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ON CONTRA –rg*b-CONTINUOUS FUNCTIONS AND APPROXIMATELY–rg*b-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

*In this paper a new class of functions called contra- rg*b-continuous functions are introduced and their properties are studied. Further the notion of approximately- rg*b -continuous functions and almost contra- rg*b-continuous functions are introduced.*

Keywords: *Contra- rg*b -continuous functions, Approximately-rg*b -continuous functions, Almost contra- rg*b-continuous functions.*

1. INTRODUCTION

In 1996, Donthev [5] introduced the notion of contra continuous functions. In 2007, Cal-das, Jafari, Noiri and Simoes [4] introduced a new class of functions called generalized contra continuous (contra g-continuous) functions. New types of contra generalized continuity such as contra αg-continuity [11] and contra gs-continuity [6] have been introduced and investigated. Recently, Nasef [14] introduced and studied so-called contra b-continuous functions. After that in 2009, Omari and Noorani [2] have studied further properties of contra b-continuous functions. Metin Akdag and Alkan Ozkan [13] introduced and investigated the notion of contra generalized b−continuity (contra gb − continuity).

The purpose of the present paper is to introduce the notion of contra regular generalized star b−continuity (contra rg*b −continuity) via the concept of rg*b−closed sets in [9] and investigate some of the fundamental properties of contra rg*b−continuous functions. It turns out that contra rg*b −continuity is stronger than contra gb−continuity and weaker than contra b−continuity. Also we study the basic properties of approximately- rg*b -continuous functions and almost contra- rg*b continuous functions.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, τ) represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no chance of confusion. We denote the family of all rg*b-closed sets in X by rg*b-C(X) and regular generalized star b-neighbourhoods by rg*b-nbhd in topological spaces.

Let us recall the following definitions which we shall require later.

Definition: 2.1 A subset A of a space (X, τ) is called
1) a regular open set[18] if A= int (cl(A)) and a regular closed set if A= cl(int (A))
2) a b-open set [4] if $A \subset cl(int(A)) \cup int(cl(A))$.
3) a regular generalized closed set (briefly, rg-closed)[15] if cl (A) ⊆ U whenever A ⊆ U and U is regular open in X.
4) a generalized b-closed (briefly gb-closed)[1] if bcl(A) ⊂ U whenever A⊂U and U is open.
5) a regular generalized b-closed set (briefly rgb-closed) [12] if bcl(A)⊆U whenever A⊆U and U is regular open in X.
6) a regular generalized star b- closed set (briefly rg*b-closed set)[9] if bcl (A) ⊆ U whenever A ⊆U and U is rg-open in X.

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Definition: 2.2 A function f: (X, τ) → (Y, σ) is called
1) b-irresolute: [8] if for each b-open set V in Y, f-1(V) is b-open in X;
2) b-continuous: [8] if for each open set V in Y, f-1(V) is b-open in X.
3) rg*b Continuous[10] if $f^{-1}(V)$ is rg*b Closed in X for every closed set V in Y.
4) rg*b irresolute[10] if the inverse image of each rg*b Closed set in Y is a rg*b Closed set in X .
5) rg*b Closed[10], if the image of each closed set in X is a rg*b Closed set in Y.
6) rg*b Open[10], if the image of each open set in X is a rg*b open in Y.
7) Pre rg*b Closed (resp. Pre rg*b open)[10], if the image of each rg*b closed (resp. rg*b open)set in X is a rg*b Closed (resp. rg*b open) set in Y.

Definition: 2.3 A function f:(X, τ) → (Y, σ) is called
1) contra –continuous[5] if $f^{-1}(V)$ is closed in X for each open set V of Y.
2) contra-b-continuous [14] if $f^{-1}(V)$ is b-closed in X for each open set V of Y.
3) contra gb-continuous[13] if $f^{-1}(V)$ is gb-closed in X for each open set V of Y.

Definition: 2.4 A space X is said to be
1) Strongly-S-closed [7] if every closed cover of X has a finite sub-cover.
2) Mildly compact [17] if every clopen cover of X has a finite sub-cover.
3) Strongly-S-Lindelof [7] if every closed cover of X has a countable sub-cover.

3. CONTRA-rg*b-CONTINUOUS FUNCTIONS

Definition: 3.1 A function f: (X, τ)→(Y, σ) is called contra-rg*b-continuous if $f^{-1}(V)$ is rg*b closed in (X, τ) for each open set V of (Y, σ).

