv) with $\operatorname{Cdf} G_{00}(y)$ and pdf $g_{00}(x)$ when the operation time is more than the threshold U and Z is more than the threshold V , in order to provide change in selling rate. When no unit is produced during operation time sales time S is used for other purpose.
4. The time threshold U has exponential distribution with parameter $\mu$ and the production size threshold V has exponential distribution with parameter $\alpha$.

Noting that the number of products produced during a period has Poisson distribution, we may derive the joint distribution of $T, V$ and $S$ as follows. The joint probability density function of $T, V$ and $S$ is

$$
\begin{align*}
\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial v} P(T & \leq x, V \leq v, S \leq y)=f(x, v, y) \\
= & \sum_{0}^{\infty}\left[f(x)\left\{e^{-\lambda x} \frac{(\lambda x)^{n}}{n!}\right\} e^{-\mu x} H_{n}(v) \alpha e^{-\alpha v} g_{11}(y)\right. \\
& +f(x)\left\{e^{-\lambda x} \frac{(\lambda x)^{n}}{n!}\right\} e^{-\mu x}\left(1-H_{n}(v)\right) \alpha e^{-\alpha v} g_{10}(y) \\
& +f(x)\left\{e^{-\lambda x} \frac{(\lambda x)^{n}}{n!}\right\}\left(1-e^{-\mu x}\right) H_{n}(v) \alpha e^{-\alpha v} g_{01}(y) \\
& \left.+f(x)\left\{e^{-\lambda x} \frac{(\lambda x)^{n}}{n!}\right\}\left(1-e^{-\mu x}\right)\left(1-H_{n}(v)\right) \alpha e^{-\alpha v} g_{00}(y)\right] . \tag{11}
\end{align*}
$$

Here, $n=0,1,2,3, \ldots$. The first term of equation (11) is the part of the pdf that the operation time is $x$, the sale time is y , the operation time is within the threshold U , the number of productions is n and the size of production Z is within the threshold level V. The second term of equation (11) is the part of the pdf that the operation time is $x$ which is within the threshold $U$, the sale time is $y$, the number of productions is $n$ and the size of production $Z$ is in excess of $V$, the threshold level. The third term of equation (11) is the part of the pdf that the operation time is x which is in excess of the threshold U , the sale time is y , the number of productions is n and the size of production Z is within V , the threshold level. The fourth term of equation (11) is the part of the pdf that the operation time is $x$ which is in excess of the threshold $U$, the sale time is $y$, the number of productions is $n$ and the size of production $Z$ is in excess of $V$, the threshold level. We find the joint Laplace transform as follows.

$$
\begin{align*}
E\left(e^{-t T} e^{-s s} e^{-w v}\right) & =\sum_{0}^{\infty} \iiint_{0}^{\infty}(f(x, v, y)) e^{-t x} e^{-s y} e^{-w v} d x d v d y \\
& =\sum_{0}^{\infty} \iiint_{0}^{\infty} f(x)\left\{e^{-\lambda x} \frac{(\lambda x)^{n}}{n!}\right\} e^{-\mu x} H_{n}(v) \alpha e^{-\alpha v} g_{11}(y) e^{-t x} e^{-s y} e^{-w v} d x d y d v \\
& +\sum_{0}^{\infty} \iiint_{0}^{\infty} f(x)\left\{e^{-\lambda x} \frac{(\lambda x)^{n}}{n!}\right\} e^{-\mu x}\left(1-H_{n}(v)\right) \alpha e^{-\alpha v} g_{10}(y) e^{-t x} e^{-s y} e^{-w v} d x d y d v \\
& +\sum_{0}^{\infty} \iiint_{0}^{\infty} f(x)\left\{e^{-\lambda x} \frac{(\lambda x)^{n}}{n!}\right\}\left(1-e^{-\mu x}\right) H_{n}(v) \alpha e^{-\alpha v} g_{01}(y) e^{-t x} e^{-s y} e^{-w v} d x d y d v \\
& +\sum_{0}^{\infty} \iiint_{0}^{\infty} f(x)\left\{e^{-\lambda x} \frac{(\lambda x)^{n}}{n!}\right\}\left(1-e^{-\mu x}\right)\left(1-H_{n}(v)\right) \alpha e^{-\alpha v} g_{00}(y) e^{-x x} e^{-s y} e^{-w v} d x d y d v, t, s, w \geq 0 . \tag{12}
\end{align*}
$$

