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On rg*b-Compactness and rg*b-Connectedness in Topological Spaces

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ABSTRACT

*This paper deals with rg*b -compact spaces and their properties by using nets, filter base and rg*b-complete accumulation points. The notion of rg*b -connectedness in topological spaces is also introduced and their properties are studied.*

Keywords: *rg*b -open set, rg*b -closed sets, rg*b -compact spaces, rg*b –complete accumulation point, rg*b – connectedness.*

1. INTRODUCTION

The notions of compactness and connectedness are useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness and connectedness. The productivity and fruitfulness of these notions of compactness and connectedness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness and connectedness have been introduced and investigated. D. Andrijevic [1] introduced a new class of generalized open sets in a topological space called b-open sets. The class of b-open sets generates the same topology as the class of pre-open sets. Since the advent of this notion, several research paper with interesting results in different respects came into existence (See [2, 3, 4, 6, 7, 8]). The aim of this paper is to introduce the concept of rg*b-compactness and rg*b -connectedness via the concept of rg*b−closed sets [9] in topological spaces and to give some characterizations of rg*b -compact spaces in terms of nets and filter bases. The notion of rg*b -complete accumulation points is introduced and is used to characterize rg*b -compactness. Further it is proved that rg*b -connectedness is preserved under rg*b -irresolute surjections.

2. PRELIMINARY NOTES

Throughout this paper (X, τ), (Y, σ) are topological spaces with no separation axioms assumed unless otherwise stated. Let $A \subseteq X$. The closure of A and the interior of A will be denoted by Cl(A) and Int(A) respectively.

Let us recall the following definitions which we shall require later.

Definition: 2.1 A subset A of a space (X, τ) is called
1) a regular open set[13] if A= int (cl(A)) and a regular closed set if A= cl(int (A))
2) a b-open set [1] if $A \subseteq cl(int(A)) \cup int(cl(A))$.
3) a regular generalized closed set (briefly, rg-closed)[9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
5) a regular generalized star b- closed set (briefly rg*b-closed set)[10] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is rg-open in X.

Definition: 2.2 A function $f: (X, \tau) \to (Y, \sigma)$ is called
1) b-irresolute: [8] if for each b-open set V in Y, $f^{-1}(V)$ is b-open in X;
2) b-continuous: [8] if for each open set V in Y, $f^{-1}(V)$ is b-open in X.
3) rg*b Continuous[11] if $f^{-1}(V)$ is rg*b Closed in X for every closed set V in Y.
4) rg*b irresolute[11] if the inverse image of each rg*b Closed set in Y is a rg*b Closed set in X .
5) rg*b Closed[11], if the image of each closed set in X is a rg*b Closed set in Y.
6) rg*b Open[11], if the image of each open set in X is a rg*b open in Y.

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7) Pre rg*b Closed(resp. Pre rg*b open)[11], if the image of each rg*b closed (resp. rg*b open)set in X is a rg*b Closed (resp. rg*b open)set in Y.
8) contra-rg*b-continuous [12] if $f^{-1}(V)$ is rg*b closed in (X, τ) for each open set V of (Y, σ).
9) approximately- rg*b -continuous (ap- rg*b -continuous) [12] if bcl (F) ⊂ $f^{-1}(U)$ whenever U is an open subset of Y and F is a rg*b -closed subset of X such that F⊂f^{-1} (U).
10) approximately- rg*b -closed (briefly ap- rg*b -closed) [12] if f (F) ⊂ bint(V) whenever V is a rg*b -open subset of Y, F is a closed subset of X and f(F) ⊂V.
11) contra- rg*b -closed (resp.contra rg*b -open) [12] if f (U) is rg*b -open (resp rg*b -closed) in Y for each closed (resp. open) set U of X.
12) approximately - rg*b -open (briefly ap- rg*b –open) [12] if bcl (F) ⊂f(U) whenever U is an open subset of X, F is a rg*b -closed subset of Y and F ⊂f(U).
13) contra- rg*b –irresolute [12] if $f^{-1}(V)$ is rg*b - closed in X for each rg*b –open set V in Y.
14) almost contra- rg*b – continuous [12] if $f^{-1}(V)$∈ rg*bC(X,τ) for each V∈RO(Y, σ).