Theorem: 3.2
(i) Every contra continuous function is contra- rg*b - continuous.
(ii) Every contra-b-continuous function is contra- rg*b - continuous.
(iii) Every contra- rg*b -continuous function is contra- gb-continuous.
(iv) Every contra- rg*b -continuous function is contra- rgb-continuous.
(v) Every contra- rg*b -continuous function is contra- g*b-continuous.

Remark: 3.3 Converses of theorem 3.2 are not true as shown in the following examples.

Example: 3.4
(i) Let X=Y= {a, b, c}, τ = {Φ,{a , b}, X }, σ = {Φ, {a},Y}.Then the identity function f: (X, τ)→ (Y, σ) is contra-rg*b- continuous but not contra –continuous, since f^{-1}({a})={a} is rg*b closed in X but not closed in X.
(ii) Let X =Y= {a, b, c, d}, τ = {Φ ,{a},{b},{a, b},{b, c},{a, b, c},X}, σ = {Φ,{b, d},Y}. Then the identity function f: (X, τ) → (Y, σ) is contra rg*b- continuous but not contra -b-continuous since f^{-1}({b, d}) ={b, d} is rg*b closed in X but not b-closed in X.
(iii) Let X=Y= {a, b, c}, τ = {Φ,{a},{a, b}, X }, σ = {Φ, {a, c},Y}.Then the identity function f: (X, τ)→ (Y, σ) is contra- gb- continuous but not contra rg*b –continuous , since f^{-1}({a, c})={a, c} is gb closed in X but not rg*b closed in X.
(iv) Let X=Y= {a, b, c}, τ = {Φ,{a},{b},{a, b},{a, c}, X }, σ = {Φ, {a},Y}. Then the identity function f: (X, τ)→ (Y, σ) is contra- rgb- continuous but not contra rg*b –continuous, since f^{-1}({a})={a} is rgb closed in X but not rg*b closed in X.
(v) Let X=Y= {a, b, c}, τ = {Φ,{a},{a, b}, X }, σ = {Φ, {a, c},Y}.Then the identity function f: (X, τ)→ (Y, σ) is contra- g*b- continuous but not contra rg*b –continuous, since f^{-1}({a, c})={a, c} is g*b closed in X but not rg*b closed in X.

Definition: 3.5 A space (X, τ) is called
(i) rg*b -space if every rg*b closed set is closed.
(ii) rg*b -locally indiscrete if every rg*b open set is closed.
(iii) a T_{rg*b} -space if every rg*b closed set is b closed.

Theorem: 3.6
(i) If a function f :(X, τ) → (Y, σ) is rg*b -continuous and (X, τ) is rg*b -locally indiscrete, then f is contra-continuous.
(ii) If a function f :(X, τ) → (Y, σ) is contra- rg*b continuous and (X, τ) is T_{rg*b} space, then f is contra-b-continuous.
(iii) If a function f :(X, τ) → (Y, σ) is contra- rg*b -continuous and (X, τ) is rg*b - space, then f is contra-continuous.

Proof:
(i) Let V be open in (Y, σ).By assumption, $f^{-1}(V)$ is rg*b -open in X. Since X is locally indiscrete, $f^{-1}(V)$ is closed in X. Hence f is contra-continuous.
(ii) Let V be open in (Y, σ). By assumption, $f^{-1}(V)$ is rg*b -closed in X. Since X is T_{rg*b} space, $f^{-1}(V)$ is b-closed in X. Hence f is contra-b-continuous.
(iii) Let V be open in (Y, σ). By assumption,$f^{-1}(V)$ is rg*b -closed in X. Since X is rg*b -space, $f^{-1}(V)$ is closed in X. Hence f is contra-continuous.

Theorem: 3.7 Let A⊂Y⊂X.
(i) If Y is open in X, then A ∈ rg*b C(X) implies A∈ rg*b C(Y).
(ii) If Y is regular open and rg*b -closed in X, then A ∈ rg*b C(Y) implies A∈ rg*b C(X)

Theorem: 3.8 Suppose rg*b O(X, τ) is closed under arbitrary union. Then the following are equivalent for a function f: (X, τ) → (Y, σ):
(i) f is contra- rg*b -continuous.
(ii) For every closed subset F of Y, $f^{-1}(F)$ ∈ rg*b O(X)
(iii) For each x∈X and each F ∈C(Y, f(x)),there exists U ∈ rg*b O(X, x) such that f(U)⊂F.

