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ON SKOLEM DIFFERENCE MEAN LABELING OF GRAPHS

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ABSTRACT

A graph $G=(V,E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,3,\ldots,p+q$ in such a way that for each edge e = uv, let f(e) = $\left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from 1, 2, 3,…, q. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph. In this paper we prove that, , $<T\,\hat{o}\,K_{1,n}>$, where T is a Tp-tree, caterpillar , $S_{m,n}$ and $C_n @ K_{1,m}$ are skolem difference mean graphs, where T is a Tp-tree, are skolem difference mean graphs.*

Key words: *Skolem difference mean labeling, extra skolem difference mean labeling.*

AMS Subject Classification: *05C78.*

1. INTRODUCTION

By a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The disjoint union of m copies of the graph G is denoted by mG. The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. A vertex of degree one is called a pendant vertex. The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.

Let T be a tree and u_0 and v_0 be two adjacent vertices in $V(T)$. Let there be two pendant vertices u and v in T such that the length of $u_o - u$ path is equal to the length of $v_o - v$ path. If the edge $u_o v_o$ is deleted from T and u, v are joined by an edge uv, then such a transformation of T is called an elementary parallel transformation (or an ept) and the edge $u_o v_o$ is called a transformable edge. If by a sequence of ept's T can be reduced to a path, then T is called a Tp-tree (transformed tree) and any such sequence regarded as a composition of mappings (ept's) denoted by P, is called a parallel transformation of T. The path, the image of T under P is denoted as $P(T)$. $S_{m,n}$ is a star graph with n spokes in which each spoke is a path of length m. Let T be a Tp- tree on m vertices. Then $< T\,\hat{o}\,K_{1,n} >$ is a graph obtained from T and m copies of $K_{1,n}$ by identifying a leaf of i^{th} copy of $K_{1,n}$ with i^{th} vertex of T. A caterpillar is a tree with the property that the removal of its pendant vertices results in a path.

Terms and notations not defined here are used in the sense of Harary [1].

A graph $G=(V,E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,3,\ldots,p+q$ in such a way that for each edge $e = uv$, let $f^*(e) = \frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting labels of the edges are distinct and are from 1, 2, 3,…, q. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph.

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Let $G=(V,E)$ be a skolem difference mean graph with p vertices and q edges. Let one of the skolem difference mean labeling of G satisfies the condition that all the labels of the vertices are odd, and then we call this skolem difference mean labeling an extra skolem difference mean labeling and the graph G as extra skolem difference mean graph.

The concept of skolem difference mean labeling is introduced by K. Murugan and A. Subramanian [3] in 2011. They have studied the skolem difference mean labeling of H – graphs. In [6], some standard results on skolem difference mean labeling was proved.

The extra skolem difference mean labeling of a Tp-tree with 14 vertices is given in Figure 1.

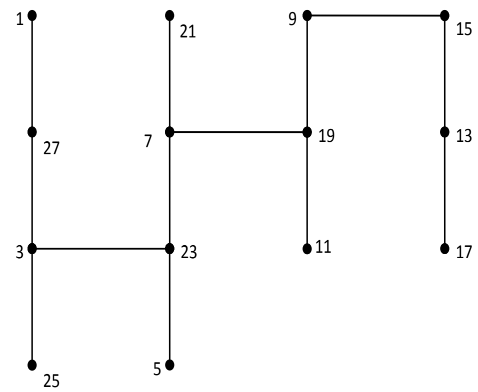


Figure: 1

2. SKOLEM DIFFERENCE MEAN LABELING

Theorem: 2.1 The graph $< T\, \hat{o}\, K_{1,n} >$ is an extra skolem difference mean graph.

Proof: Let T be a Tp- tree with m vertices. By the definition of a Tp - tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ and (ii)$E(P(T)) = (E(T) \setminus E_d) \cup E_P$, where E_d is the set of edges deleted from T and E_P is the set of edges newly added through the sequence $P = (P_1, P_2, ..., P_k)$ of the epts P used to arrive at the path $P(T)$. Clearly E_d and E_P have the same number of edges.

