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A NOTE ON δ– H -SETS IN GTS WITH HEREDITARY CLASSES

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ABSTRACT

We characterize $\delta-H$- sets, discuss the relation between δ- sets, $\delta-H$- sets and $H\delta$ - sets and generalize some of the results in GTS with hereditary classes.

Keywords: *δ-set, $\delta-H$- sets, $H\delta$ - sets, $\mu-$codense and completely $\mu-$codense hereditary classes.*

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1. INTRODUCTION

In 2007, Csaszar [6] defined a nonempty class of subsets of a nonempty set, called hereditary class and studied modification of generalized topology via hereditary classes. Also it is studied in [10]. The aim of the paper is to ex- tend the study of the properties of the generalized topologies via hereditary classes. A subfamily μ of $\wp(X)$ is called a generalized topology (GT) [6] if $\emptyset \in \mu$ and μ is closed under arbitrary union. The pair (X,μ) is called a generalized topological space (GTS). Members of μ are called μ - open sets and its complement is called a μ - closed set. The largest μ - open set contained in a subset A of X is denoted by $i_\mu(A)$ [4] and is called the μ - interior of A. The smallest μ- closed set containing A is called the μ -closure of A and is denoted by $c_\mu(A)$[2].

A generalized topology μ is said to be a quasi-topology if μ is closed under fi nite intersection. Let X be a nonempty set. A hereditary class H of X is a nonempty collection of subset of X such that $A\subset B$, $B\in H$ implies $A\in H$ [6]. A hereditary class of X is an ideal [9] if $A\cup B \in H$ whenever $A \in H$ and $B \in H$.

An ideal I in a topological space (X,τ) is said to be condense if $\tau \cap I = \{\emptyset\}$. With respect to the generalized topology μ of all μ - open sets and a hereditary class H, for each subset A of X, a subset $A^*(H)$ or simply A^* of X is defined by $A^* = \{x \in X | M \cap A \in H$ for every $M\in\mu$ such that $x\in M\}$ [8]. H is said to be $\mu-$codense if $\mu \cap H = \{\emptyset\}$ [8] and is said to be completly $\mu-$ condense [8] if $\pi(X)\cap H = \{\emptyset\}$, where $\pi(X)$ is the family of all preopen sets in (X,μ). A subset A is said to be preopen if $A \subset i_\mu(c_\mu(A))$, every completely $\mu-$ condense is $\mu-$ condense. If $c^*(A) = A\cup A^*$ for every subset A of X, with respect to μ and a hereditary class H of subsets of X, then $\mu^* = \{A \subset X | c^*(X-A)=X-A\}$ is a generalized topology[3], and $i^*(A)$ will denote the interior of A in (X,μ^*). The following Lemmas will be useful in the sequel.

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Lemma: 1.1[11, Lemma 1.2] If (X,μ) is a quasi topology with a hereditary class H, then the following are hold.

(a) H is $\mu-$ condense if and only if $A \subset A^*$ for every $A \in \mu$.

(b) If $A \subset A^*$, then $A^* = c_\mu(A^*) = c_\mu(A) = c^*(A)$.

Lemma: 1.2 [8, Theorem 4.13]. Let (X,μ) be a GTS with a hereditary class H. Then H is completely $\mu-$ codense if and only if $H \subset N$ where N is the hereditary class of all nowhere dense sets in (X,μ).

2. δ −H− SETS

Let (X,μ) be a generalized topological space and H be a hereditary class. A subset A of X is a δ -set [3] if $i_\mu c_\mu(A) \subset c_\mu i_\mu(A)$). We will denote the family of all δ -sets in (X,μ) by $\delta(X,\mu)$ or simply $\delta(X)$. Clearly, every nowhere dense subset of X is a δ -set. A subset A of X is a H_δ -set [2] if $i_\mu(A^*) \subset (i_\mu(A))^*$. We will denote the family of all H_δ-sets in (X,μ) with a hereditary class H by $H\delta(X,\mu)$ or simply $H\delta(X)$. A subset A of X is a $\delta-H$ -set [1] if $i_\mu(c^*(A)) \subset c^*(i_\mu(A))$.