Proof: (i) ⇔ (ii) and (ii) ⇒ (iii) is obvious. (iii)⇒ (ii): Let F be any closed set of Y and x∈$f^{-1}(F)$.Then f(x)∈F and there exists U_x ∈ rg*b O(X) such that f(U_x)⊂F. Therefore we obtain $f^{-1}(F) = \cup\{ U_x : x\in f^{-1}(F)\}$ and $f^{-1}(F)$ is rg*b -open.

Theorem: 3.9 Suppose rg*b O(X,τ) is closed under arbitrary intersections. If a function f: (X, τ) → (Y, σ) is contra rg*b continuous and U is open in X, then f / U: (U, τ)→(Y, σ) is contra- rg*b -continuous.

Proof: Let V be closed in Y. Since f: (X, τ) → (Y, σ) is contra- rg*b -continuous, $f^{-1}(V)$ is rg*b -open in (X, τ). $(f/U)^{-1}(V) = f^{-1}(V) \cap U$ is rg*b -open in X. By theorem 3.7 (i) $(f/U)^{-1}(V)$ is rg*b -open in U.

Theorem: 3.10 Suppose rg*b O(X, τ) is closed under arbitrary unions. Let f: (X, τ)→ (Y, σ) be a function and $\{U_i : i\in I\}$ be a cover of X such that U_i ∈ rg*b C(X) and regular open for each i∈I. If f / U_i : (U_i, τ /U_i)→(Y, σ) is contra- rg*b -continuous for each i∈I, then f is contra- rg*b -continuous.

Proof: Suppose that F is any closed set of Y. We have $f^{-1}(F) =\cup \{f^{-1}(F) \cap U_i : i\in I\} = \cup\{(f/U_i)^{-1}(F): i\in I\}$. Since f/$U_i$ is contra- rg*b -continuous for each i∈I, it follows $(f/Ui)^{-1}(F)$ ∈ rg*b O (U_i). By theorem 3.7 (ii), it follows that $f^{-1}(F)$ ∈ rg*b O(X). Therefore f is contra- rg*b -continuous.

Theorem: 3.11 Suppose rg*b O(X, τ) is closed under arbitrary unions. If f:(X, τ)→ (Y, σ) is contra- rg*b -continuous and Y is regular open, then f is rg*b -continuous.

Proof: Let x be an arbitrary point of X and V an open set of Y containing f(x).Y is regular implies that there exists an open set W in Y containing f(x) such that cl (W) ⊂V. Since f is contra - rg*b -continuous, by theorem 3.8, there exists U∈ rg*b O (X, x) such that f(U)⊂ cl(W)⊂V. Hence f is rg*b -continuous.

4. APPROXIMATELY - rg*b -CONTINUOUS FUNCTIONS

Definition: 4.1 A map f: X→Y is said to be approximately- rg*b -continuous (ap- rg*b -continuous) if bcl (F) ⊂ $f^{-1}(U)$ whenever U is an open subset of Y and F is a rg*b -closed subset of X such that F⊂f^{-1} (U).

Definition: 4.2 A map f: X→Y is said to be approximately - rg*b -closed (briefly ap- rg*b -closed) if f (F) ⊂ bint(V) whenever V is a rg*b -open subset of Y, F is a closed subset of X and f(F) ⊂V.

Definition: 4.3 A map f: X→Y is said to be approximately - rg*b -open (briefly ap- rg*b -open if bcl (F) ⊂f(U) whenever U is an open subset of X, F is a rg*b -closed subset of Y and F ⊂f(U).

Definition: 4.4 A map f: X→Y is said to be contra - rg*b -closed (resp.contra rg*b -open) if f (U) is rg*b -open (resp rg*b -closed) in Y for each closed (resp. open) set U of X.

Theorem: 4.5 Let f: X→Y be a function, then
(1) If f is contra -b- continuous, then f is an ap- rg*b -continuous.
(2) If f is contra- b-closed, then f is ap- rg*b -closed.
(3) If f is contra -b-open, then f is ap- rg*b -open.