Now denote the vertices of $P(T)$ successively as $v_1, v_2, v_3, ..., v_m$ starting from one pendant vertex of $P(T)$ right up to other. Now, denote the vertices of $P(T)$ successively as $v_1, v_2, v_3, ..., v_m$ starting from one pendant vertex of $P(T)$ right up to the other. Let $u_0^i, u_1^i, u_2^i, ..., u_n^i$ be the vertices of the i^th^ copy of $K_{1,n}$. Identify the vertex u_1^i with v_i for $1 \le i \le m$ to get $<T\, \hat{o}\, K_{1,n}>$.

Define $f : V(<T\, \hat{o}\, K_{1,n}>) \to \{1,2,3,..., p+q = 2m(n+1)-1\}$ as follows:

$f(u_0^j) = (n+1)(2m-j)+n$ for j is odd , $1 \le j \le m$,

$f(u_0^j) = (n+1)j-1$ for j is even , $1 \le j \le m$,

$f(u_i^j) = (n+1)(j-1)+2i-1$ for j is odd , $1 \le j \le m$, $1 \le i \le n$,

$f(u_{n+1-i}^j) = (n+1)[2(m+1)-j]-(2i+1)$ for j is even , $1 \le j \le m$, $1 \le i \le n$,

$f(v_j) = (n+1)j-n$ for j is odd , $1 \le j \le m$,

$f(v_j) = (n+1)[2(m+1)-j]-2n-1$ for j is even , $1 \le j \le m$.

Let v_iv_j be an edge of T for some indices i and j, $1 \le i < j \le n$ and let P_1 be the ept that deletes this edge and adds the edge $v_{i+t}\, v_{j-t}$ where t is the distance from v_i to v_{i+t} and also the distance from v_j to v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts. Since $v_{i+t}v_{j-t}$ is an edge of the path $P(T)$, it follows that $i + t + 1 = j - t$ which implies $j = i + 2t + 1$. The induced label of the edge v_iv_j is given by,

$$f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = \left\lceil \frac{|f(v_i) - f(v_{i+2t+1})|}{2} \right\rceil = (n+1)|m-i-t| \quad (1)$$

and $f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = \left\lceil \frac{|f(v_{i+t}) - f(v_{i+t+1})|}{2} \right\rceil = (n+1)|m-i-t|$ (2)

Therefore from (1) and (2), $f^*(v_iv_j) = f^*(v_{i+t}v_{j-t})$.

Let $e_i^j = u_0^j u_i^j (1 \le j \le m, 1 \le i \le n)$, $e_j = v_jv_{j+1} (1 \le j \le m-1)$ be the edges of $<T \hat{o} K_{1,n}>$.

For each vertex label f, the induced edge labeling f^* is as follows:

$f^*(e_i^j) = (n+1)(m-j+1) - i$ for j is odd, $1 \le j \le m, 1 \le i \le n$,

$f^*(e_i^j) = (n+1)(m-j) + i$ for j is even, $1 \le j \le m, 1 \le i \le n$,

$f^*(e_j) = (n+1)(m-j)$ for $1 \le j \le m-1$.

It can be verified that f is an extra skolem difference mean labeling of $<T \hat{o} K_{1,n}>$. Hence, $<T \hat{o} K_{1,n}>$ is an extra skolem difference mean graph.

For example, the extra skolem difference mean labeling of $<T \hat{o} K_{1,3}>$, where T is a T_P-tree with 7 vertices is given in Figure 2.

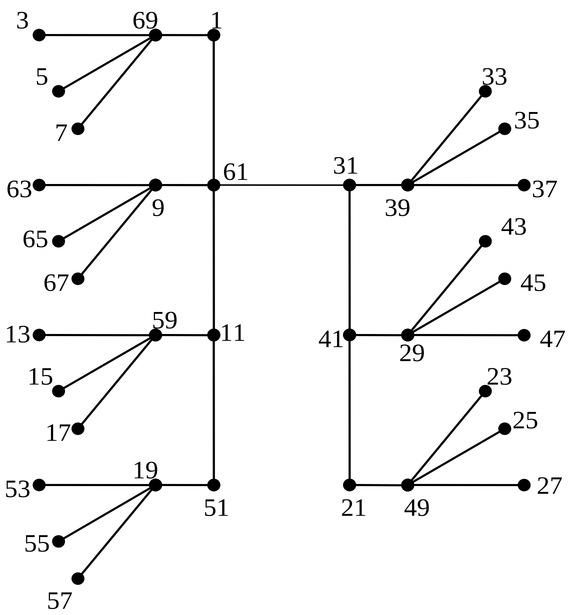


Figure: 2

Theorem: 2.2 The caterpillar graph $S(k_1, k_2, ..., k_n)$ is an extra skolem difference mean graph.