We will denote the family of all δ−H-sets in (X,μ) by $\delta H(X,\mu)$ or simply $\delta H(X)$. δ set and δ − H-set are independent concepts [1, Remark 2.2]. The following Theorem 2.1 shows that every δ -set is a δ−H-set, if H is $\mu-$ condense and Example 2.2 says that the reverse inclusion is not true, even if H is $\mu-$ condense. Theorem 2.3 below shows that for completely $\mu-$ codense, $\delta-H$ -sets coincide with δ -sets.

Theorem: 2.1 Let (X,μ) be a quasi topology with a hereditary class H and H be $\mu-$ codense. Then every δ -set is a $\delta-H$ -set and so $N \subset \delta(X) \subset \delta H(X)$.

Proof: Suppose H is $\mu-$codense and A is a δ -set. Then $i_\mu(c_\mu(A)) \subset c_\mu(i_\mu(A))$.

Now $i_\mu c^*(A) \subset i\mu(c\mu(A)) \subset c\mu(i\mu(A)) = c^*(i\mu(A))$, by Lemma 1.1. Therefore, A is a $\delta-H$ -set.

Theorem: 2.2 Let (X,μ) be a quasi topology with a hereditary class H and H be $\mu-$ completely condense. Then every $\delta-H$ -sets coincide with δ -sets.

Proof: Suppose A is a δ−H -set. Then $i_\mu(c^*(A)) \subset c^*(i_\mu(A))$. Since H be $\mu-$ codense $c^*(i_\mu(A)) = c_\mu(i_\mu(A))$ by Lemma 1.1 and Lemma 1.2. Since H be completely $\mu-$ codense, by lemma 1.2, $H \subset N$ and so $A^*(N) \subset A^*(H)$. Since $A^*(N) = c_\mu(i_\mu(c_\mu(A)))$ [13], $c_\mu(i_\mu(c_\mu(A))) \subset A^*(H)$. It follows that $i_\mu(c_\mu(A)) \subset A^*(H) \subset c_\mu(i_\mu(A))$. Therefore, A is a δ -set. The reverse direction follows from Theorem 2.1, since every completely $\mu-$ condense is $\mu-$ condense.

Corollary: 2.3 (1, Proposition 2.6). Let (X,μ) be a GTS with a hereditary class H. If H is either $\{\emptyset\}$ or N, then a subset A of X is a $\delta-H$ -set if and only if A is a δ -set. The following theorem gives a characterization of $\delta-H$ -sets.

Theorem: 2.4. Let (X,μ) be a GTS with a hereditary class H. Then $A \subset X$ is a $\delta-H$ -set if and only if $i_\mu(c^*(A)) = i_\mu(c^*(i_\mu(A)))$.

Proof: Suppose A is a δ −H -set. Then $i\mu(c^*(A)) \subset c^*(i\mu(A))$ and so $i\mu(c^*(A)) \subset i\mu(c^*(i\mu(A)))$. Since $c^*(i\mu(A)) \subset c^*(A)$, $i\mu(c^*(i\mu(A))) \subset i\mu(c^*(A))$ and so $i\mu(c^*(A)) = i\mu(c^*(i\mu(A)))$. The converse is clear.

The following Theorem 2.5 shows that every $\delta-H$ -set is a H_δ-set. Example 2.6 below shows that the reverse direction is not true.

Theorem: 2.5. Let (X,μ) be a GTS with a hereditary class H. Then every δ–H set is a $H\delta$ -set.

Proof: Suppose A is a δ −H -set. Then $i\mu(c^*(A)) \subset c^*(i\mu(A))$. Now $i\mu(A^*) \subset i\mu(c^*(A)) \subset c^*(i\mu(A)) = i\mu(A) \cup ((i\mu(A))^*$. Suppose $x \in i\mu(A^*)$. Then $x \in i\mu(A^*)$. Then $x \in i\mu(A) \cup (i\mu(A))^*$. Since $x \in i\mu(A^*)$, there is a μ– open set G containing x such that $G \subset A^*$. If $x \notin (i\mu(A))^*$, then there is a μ− open set I containing x such that $I \cap i\mu(A) \in H$. Now $I \cap i\mu(A) = I \cap (i\mu(A) \cap (A)) = (I \cap i\mu(A)) \cap A \in H$ and so $((I \cap i\mu(A)) \cap A)^* = \emptyset$ which implies that $(I \cap i\mu(A)) \cap A^* = \emptyset$. Since $G \subset A^*$, $(I \cap i\mu(A)) \cap G = \emptyset$ and so $(I \cap G) \cap i\mu(A) = \emptyset$ which implies that $x \notin i\mu(A)$, a contradiction to the fact that $x \in i\mu(A) \cup (i\mu(A))^*$. Therefore, $x \in (i\mu(A))^*$ and so $i\mu(A^*) \subset (i\mu(A))^*$. Hence $A \in H\delta(X)$.