Proof:

(1) Let F⊂ f^{-1} (U) where U is a open subset in Y and F is a rg*b -closed subset of X. Then bcl(F) ⊂ bcl (f^{-1}(U)).Since f is contra-b- continuous, bcl(F) ⊂bcl (f^{-1}(U))= f^{-1}(U).This implies f is ap- rg*b - continuous.

(2) Let f (F) ⊂V, where F is a closed subset of X and V is a rg*b -open subset of Y. Therefore f (F) = bint(f(F)) ⊂ bint (V).Thus f is ap- rg*b -closed.

(3) Let F⊂f(U) where F is rg*b -closed subset of Y and U is an open subset of X. Since f is contra -b-open, f(U) is b-closed in Y for each open set U of X. bcl(F)⊂bcl(f(U)=f(U).Thus f is ap- rg*b -open.

Theorem: 4.6 Let f: (X, τ) → (Y, σ) be a map.

(1) If the open and b-closed sets of (X, τ) coincide, then f is a ap- rg*b -continuous if and only if f is contra-b-continuous.
(2) If the open and b-closed sets of (Y, σ) coincide, then f is ap- rg*b -closed if and only if f is contra - b-closed.
(3) If the open and b-closed sets of (Y, σ) coincide, then f is ap- rg*b -open if and only if f is contra-b-open.

Proof:

(1) Assume f is ap- rg*b -continuous. Let A be an arbitrary subset of (X, τ) such that A ⊂U, where U is rg-open in X. Then bcl (A) ⊂ bcl (U) = U .Therefore all subsets of (X, τ) are rg*b -closed (hence all are rg*b -open). So for any open set V in (Y, σ), we have f^{-1}(V) is rg*b -closed in (X, τ). Since f is ap- rg*b -continuous, bcl(f^{-1}(V)) ⊂f^{-1}(V).Therefore f^{-1}(V)) is b-closed in (X, τ) and f is contra-b-continuous. Converse is obvious from theorem 4.5.

(2) Assume f is ap- rg*b -closed. As in (1), we get that all subsets of (Y, σ) are rg*b -open. Therefore for any closed subset F of (X, τ),f(F) is rg*b -open in Y. Since f is ap- rg*b -closed, f(F) ⊂bint f(F).Hence f(F) is b-open and thus f is contra b-closed. Converse is obvious from theorem 4.5.

(3) Assume f is ap- rg*b -open. As in (1) all subsets of Y are rg*b -closed. Therefore for any open subset F of (X, τ), f(F) is rg*b -closed in (Y, σ).Since f is ap- rg*b -open, bcl(F)⊂f(F).Hence f(F) is b-closed and thus f is contra b-open. Converse is obvious from theorem 4.5.

Theorem: 4.7 If a map f: X→Y is ap- rg*b -continuous and b-closed map, then the image of each rg*b -closed set in X is rg*b -closed set in Y.

Proof: Let F be a rg*b -closed subset of X. Let f(F) ⊂V where V is an open subset of Y. Then F⊂ f^{-1} (V) holds. Since f is ap- rg*b - continuous, bcl(F) ⊂ f^{-1}(V).Thus f(bcl(F)) ⊂V. Therefore we have bcl(f(F))⊂ bcl(f(bcl(F))=f(bcl(V))⊂V. Hence f(F) is rg*b -closed set in Y.

Definition: 4.8 A map f: X→Y is said to be contra- rg*b -irresolute if f^{-1}(V) is rg*b - closed in X for each rg*b –open set V in Y.

Definition: 4.9 A space X is said to be rg*b -Lindelof if every cover of X by rg*b -open sets has a countable sub cover.

Theorem: 4.10 Let f: X→Y and g: Y→Z be two maps such that gof: X→Z.
(i) If g is rg*b -continuous and f is contra - rg*b - irresolute, then gof is contra rg*b - continuous. (ii) If g is rg*b -irresolute and f is contra- rg*b irresolute, then gof is contra- rg*b - irresolute.

Proof:

(i) Let V be closed set in Z. Then g^{-1}(V) is rg*b -closed in Y. Since f is contra - rg*b - irresolute,f^{-1}(g^{-1}(V)) is rg*b -open in X. Hence gof is contra rg*b - continuous.
(ii) Let V be rg*b -closed in Z. Then g^{-1}(V) is rg*b -closed in Y. Since f is contra - rg*b - irresolute, f^{-1}(g^{-1}(V)) is rg*b -open in X. Hence gof is contra rg*b - irresolute.