Proof: Let $v_1, v_2, v_3, ..., v_n$ be the vertices of the path and $u_1^j, u_2^j, u_3^j, ..., u_{k_j}^j$ be the pendant vertices attached with the vertex $v_j (1 \le j \le n)$.

Define $f: V(S(k_1, k_2, ..., k_n)) \to \{1, 2, 3, ..., p+q = 2(k_1 + k_2 + ... + k_n) + 2n - 1\}$ as follows:

$f(u_i^1) = 2i - 1$ for $1 \le i \le k_1$,

$f(u_i^j) = 2(k_1 + k_3 + k_5 + ... + k_{j-2}) + 2(i-1) + j$ for j isodd, $3 \le j \le n, 1 \le i \le k_j$,

$f(u_i^j) = 2(k_1 + k_3 + k_5 + ... + k_{j-1} + k_j + k_{j+1} + ... + k_n) + 2(n-i) + 1 - j$ for j is even, $1 \le i \le k_j$, $1 \le j \le n$,

$f(v_j) = 2(k_1 + k_3 + ... + k_{j-2} + k_j + k_{j+1} + ... + k_n) + 2n - j$ for j is odd, $1 \le j \le n$, $f(v_j) = 2(k_1 + k_3 + ... + k_{j-1}) + (j-1)$ for j is even, $1 \le j \le n$.

Let $e_i^j = v_ju_i^j (1 \le j \le n, 1 \le i \le k_j)$, $e_j = v_jv_{j+1} (1 \le j \le n-1)$ be the edges of $S(k_1, k_2, ..., k_n)$.

For each vertex label f, the induced edge label f^* is defined as follows:

$f^*(e_i^j) = k_j + k_{j+1} + k_{j+2} + \cdots + k_n + (n-j+1) - i$ for j is odd , $1 \le j \le n$, $1 \le i \le k_j$,

$f^*(e_{n+1-i}^j) = k_j + k_{j+1} + k_{j+2} + \cdots + k_n + (n-j+1) - i$ for j is even, $1 \le j \le n$, $1 \le i \le k_j$,

$f^*(e_j) = k_{j+1} + k_{j+2} + \cdots + k_n + n - j$ for $1 \le j \le n-1$.

It can be verified that f is an extra skolem difference mean labeling of $S(k_1, k_2, ..., k_n)$. Hence, $S(k_1, k_2, ..., k_n)$ is an extra skolem difference mean graph.

For example, the extra skolem difference mean labeling of $S(5,4,4,6,2)$ is given in Figure 3.

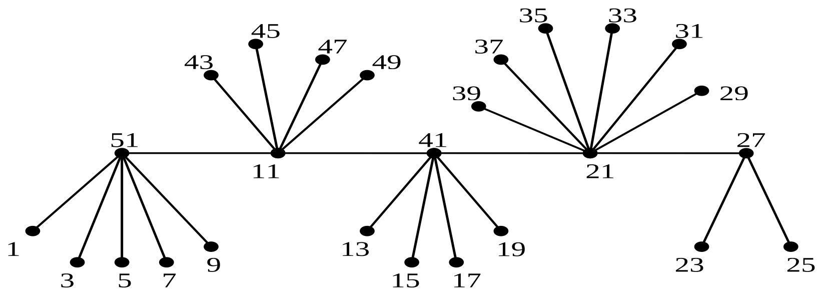


Figure: 3

Theorem: 2.3 The graph $C_n@K_{1,m}$ $(n \ge 3, m \ge 1)$ is a skolem difference mean graph.

Proof: we prove this theorem in two cases.

Case: (i) n is odd

Let $n = 2k + 1$.

Let $u_1, u_2, u_3, \ldots, u_k, v_k, v_{k-1}, \ldots, v_1, v_0$ be the vertices of the cycle C_{2k+1} and let $w, w_1, w_2, \ldots w_m$ be the vertices of $K_{1,m}$. The graph $C_n@K_{1,m}$ is obtained by identifying w of $K_{1,m}$ with v_0.