Definition: 2.5 A space (X, τ) is called
(i) rg*b –space [12] if every rg*b closed set is closed.
(ii) rg*b -locally indiscrete [12] if every rg*b open set is closed.
(iii) a T_{rg*b} –space [12] if every rg*b closed set is b closed.
(iv) rg*b –Lindelof [12] if every cover of X by rg*b -open sets has a countable sub cover.

3. rg*b-COMPACTNESS

Definition: 3.1 A collection $\{A_i: i \in \wedge\}$ of rg*b-open sets in a topological space X is called a rg*b-open cover of a subset B of X if B ⊂ ∪$\{A_i: i \in \wedge\}$ holds.

Definition: 3.2 A topological space X is rg*b -compact if every rg*b -open cover of X has a finite sub-cover.

Definition; 3.3 A subset B of a topological space X is said to be rg*b -compact relative to X if, for every collection $\{A_i : i \in \wedge\}$ of rg*b -open subsets of X such that B ⊂ ∪{Ai : i ∈ ∧} there exists a finite subset $\wedge_0$ of ∧ such that B ⊆ ∪ $\{A_i : i \in \wedge_0\}$

Definition: 3.4 A subset B of a topological space X is said to be rg*b –compact if B is rg*b -compact as a subspace of X.

Theorem: 3.5 Every rg*b -closed subset of a rg*b -compact space is rg*b -compact relative to X.

Proof: Let A be rg*b -closed subset of rg*b -compact space X. Then A^C is rg*b –open in X. Let M = $\{G_\alpha : \alpha \in \wedge\}$ be a cover of A by rg*b -open sets in X. Then M∗ = M ∪ A^c is a rg*b -open cover of X. Since X is rg*b -compact M∗ is reducible to a finite subcover of X, say X = $G_{\alpha1}$ ∪$G_{\alpha2}$ ∪· · ·∪$G_{\alpha m}$ ∪A^C, $G_{\alpha k}$ ∈ M. But A and A^C are disjoint hence A ⊂ $G_{\alpha1}$ ∪ · · · ∪ $G_{\alpha m}$, $G_{\alpha k}$ ∈ M, which implies that any rg*b -open cover M of A contains a finite sub-cover. Therefore A is rg*b -compact relative to X.

Thus every rg*b -closed subset of a rg*b -compact space X is rg*b -compact.

Definition: 3.6 A function f: X → Y is said to be rg*b -continuous [11] if f^{-1} (V) is rg*b -closed in X for every closed set V of Y .

Definition: 3.7 A function f: X → Y is said to be rg*b -irresolute [11] if f^{-1} (V) is rg*b -closed in X for every rg*b-closed set V of Y .

Theorem: 3.8 A rg*b -continuous image of a rg*b -compact space is compact.

Proof: Let f : X → Y be a rg*b -continuous map from a rg*b -compact space X onto a topological space Y . Let $\{A_i : i \in \wedge\}$ be an open cover of Y . Then $\{f^{-1}(A_i) : i \in \wedge\}$ is a rg*b -open cover of X. Since X is rg*b -compact it has a finite sub-cover say $\{f^{-1}(A_1), f^{-1}(A_2), \cdots, f^{-1}(A_n)\}$. Since f is onto $\{A_1, \cdots, A_n\}$ is a cover of Y , which is finite. Therefore Y is compact.

Theorem: 3.9 If a map f: X → Y is rg*b -irresolute and a subset B of X is rg*b -compact relative to X, then the image f(B) is rg*b -compact relative to Y .

Proof: Let $\{A_\alpha : \alpha \in \wedge\}$ be any collection of rg*b -open subsets of Y such that $f(B) \subset \cup\{A_\alpha : \alpha \in \wedge\}$. Then $B \subset \cup\{f^{-1}(A_\alpha) : \alpha \in \wedge\}$ holds. Since by hypothesis B is rg*b -compact relative to X there exists a finite subset $\wedge_0$ of $\wedge$ such that $B \subset \cup \{f^{-1}(A_\alpha) : \alpha \in \wedge_0\}$ Therefore we have $f(B) \subset \cup\{A_\alpha : \alpha \in \wedge_0\}$, which shows that f(B) is rg*b compact relative to Y .