Theorem: 4.11 Let f: X→Y and g: Y→Z be two maps such that (gof): X→Z.

(i) If f is closed and g is ap- rg*b - closed, then (gof) is ap- rg*b - closed.
(ii) If f is ap- rg*b - closed and g is rg*b -open and g^{-1} preserves rg*b -open sets, then (gof) is ap- rg*b - closed.
(iii) If f is ap- rg*b-continuous and g is continuous, then gof is ap- rg*b -continuous

Proof:

(i) Suppose B is an arbitrary closed subset in X and A is a rg*b - open subset of Z for which (gof) (B) ⊆A. Then f(B) is closed in Y because f is closed. Since g is ap- rg*b -closed, g (f(B)) ⊂ bint (A) .This implies (gof) is ap- rg*b -closed.

(ii) Suppose B is an arbitrary closed subset of X and A is a rg*b -open subset of Z for which (gof) (B) ⊆ A. Hence f(B) ⊂ g^{-1} (A).Then f(B) ⊂ bint (g^{-1} (A)) because g^{-1} (A) is rg*b -open and f is ap- rg*b -closed. Hence (gof) (B) = g(f(B)) ⊆g[bint (g^{-1} (A)] ⊆ bint (g (g^{-1} (A))⊆ bint (A).This implies that (gof) is ap- rg*b -closed.

(iii) Suppose F is an arbitrary rg*b -closed subset of X and U is open in Z for which F ⊂ $(gof)^{-1}$ (U).Then g^{-1} (U) is open in Y , because g is continuous. Since f is ap- rg*b - continuous, then we have bcl (F) ⊆ f^{-1} (g^{-1} (U))= $(gof)^{-1}$(U).This shows that gof is ap- rg*b -continuous.

5. ALMOST CONTRA- rg*b -CONTINUOUS FUNCTIONS

Definition: 5.1 A function f: (X, τ) → (Y, σ) is said to be almost contra- rg*b - continuous if f^{-1}(V)∈ rg*bC(X, τ) for each V∈RO(Y,σ).

Theorem: 5.2 Suppose rg*bO(X, τ) is closed under arbitrary unions. Then the following statements are equivalent for a function f: (X, τ) → (Y,σ).

(i) f is almost contra- rg*b - continuous.

(ii) f^{-1}(F) ∈ rg*bO(X,τ) for every F ∈RC(Y,σ).

(iii) For each x∈X and each regular closed set F in Y containing f(x), there exists a rg*b -open set U in X containing x such that f(U)⊂F.

(iv) For each x∈X, and each regular open set V in Y not containing f(x),there exists a rg*b -closed set K in X not containing x such that f^{-1}(V)⊂K.

(v) f^{-1}(int(cl(G)))∈ rg*b C(X,τ) for every open subset G of Y.

(vi) f^{-1}(int(cl(F)))∈ rg*b O(X,τ) for every closed subset F of Y.

Proof:

(i)⇒ (ii): Let F∈RC(Y, σ).Then Y-F∈RO(Y, σ) by assumption. Hence f^{-1}((Y-F)=X-f^{-1}(F)∈ rg*b C(X, τ).This implies f^{-1}(F)∈ rg*b O(X, τ).

(ii) ⇒ (i): Let V∈RO(Y, σ).Then by assumption (Y-V) ∈RC(Y,σ). Hence f^{-1}((Y-F) =X-f^{-1}(F) ∈ rg*b O(X, τ).This implies f^{-1}(F)∈ rg*b C(X, τ).

(ii)⇒ (iii): Let F be any regular closed set in Y containing f(x). f^{-1}(F)∈ rg*b O(X, τ) and x∈f^{-1}(F) (by(ii)). Take U= f^{-1}(F).Then f(U)⊂F.

(iii)⇒ (ii): Let F∈RC(Y, σ) and x∈f^{-1}(F).From (iii), there exists a rg*b -open set U_x in X containing x such that U_x ⊂ f^{-1}(F). We have f^{-1}(F) =∪ {U_x: x∈f^{-1}(F)}.Then f^{-1}(F) is rg*b -open.