Then $E(C_n@K_{1,m}) = \{ww_i, u_ju_{j+1}, v_jv_{j+1}, wu_1, wv_1, u_kv_k,\ 1 \le i \le m, 1 \le j \le k-1\}$.

Define $f: V(C_{2k+1}@K_{1,m}) \to \{1,2,3, \ldots, p+q = 2n+2m\}$ as follows:

$f(w) = 1$,

$f(u_{2i-1}) = 2n + 2m - 4(i-1)$ for $1 \le i \le \left\lceil \frac{k}{2} \right\rceil$,

$f(v_{2i-1}) = 2n + 2m - 4i + 3$ for $1 \le i \le \left\lceil \frac{k}{2} \right\rceil$,

$f(u_{2i}) = 4i$ for $1 \le i \le \left\lfloor \frac{k}{2} \right\rfloor$,

$f(v_{2i}) = 4i + 2$ for $1 \le i \le \left\lfloor \frac{k}{2} \right\rfloor$,

$f(w_i) = 3 + 2i$ for $1 \le i \le m$.

For each vertex label f, the induced edge label f^* is defined as follows:

$f^*(u_kv_k) = 1$,

$f^*(ww_i) = 1 + i$, $1 \le i \le m$,

$f^*(wu_1) = n + m$,

$f^*(wv_1) = n + m - 1$,

$f^*(u_iu_{i+1}) = n + m - 2i$ for $1 \le i \le k-1$,

$f^*(v_iv_{i+1}) = n + m - 2i - 1$ for $1 \le i \le k-1$.

It can be verified that f is a skolem difference mean labeling of $C_n@K_{1,m}$.

Case: (ii) *n* is even

Let $n = 2k$

Let $u_0, u_1, u_2, \ldots, u_{k-1}, w, v_{k-1}, \ldots v_1$ be the vertices of C_{2k}.

Define $f: V(C_{2k}@K_{1,m}) \to \{1,2,3, \ldots, p+q = 2n+2m\}$ as follows:

Sub case: (i) *k* is even

$f(w) = 1,$

$f(w_i) = 5 + 2i$ for $1 \le i \le m.$

$f(u_{2i-1}) = 2n + 2m - 4(i-1)$ for $1 \le i \le \frac{k}{2},$

$f(v_{2i-1}) = 2n + 2m - 4i + 3$ for $1 \le i \le \frac{k}{2},$

$f(u_{2i}) = 4i$ for $1 \le i \le \frac{k-2}{2},$

$f(v_{2i}) = 4i + 2$ for $1 \le i \le \frac{k-2}{2},$

$f(u_0) = 2n + 2m - 2k + 1.$

For each vertex label f, the induced edge label f^* is defined as follows:

$f^*(ww_i) = 2 + i$ for $1 \le i \le m,$

$f^*(wu_1) = n + m,$

$f^*(wv_1) = n + m - 1,$

$f^*(u_i u_{i+1}) = n + m - 2i$ for $1 \le i \le k-2,$

$f^*(v_i v_{i+1}) = n + m - 2i - 1$ for $1 \le i \le k-2,$

$f^*(u_0 u_k) = 2,$

$f^*(v_0 v_k) = 1.$

Subcase: (ii) *k* is odd

$f(w) = 1$

$f(w_i) = 1 + 2i$ for $1 \le i \le m,$

$f(u_{2i-1}) = 2n + 2m - 4(i-1)$ for $1 \le i \le \frac{k-1}{2},$

$f(v_{2i-1}) = 2n + 2m - 4i + 3$ for $1 \le i \le \frac{k-1}{2},$

$f(u_{2i}) = 4i$ for $1 \le i \le \frac{k-1}{2},$

$f(v_{2i}) = 4i + 2$ for $1 \le i \le \frac{k-1}{2},$

$f(u_0) = 2(n + m - k + 1).$

For each vertex label f, the induced edge label f^* is defined as follows:

$f^*(ww_i) = i$ for $1 \le i \le m,$

$f^*(wu_1) = n + m,$

$f^*(wv_1) = n + m - 1,$

$f^*(u_i u_{i+1}) = n + m - 2i$ for $1 \le i \le k-2,$

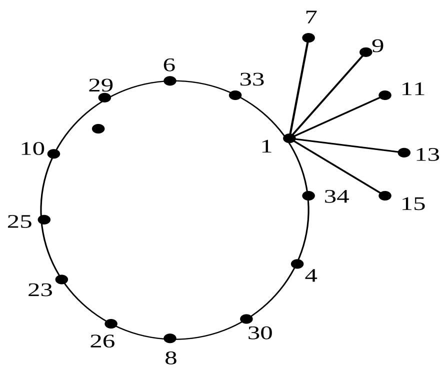
$f^*(v_i v_{i+1}) = n + m - 2i - 1$ for $1 \le i \le k-2,$