This on simplification gives the joint transform as

$$
\begin{aligned}
E\left(e^{-t T} e^{-s S} e^{-w V}\right) & =\left(\frac{\alpha}{\alpha+w}\right) g_{11}^{*}(s) f^{*}\left[t+\mu+\lambda\left(1-h^{*}(w+\alpha)\right)\right] \\
& +\left(\frac{\alpha}{\alpha+w}\right) g_{10}^{*}(s)\left[f^{*}(t+\mu)-f^{*}\left[t+\mu+\lambda\left(1-h^{*}(w+\alpha)\right)\right]\right] \\
& +\left(\frac{\alpha}{\alpha+w}\right) g_{01}^{*}(s)\left[f^{*}\left[t+\lambda\left(1-h^{*}(\alpha+w)\right)\right]-f^{*}\left[t+\mu+\lambda\left(1-h^{*}(w+\alpha)\right)\right]\right. \\
& +\left(\frac{\alpha}{\alpha+w}\right) g_{00}^{*}(s)\left[f^{*}(t)-f^{*}(t+\mu)-f^{*}\left[t+\lambda\left(1-h^{*}(\alpha+w)\right)\right]\right. \\
& +f^{*}\left[t+\mu+\lambda\left(1-h^{*}(w+\alpha)\right)\right]
\end{aligned}
$$

Collecting like terms, we obtain

$$
\begin{align*}
E\left(e^{-t T} e^{-w V} e^{-s S}\right)= & \left(\frac{\alpha}{\alpha+w}\right)\left[g_{11}^{*}(s)-g_{10}^{*}(s)-g_{01}^{*}(s)+g_{00}^{*}(s)\right] f^{*}\left[t+\mu+\lambda\left(1-h^{*}(w+\alpha)\right)\right] \\
& +\left(\frac{\alpha}{\alpha+w}\right)\left[g_{01}^{*}(s)-g_{00}^{*}(s)\right] f^{*}\left[t+\lambda\left(1-h^{*}(\alpha+w)\right)\right] \\
& +\left(\frac{\alpha}{\alpha+w}\right)\left[g_{10}^{*}(s)-g_{00}^{*}(s)\right] f^{*}(t+\mu)+\left(\frac{\alpha}{\alpha+w}\right) g_{00}^{*}(s) f^{*}(t) \tag{13}
\end{align*}
$$

$$
E\left(e^{-t T}\right)=f^{*}(t)
$$

$$
E\left(e^{-s s}\right)=\left[g_{11}^{*}(s)-g_{10}^{*}(s)-g_{01}^{*}(s)+g_{00}^{*}(s)\right] f^{*}\left[\mu+\lambda\left(1-h^{*}(\alpha)\right)\right]+\left[g_{01}^{*}(s)-g_{00}^{*}(s)\right]\left[f^{*}\left(\lambda\left(1-h^{*}(\alpha)\right)\right]\right.
$$

$$
\begin{equation*}
+\left[g_{10}^{*}(s)-g_{00}^{*}(s)\right] f^{*}(\mu)+g_{00}^{*}(s) \tag{14}
\end{equation*}
$$

Using differentiation we get

$$
\begin{align*}
E(S)=\left[E\left(S_{11}\right)\right. & \left.-E\left(S_{10}\right)-E\left(S_{01}\right)+E\left(S_{00}\right)\right] f^{*}\left[\mu+\lambda\left(1-h^{*}(\alpha)\right)\right] \\
& +\left[E\left(S_{01}\right)-E\left(S_{00}\right)\right]\left[f^{*}\left(\lambda\left(1-h^{*}(\alpha)\right)\right]+\left[E\left(S_{10}\right)-E\left(S_{00}\right)\right] f^{*}(\mu)+E\left(S_{00}\right)\right. \tag{15}
\end{align*}
$$

setting $w=0$,

$$
\begin{align*}
E\left(e^{-t T} e^{-s S}\right)= & {\left[g_{11}^{*}(s)-g_{10}^{*}(s)-g_{01}^{*}(s)+g_{00}^{*}(s)\right] f^{*}\left[t+\mu+\lambda\left(1-h^{*}(\alpha)\right)\right] } \\
& +\left[g_{01}^{*}(s)-g_{00}^{*}(s)\right]\left[f^{*}\left[t+\lambda\left(1-h^{*}(\alpha)\right)\right]+\left[g_{10}^{*}(s)-g_{00}^{*}(s)\right] f^{*}(t+\mu)+g_{00}^{*}(s) f^{*}(t)\right. \tag{16}
\end{align*}
$$