Now we introduce the following notions which will be used in this paper.

Definition: 3.10 Let $\wedge$ be a directed set. A net $\xi = \{x_\alpha : \alpha \in \wedge\}$ rg*b accumulates at a point $x \in X$ if the net is frequently in every $U \in$ rg*bO(X, x), i.e., for each $U \in$ rg*b O(X, x) and for each $\alpha_0 \in \wedge$, there is some $\alpha \geq \alpha_0$ such that $x_\alpha \in U$. The net ξ rg*b -converges to a point x of X if it is eventually in every $U \in$ rg*b O(X, x).

Definition: 3.11 We say that a filter base $\Theta = \{F_\alpha : \alpha \in \Gamma\}$ rg*b -accumulates at a point $x \in X$ if $x \in D \cap_{\alpha\in\Gamma}$ rg*b Cl (F_α). A filter base $\Theta = \{F_\alpha : \alpha \in \Gamma\}$ rg*b converges to a point x in X if for each $U \in$ rg*b O(X, x), there exists an F_α in Θ such that $F_\alpha \subset U$.

Remark: 3.12 Since openness implies rg*b -openness, it follows that rg*b -compactness implies compactness. In the following example we show that compactness does not imply rg*b -compactness.

Example: 3.13 Let X be an infinite set with indiscrete topology. Then X is compact, but fails to be rg*b -compact. To see this, we note that every singleton set in X is rg*b -open. Thus a cover of X by singletons does not have a finite sub cover.

Definition: 3.14 A point x in a space X is said to be rg*b -complete accumulation point of a subset S of X if Card $(S \cap U)$ = Card(S) for each $U \in$ rg*b O(X, x), where Card(S) denotes the cardinality of S.

Definition: 3.15 In a topological space X, a point x is said to be a rg*b adherent point of a filter base Θ on X if it lies in the rg*b -closure of all sets of Θ.

Theorem: 3.16 A space X is rg*b -compact if and only if each infinite subset of X has a rg*b -complete accumulation point.

Proof: Let the space X be rg*b -compact and let S be an infinite subset of X. Let K be the set of points x in X which are not rg*b -complete accumulation points of S. Now it is obvious that for each x in K, we are able to find $U(x) \in$ rg*b O(X, x) such that $Card(S \cap U(x)) \neq Card(S)$. If K is the whole space X, then $\Theta = \{U(x): x \in X\}$ is a rg*b-cover of X. By the hypothesis X is rg*b -compact, so there exists a finite sub-cover $\psi = \{U(x_i)\}$, where i = 1, 2, · · ·, n such that $S \subset \{U(x_i) \cap S : i = 1, 2, \cdots, n\}$. Then Card(S) = max {Card (U (xi) ∩ S)}, where i = 1, 2, · · ·, n which does not agree with what we assumed. This implies that S has rg*b -complete accumulation point.

Conversely, suppose that X is not rg*b -compact and that every infinite subset $S \subset X$ has a rg*b -complete accumulation point in X. It follows that there exists a rg*b -cover E with no finite sub-cover. Set $\delta = min\ \{Card(\Phi) : \Phi \subset E\}$, where Φ is a rg*b -cover of X. Fix $\psi \subset E$ for which $Card(\psi) = \delta$ and $\cup\{U : U \in \psi\} = X$. Let N denote the set of natural numbers. Then by hypothesis $\delta \geq Card\ (N)$. By well ordering of ψ by some minimal well ordering $\sim$ suppose that U is any member of ψ. By minimal well ordering $\sim$ we have $Card\ (\{V: V \in \psi,\ V \sim U\}) < Card\ (\{V: V \in \psi\})$. Since ψ cannot have any sub-cover with cardinality less than δ, then for each $U \in \psi$ we have $X \neq \cup\{V : V \in \psi,\ V \sim U\}$. For each $U \in \psi$ choose a point $x(U) \in X - U\ \{V \cup \{x(V)\} : V \in \psi,\ V \sim U\}$. We are always able to do this if not one can choose a cover of smaller cardinality from ψ. If $H = \{x(U) : U \in \psi\}$, then to finish the proof we will show that H has no rg*b -accumulation points in X. Suppose that z is a point of X. Since ψ is a rg*b -cover of X, then z is a point of some set W in ψ. By the fact that $U \sim W$, we have $x(U) \in W$. But $Card\ (T) < \delta$. Therefore, $Card\ (H \cap W) < \delta$. But $Card\ (H) = \delta \geq Card\ (N)$, since for two distinct points U and W in ψ, we have $x(U) \neq x(W)$. This means that H has no rg*b -complete accumulation point in X which contradicts our assumptions. Therefore X is rg*b -compact.