Example: 2.6 Let X = {a, b, c, d}, $\mu = \{\emptyset, \{b\}\}$ and $H = \{\emptyset, \{b\}\}$. If A = {b, c}, then A is not a δ− H-set. Since $A^* = \emptyset, i\mu(A^*) = \emptyset$ and so A is a $H\delta$ -set.

Theorem: 2.7 Let (X, μ) be a quasi topology with a hereditary class H. If H is μ– condense, then $\delta H(X) = H\delta(X)$.

Proof: Suppose H is μ– co ndense. By Theorem 2.5, $\delta H(X) \subset H\delta(X)$. On the other hand, if $A \in H\delta(X)$, then $i\mu(A^*) \subset (i\mu(A))^*$. Now $i\mu(c^*(A)) = i\mu(A \cup A^*) \subset i\mu(A) \cup A^*$, since A^* is closed. Since H −is μ ndense, $i\mu(A) \subset (i\mu(A))^*$.Therefore, $i\mu(c^*(A)) \subset (i\mu(A))^* \cup A^* = (i\mu(A) \cup A)^* = A^*$ and so $i\mu(c^*(A)) \subset i\mu(A^*) \subset (i\mu(A))^* \subset c^*(i\mu(A))$ which implies that $A \in \delta H(X)$. Hence $H\delta(X) \subset \delta H(X)$ and so $\delta H(X) = H\delta(X)$.

A subset A of a GTS (X,μ) with a hereditary class H is said to be H− dense [8] if $A^* = X$. Clearly, every H− dense is a μ^*− dense but the converse is not true. The following theorem gives some properties of δ− H -sets.

Theorem: 2.8 Let (X, μ) be a GTS with a hereditary class H and $A \subset X$ be a δ− H -set. If $A \subset B \subset A^*$, then B, A^* and B^* are δ − H -sets.

Proof: Since $B \subset A^*$ implies $B \subset c^*(A)$, B is a δ − H -set, by Proposition 2.1 and Corollary 2.6 of [6]. Since $A \subset B \subset A^*$ implies that $A^* = B^*$, A^* and $B^*$are δ− H -sets.

The following theorem gives a necessary condition for a set to be a δ − H-set. Example 2.10 below shows that the converse of Theorem 2.9 is not true.

Theorem: 2.9 Let (X, μ) be a GTS with a hereditary class H and A be a δ −H-set. Then $A = B \cup C$, where $B \in \mu$, $i\mu(c^*(C)) = \emptyset$ and $B \cap C = \emptyset$

Proof: Suppose A is a δ − H -set. If $B = i\mu(A)$ and $C = A - i\mu(A)$, then B is a μ– open set such that $A = B \cup C$, and $B \cap C = \emptyset$ Now $C \subset A$ implies $c^*(C) \subset c^*(A)$ and so that $i\mu(c^*(C)) \subset i\mu(c^*(A)) \subset c^*(i\mu(A))$. Again $B \cap C = \emptyset$ implies $B \cap c^*(C) = \emptyset$ implies $B \cap i\mu(c^*(C)) = \emptyset$ implies $c^*(B) \cap i\mu(c^*(C)) = \emptyset$ implies $c^*(i\mu(A)) \cap i\mu(c^*(C)) = \emptyset$ implies $i\mu(c^*(C)) = \emptyset$.

Example: 2.10 Consider the GTS (X, μ) with a hereditary class H of Example 2.6. If A = {b, c}, then A is not a δ − H -set If B = {b} and C = {c}, then $A = B \cup C$, where B is μ– open and $B \cap C = \emptyset$. Now $i\mu(c^*(C)) = i\mu(\{c\} \cup \{c\}^*)) = i\mu(\{c\}) = \emptyset$. This shows that the converse of Theorem 2.9 is not true.

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