$$
\begin{align*}
E(T S)= & -\left[E\left(S_{11}\right)-E\left(S_{10}\right)-E\left(S_{01}\right)+E\left(S_{00}\right)\right] f^{*^{\prime}}\left[\mu+\lambda\left(1-h^{*}(\alpha)\right)\right] \\
& -\left[E\left(S_{01}\right)-E\left(S_{00}\right)\right]\left[f^{*^{\prime}}\left(\lambda\left(1-h^{*}(\alpha)\right)\right]-\left[E\left(S_{10}\right)-E\left(S_{00}\right)\right]\left[f^{*^{\prime}}(t+\mu)\right]+E\left(S_{00}\right) E(T)\right. \tag{17}
\end{align*}
$$

$$
\begin{align*}
\operatorname{Cov}(T, S)=-[ & \left.E\left(S_{11}\right)-E\left(S_{10}\right)-E\left(S_{01}\right)+E\left(S_{00}\right)\right] f^{*^{\prime}}\left[\mu+\lambda\left(1-h^{*}(\alpha)\right)\right] \\
& +E(T) f^{*}\left[\mu+\lambda\left(1-h^{*}(\alpha)\right)\right]-\left[E\left(S_{01}\right)-E\left(S_{00}\right)\right]\left[f^{*^{\prime}}\left(\lambda\left(1-h^{*}(\alpha)\right)\right)\right] \\
& +E(T) f^{*}\left(\lambda\left(1-h^{*}(\alpha)\right)-\left[E\left(S_{10}\right)-E\left(S_{00}\right)\right]\left[f^{*^{\prime}}(\mu)+E(T) f^{*}(\mu)\right]\right. \tag{18}
\end{align*}
$$

We may find that

$$
\begin{align*}
E\left(e^{-w V} e^{-s s}\right)=( & \left.\frac{\alpha}{\alpha+w}\right)\left[g_{11}^{*}(s)-g_{10}^{*}(s)-g_{01}^{*}(s)+g_{00}^{*}(s)\right] f^{*}\left[\mu+\lambda\left(1-h^{*}(w+\alpha)\right)\right] \\
& +\left(\frac{\alpha}{\alpha+w}\right)\left[g_{01}^{*}(s)-g_{00}^{*}(s)\right] f^{*}\left[\lambda\left(1-h^{*}(\alpha+w)\right)\right] \\
& +\left(\frac{\alpha}{\alpha+w}\right)\left[g_{10}^{*}(s)-g_{00}^{*}(s)\right] f^{*}(\mu)+\left(\frac{\alpha}{\alpha+w}\right) g_{00}^{*}(s) \tag{19}
\end{align*}
$$

From (19) , we get using differentiation

$$
\begin{aligned}
& E(V S)=\left[\frac{1}{\alpha} f^{*}\left[\mu+\lambda\left(1-h^{*}(\alpha)\right)\right]+f^{* \prime}\left[\mu+\lambda\left(1-h^{*}(\alpha)\right)\right] \lambda h^{* \prime}(\alpha)\right. \\
& {\left[E\left(S_{11}\right)-E\left(S_{10}\right)-E\left(S_{01}\right)+E\left(S_{00}\right)\right]+\left[\frac{1}{\alpha} f^{*}\left[\lambda\left(1-h^{*}(\alpha)\right)\right]\right.} \\
&+f^{*^{\prime}}\left[\lambda\left(1-h^{*}(\alpha)\right)\right] \lambda h^{*^{\prime}}(\alpha)\left[E\left(S_{01}\right)-E\left(S_{00}\right)\right]+\frac{1}{\alpha}\left[E\left(S_{10}\right)-E\left(S_{00}\right)\right] f^{*}(\mu)+\frac{1}{\alpha} E\left(S_{00}\right)
\end{aligned}
$$

$$
\begin{align*}
\operatorname{Cov}(V, S)=f^{*^{\prime}} & {\left[\mu+\lambda\left(1-h^{*}(\alpha)\right)\right] \lambda h^{*^{\prime}}(\alpha)\left[E\left(S_{11}\right)-E\left(S_{10}\right)-E\left(S_{01}\right)+E\left(S_{00}\right)\right] } \\
+ & {\left[f^{*^{\prime}}\left[\lambda\left(1-h^{*}(\alpha)\right)\right] \lambda h^{*^{\prime}}(\alpha)\left[E\left(S_{01}\right)-E\left(S_{00}\right)\right]\right.} \tag{20}
\end{align*}
$$

When T is exponential with parameter a , we get
$\operatorname{Cov}(T, S)=-\left[E\left(S_{1}\right)-E\left(S_{2}\right)\right] \frac{\left[\lambda\left(1-h^{*}(\mu)\right)\right]}{\left[a+\lambda\left(1-h^{*}(\mu)\right)\right]^{2}}$
and $\operatorname{Cov}(T, N)=\frac{\lambda}{a^{2}}$.