Theorem: 3.17 For a space X the following are equivalent
(1) X is rg*b -compact.
(2) Every net in X with a well ordered directed set as its domain rg*b –accumulates to some point of X.

Proof:
(1) ⇒ (2): Suppose that (X, τ) is rg*b -compact and $\xi = \{x\alpha : \alpha \in \Lambda\}$ a net with a well ordered directed set Λ as domain. Assume that ξ has no rg*b adherent point in X. Then for each point x in X, there exist $V(x) \in$ rg*b $O(X, x)$ and an $\alpha(x) \in \Lambda$ such that $V(x) \cap \{x\alpha : \alpha \geq \alpha(x)\} = \varphi$. This implies that $\{x\alpha : \alpha \geq \alpha(x)\}$ is a subset of $X - V(x)$. Then the collection $C = \{V(x) : x \in X\}$ is a rg*b -cover of X. By hypothesis of theorem, X is rg*b -compact and so C has a finite subfamily $\{V(xi)\}$, where $i = 1, 2, \cdots, n$ such that $X = \cup\{V(xi)\}$. Suppose that the corresponding elements of Λ are $\{\alpha(xi)\}$,where $i = 1, 2, \cdots, n$. Since Λ is well ordered and $\{\alpha(xi)\}$, where $i = 1, 2, \cdots, n$ is finite, the largest element of $\{\alpha(xi)\}$ exists. Suppose it is $\{\alpha(xl)\}$, then for $\gamma \geq \{\alpha(xl)\}$. We have $\{x\delta : \delta \geq \gamma\} \subset D \bigcap_{i=1}^{n}(X - V(xi))$ =X- $\bigcup_{i=1}^{n} V(xi) = \varphi$ which is impossible.

This shows that ξ has at least one rg*b-adherent point in X.

(2) ⇒ (1): Now by the last Theorem 3.16, it is enough to prove that each infinite subset has a rg*b -complete accumulation point. Suppose that $S \subset X$ is an infinite subset of X. According to Zorn's lemma, the infinite set S can be well ordered. This means that we can assume S to be a net with a domain, which is a well ordered index set. It follows that S has a rg*b -adherent point z.

Therefore z is a rg*b -complete accumulation point of S. This shows that X is rg*b -compact.

Theorem: 3.18 A space X is rg*b -compact if and only if each family of rg*b closed subsets of X with the finite intersection property has a non-empty intersection.

Proof: Given a collection A of subsets of X, let $C = \{X - A: A \in A\}$ be the collection of their complements. Then the following statements hold.

(a) A is a collection of rg*b -open sets if and only if C is a collection of rg*b –closed sets.
(b) The collection A covers X if and only if the intersection $\cap_{c \in C} C$ of all the elements of C is non-empty.
(c) The finite sub collection $\{An \cdots An\}$ of A covers X if and only if the intersection of the corresponding elements $C_i = X - A_i$ of C is empty. The statement (a) is trivial, while the (b) and (c) follow from DeMorgan's law. $X - (D \cup_{\alpha \in J} A_\alpha) = D \cap_{\alpha \in J} (X - A_\alpha)$ .The proof of the theorem now proceeds in two steps, taking the contrapositive of the theorem and then the complement.