$f^*(u_0 u_k) = n + m - 2k + 1,$

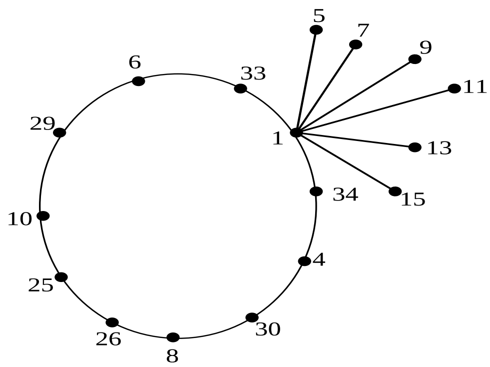
$f^*(v_0 v_k) = n + m - 2k + 2.$

It can be verified that f is a skolem difference mean labeling of $C_n@K_{1,m}$. Hence, $C_n@K_{1,m}$ is a skolem difference mean graph.

For example, the skolem difference mean labeling of $C_{12}@K_{1,5}$ and $C_{11}@K_{1,6}$ are given in Figure 4.



$C_{11}@K_{1,6}$



$C_{12}@K_{1,5}$

Figure: 4

Theorem: 2.4 The graph $S_{m,n}$, $(m \geq 1, n \geq 1)$ is an extra skolem difference mean graph.

Proof: Let u_0, u_i^j $(1 \leq i \leq m,\ 1 \leq j \leq n)$ be the vertices of two copies of $S_{m,n}$.

We define $f: V(S_{m,n}) \rightarrow \{1,2,3,\ldots, p+q = 2mn+1\}$ as follows:

$f(u_i^{n-j}) = m(n+j+1)+2-i$ for i is odd, $1 \leq i \leq m$, j is odd and $1 \leq j \leq n$,

$f(u_i^{n-j}) = m(n-j)+2-i$ for i is odd, $1 \leq i \leq m$, j is even and $1 \leq j \leq n$,

$f(u_i^{n-j}) = m(n-j-1)+1+i$ for i is even, $1 \leq i \leq m$, j is odd and $1 \leq j \leq n$,

$f(u_i^{n-j}) = m(n+j)+1+i$ for i is even, $1 \leq i \leq m$, j is even and $1 \leq j \leq n$,

$f(u_0) = 2mn - m + 2$ for m is odd,

$f(u_0) = m + 1$ for m is even.

Let $e_i^j = u_i^j u_{i+1}^j (1 \leq i \leq m-1, 1 \leq j \leq n)$ and $e_m^j = u_0 u_m^j$ be the edges of $S_{m,n}$.

For each vertex label f, the induced edge label f^* is defined as follows:

$f^*(e_i^1) = mn - i + 1$ for $1 \leq i \leq m$,

$f^*(e_{m+1-i}^j) = m(n-j+1)+1-i$ for j is even, $2 \leq j \leq n$ and $1 \leq i \leq m-1$,

$f^*(e_m^j) = mn - \frac{mj}{2}$ for j is even, $1 \leq j \leq n$,

$f^*(e_i^j) = m(n-j+1) - i$ for j is odd, $3 \leq j \leq n$ and $1 \leq i \leq m-1$,

$f^*(e_m^j) = \frac{(j-1)m}{2}$ for j is even, $3 \leq j \leq n$.

For example, the skolem difference mean labeling of $S_{5,4}$ is given in Figure 5.

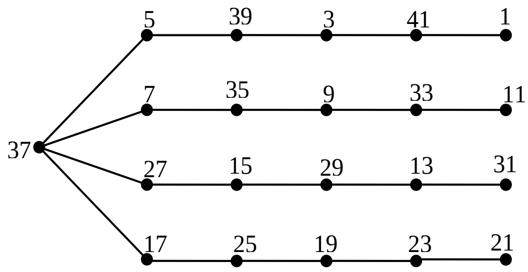


Figure: 5

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