## 4. NUMERICAL ILLUSTRATIONS

## MODELS A AND B:

We present the usefulness of the results obtained by presenting numerical examples. We consider numerical examples for two models A and B together. The results of the two models co-inside when we identify all distributions are exponentials. We consider that the operation time is exponential with parameter a in Model 2 . We give numerical values for the operation time parameter $a=2$, the inter production time parameter $\lambda=10$, Sales time $S_{11}, S_{10}, S_{01}$ and $S_{00}$ are exponentials with parameters $1,2,4$ and 8 . We note the time threshold distribution is exponential with parameter $\mu$. We further consider the individual production size is exponential with parameter $\beta=4$ so that $h^{*}(\alpha)=\frac{4}{4+\alpha}$. Assigning different values for threshold parameters $\mu$ and $\alpha$, we find the statistical values $E(S), \operatorname{Cov}(T, S)$ and $\operatorname{Cov}(V, S)$ as given below in the tables.

TABLE ( $I$ ): THE EFFECT OF VARIATION OF $\mu$ AND $\alpha$ ON E(S)

| $\mu / \alpha$ <br> $/ \alpha$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | .75 | .6875 | .65909 | .642857 | .63235 | .625 |
| 1 | .75 | .5875 | .54029 | .51771 | .504464 | .49575 | .48975 |
| 2 | .625 | .5 | .46164 | .44279 | .431548 | .42407 | .41875 |
| 3 | .55 | .44464 | .41187 | .39554 | .38571 | .37914 | .37443 |
| 4 | .5 | .40625 | .37723 | .36268 | .353896 | .34799 | .34375 |
| 5 | .4642 | .37797 | .35159 | .33837 | .330357 | .32496 | .32108 |
| 6 | .4375 | .35625 | .33180 | .31956 | .312157 | .30716 | .30357 |

## TABLE (II): THE EFFECT OF VARIATION OF $\mu$ AND $\alpha$ ON COV[T,S]

| $\mu / \alpha$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | -.0625 | -.0585 | -.0542 | -.0510 | -.0486 | -.0468 |
| 1 | -.0833 | -.1022 | -.0968 | -.0925 | -.0895 | -.0874 | -.0857 |
| 2 | -.0937 | -.1041 | -.0987 | -.0947 | -.0920 | -.0900 | -.0885 |
| 3 | -.09 | -.0988 | -.0938 | -.0902 | -.0877 | -.0859 | -.0846 |
| 4 | -.0833 | -.0924 | -.0878 | -.0845 | -.0823 | -.0806 | -.0794 |
| 5 | -.0765 | -.0898 | -.0821 | -.0791 | -.0770 | -.0755 | -.0743 |
| 6 | -.0703 | -.0807 | -.0770 | -.0742 | -.0723 | -.0709 | -.0698 |

TABLE (III): THE EFFECT OF VARIATION OF $\mu$ AND $\alpha$ ON COV[V,S]

| $\mu / \alpha$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | .123 | .0501 | .0270 | .0168 | .0115 | .0083 |
| 2 | .1083 | .0447 | .0244 | .0153 | .0105 | .0076 |
| 3 | .0994 | .0412 | .0225 | .0142 | .0098 | .0071 |
| 4 | .0937 | .0388 | .0212 | .0134 | .0092 | .0067 |
| 5 | .0898 | .0371 | .0203 | .0128 | .0088 | .0064 |
| 6 | .087 | .0357 | .0195 | .0123 | .0085 | .0062 |

From the Tables (I) and (II) we note that the increase in the values of $\mu$ and $\alpha$ (the decrease expected thresholds) increases expected sales times, increases its co variances to zero. Since the mean of $S_{11}$ is greater than the mean of $S_{10}$, and the mean of $S_{01}$ is greater than $S_{00}$ the covariance between T and S (correlation) is negative. From table III ,As $\mu$ and $\alpha$ increase we note that the co-variance decreases and when V is large, $S_{01}$ may not be replaced by $S_{00}$ and $S$ becomes large, indicating the variables are positively correlated.

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