The statement X is rg*b -compact is equivalent to: Given any collection A of rg*b -open subsets of X, if A covers X, then some finite sub collection of A covers X. This statement is equivalent to its contrapositive, which is the following.

Given any collection A of rg*b -open sets, if no finite sub-collection of A covers X, then A does not cover X. Letting C be as earlier, the collection $\{X - A : A \in A\}$ and applying (a) to (c), we see that this statement is in turn equivalent to the following:

Given any collection C of rg*b -closed sets, if every finite intersection of elements of C is non-empty, then the intersection of all the elements of C is non-empty. This is just the condition of our theorem.

Theorem: 3.19 A space X is rg*b -compact if and only if each filter base in X has at least one rg*b -adherent point.

Proof: Suppose that X is rg*b -compact and $\theta = \{F\alpha: \alpha \in \Gamma\}$ a filter base in it. Since all finite intersections of $F\alpha$'s are non-empty, it follows that all finite intersections of rg*b Cl $(F\alpha)$'s are also non-empty. Now it follows from Theorem 3.18 that $D \cap_{\alpha \in \Gamma}$ rg*b Cl $(F\alpha)$ is non-empty. This means that θ has at least one rg*b -adherent point.

Conversely, suppose θ is any family of rg*b -closed sets. Let each finite intersection be non-empty. The sets $(F\alpha)$ with their finite intersection establish a filter base θ. Therefore θ rg*b -accumulates to some point z in X. It follows that $z \in D \cap_{\alpha \in \Gamma} (F\alpha)$. Now by Theorem 3.18, we have that X is rg*b -compact.

Theorem: 3.20 A space X is rg*b -compact if and only if each filter base on X with at most one rg*b -adherent point is rg*b -convergent.

Proof: Suppose that X is rg*b -compact, x a point of X and a filter base on X. The rg*b -adherence of θ is a subset of $\{x\}$. Then the rg*b -adherence of θ is equal to $\{x\}$ by Theorem 3.19. Assume that there exists $V \in$ rg*b $O(X, x)$ such that for all $F \in \theta$, $F \cap (X - V)$ is non-empty. Then $\psi = \{F - V: F \in \theta\}$ is a filter base on X. It follows that the rg*b -adherence of θ is non-empty. However, $D \cap_{F \in \theta}$ rg*b $Cl(F - V) \subset (D \cap_{F \in \theta}$ rg*b$Cl(F) \cap (X - V)) = \{x\} \cap \{X - V\} = \varphi$. But this is a contradiction. Hence for each $V \in$ rg*b $O(X, x)$ there exists an $F \in \theta$ with $F \subset V$.
This shows that θ rg*b -converges to x.

To prove the converse, it suffices to show that each filter base in X has at least one rg*b - accumulation point. Assume that θ is a filter base on X with no rg*b -adherent point. By hypothesis, θ rg*b -converges to some point z in X. Suppose $F\alpha$ is an arbitrary element of θ. Then for each $V \in$ rg*b $O(X, z)$, there exists $F_\beta \in \theta$ such that $F_\beta \subset V$. Since θ is a filter base, there exists a γ such that $F_\gamma \subset F_\alpha \cap F_\beta \subset F_\alpha \cap V$, where F_γ non-empty. This means that $F_\alpha \cap V$ is non-empty for every $V \in$ rg*b $O(X, z)$ and correspondingly for each α, z is a point of rg*b Cl (F_α) it follows that $z \in D \cap_\alpha$ rg*b Cl $(F\alpha)$. Therefore z is a rg*b -adherent point of θ which is a contradiction. This shows that X is rg*b -compact.

4. rg*b-CONNECTEDNESS

Definition 4.1 A topological space X is said to be rg*b -connected if X cannot be expressed as a disjoint union of two non-empty rg*b open sets. A subset of X is rg*b -connected if it is rg*b -connected as a subspace.

Example: 4.2 Let X = {a, b} and let τ = {φ, {a}, X}. Then rg*bO= {φ, {a}, {a, b}, {a, c}, X} and it is rg*b connected.

Remark: 4.3 Every rg*b -connected space is connected but the converse need not be true in general, which follows from the following example.