(iii)⇒(iv): Let V be any regular open set in Y containing f(x).Then Y-V is a regular closed set containing f(x).By (iii), there exists a rg*b -open set U in X containing x such that f(U)⊂ Y-V. Hence U⊂ f^{-1}(Y-V) ⊂ X- f^{-1}(V).Then f^{-1}(V) ⊂X-U. Take K=X-U. We obtain a rg*b -closed set in X not containing x such that f^{-1}(V) ⊂K.

(iv)⇒ (iii): Let F be regular closed set in Y containing f(x).Then Y-F is regular open set in Y containing f(x).By (iv), there exists a rg*b -closed set K in X not containing x such that f^{-1}(Y-F) ⊂K. Then X- f^{-1}(F) ⊂ K implies X-K ⊂f^{-1}(F).Hence f (X-K) ⊂F. Take U=X-K. Then U is a rg*b -open set U in X containing x such that f(U)⊂F.

(i)⇒ (v): Let G be a open subset of Y. Since int(cl(G)) is regular open, then by (i),f^{-1} (int(cl(G))∈ rg*b C(X, τ).

(v)⇒(i): Let V∈RO(Y,σ).Then V is open in Y. By (v), f^{-1}(int(cl(G))∈ rg*b C(X,τ).This implies f^{-1}(V)∈ rg*b C(X, τ)
(ii)⇔ (vi) is similar as (i)⇔(v).

Theorem: 5.3 If f: (X, τ) → (Y, σ) is an almost contra- rg*b - continuous function and A is a open subset of X, then the restriction f/A:A→Y is almost contra- rg*b - continuous.

Proof: Let F ∈RC(Y). Since f is almost contra- rg*b - continuous, f^{-1}(F)∈rg*b O(X).Since A is open, it follows that $(f/A)^{-1}$(F)= A∩f^{-1}(F)∈rg*b O(A).Therefore f/A is an almost contra- rg*b- continuous.

Theorem: 5.4 Let f: (X, τ) →(Y, σ) be an almost contra- rg*b -continuous surjection. Then the following statements hold.

(i) If X is rg*b -closed, then Y is nearly compact.
(ii) If X is rg*b -Lindelof,then Y is nearly Lindelof.
(iii) If X is countably- rg*b -closed,then Y is nearly countably compact.
(iv) If X is rg*b O-compact,then Y is S-closed.
(v) If X is rg*b -Lindelof,then Y is S- Lindelof.
(vi) If X is countable rg*b -compact, then Y is countably S-closed compact.

Proof: (i) Let {V_b: b∈I} be regular open cover of Y. Then f is almost contra- rg*b -continuous implies {f^{-1} (V_b): b∈I} is a rg*b -closed cover of X. Since X is rg*b -closed, there exists a finite subset I_0 of I such that X=∪ {f^{-1} (V_b): b∈I_0}.Then we have Y=∪{V_b: b∈I_0}.Hence Y is nearly compact.

Proof of (ii) and (iii) is similar to that of (i).

(iv) Let {V_b :b∈I} be regular closed cover of Y. Then f is almost contra- rg*b -conntinuous implies {f^{-1} (V_b):b∈I} is a rg*b -open cover of X. By assumption, there exists a finite subset I_0 of I such that X=∪ {f^{-1} (V_b):b∈I_0}.Then we have Y=∪{V_b :b∈I_0}.Hence Y is nearly compact. Proof of (v) and (vi) is similar to that of (iv).

Theorem: 5.5 If f: (X, τ) →(Y, σ) is almost contra- rg*b -continuous and almost rg*b-continuous surjection. Then

(i) If X is mildly rg*b -compact, then Y is nearly compact.
(ii) If X is mildly countably - rg*b -compact, then Y is nearly countably compact.
(iii) If X is mildly rg*b -Lindelof,, then Y is nearly Lindelof.

Proof: (i) Let V∈RO(Y).Since f is almost contra- rg*b -continuous and almost rg*b -continuous, f^{-1}(V) is rg*b -closed and rg*b -open in X respectively. Then f^{-1} (V) is rg*b -clopen in X. Let {V_b: b∈I} be any regular open cover of Y. Then {f^{-1} (V_b):b∈I} is rg*b -clopen in X. Since X is mildly rg*b -compact, there exists a finite subset I_0 of I such that X=∪ {f^{-1} (V_b):b∈I_0}.Since X is surjective, we obtain Y=∪{V_b :b∈I_0}.Hence Y is nearly compact. Proof of (ii) and (iii) is similar to that of (i).

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