Example: 4.4 Let X = {a, b} and let τ = {X, φ}. Clearly (X, τ) is connected. The rg*b -open sets of X are {X, φ, {a}, {b}}. Therefore (X, τ) is not a rg*b -connected space, because X = {a} ∪ {b} where {a} and {b} are non-empty rg*b -open sets.

Theorem: 4.5 For a topological space X the following are equivalent.
(i) X is rg*b -connected.
(ii) X and φ are the only subsets of X which are both rg*b -open and rg*b -closed.
(iii) Each rg*b -continuous map of X into a discrete space Y with at least two points is a constant map.

Proof:
(i) ⇒ (ii): Let *O* be any rg*b -open and rg*b -closed subset of *X*. Then O^C is both rg*b –open and rg*b -closed. Since *X* is disjoint union of the rg*b -open sets *O* and O^C implies from the hypothesis of (i) that either $O = \varphi$ or $O = X$.

(ii) ⇒ (i): Suppose that X = A ∪ B where A and B are disjoint non-empty rg*b -open subsets of X. Then A is both rg*b -open and rg*b -closed. By assumption A = φ or X. Therefore X is rg*b -connected.

(ii) ⇒ (iii): Let f: X → Y be a rg*b -continuous map. Then X is covered by rg*b -open and rg*b -closed covering $\{f^{-1}(Y) : y \in (Y)\}$. By assumption $f^{-1}(y) = \varphi$ or X for each y ∈ Y. If $f^{-1}(y) = \varphi$ for all y ∈ Y , then f fails to be a map. Then there exists only one point y ∈ Y such that $f^{-1}(y) \neq \varphi$ and hence $f^{-1}(y) = X$. This shows that f is a constant map.

(iii) ⇒ (ii): Let O be both rg*b -open and rg*b -closed in X. Suppose O ≠ φ. Let f : X → Y be a rg*b -continuous map defined by f(O) = y and $f(O^C) = \{w\}$ for some distinct points y and w in Y . By assumption f is constant. Therefore we have $O = X$.

Theorem: 4.6 If f: X → Y is a rg*b -continuous and X is rg*b -connected, then Y is connected.

Proof: Suppose that Y is not connected. Let Y = A∪B where A and B are disjoint non-empty open sets in Y . Since f is rg*b -continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty rg*b open sets in X. This contradicts the fact that X is rg*b -connected. Hence Y is connected.

Theorem: 4.7 If f: X → Y is a rg*b -irresolute surjection and X is rg*b connected, then Y is rg*b -connected.

Proof: Suppose that Y is not rg*b -connected. Let Y = A ∪ B where A and B are disjoint non-empty rg*b -open sets in Y. Since f is rg*b -irresolute and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty rg*b -open sets in *X*. This contradicts the fact that *X* is rg*b -connected. Hence *Y* is rg*b connected.

Theorem: 4.8 In a topological space (X, τ) with at least two points, if bO(X, τ) = bCL(X, τ) , then X is not rg*b -connected.

Proof: By hypothesis we have bO(X, τ) = bCL(X, τ) and we have every b closed set is rg*b –closed (By [10]), there exists some non-empty proper subset of X which is both rg*b -open and rg*b -closed in X. So by last Theorem 4.5 we have X is not rg*b -connected.

Definition: 4.9 A topological space X is said to be rg*b -space if every rg*b closed subset of X is closed subset of X (By [12]).

Theorem: 4.10 suppose that X is a rg*b -space then X is connected if and only if it is rg*b -connected.

Proof: Suppose that X is connected. Then X cannot be expressed as disjoint union of two non-empty proper subsets of X. Suppose X is not a rg*b -connected space. Let A and B be any two rg*b -open subsets of X such that X = A∪B, where A ∩ B = φ and A ⊂ X, B ⊂ X. Since X is rg*b -space and A, B are rg*b -open, A, B are open subsets of X, which contradicts that X is connected. Therefore X is rg*b -connected.

Conversely, every open set is rg*b -open. Therefore every rg*b -connected space is connected.

Theorem: 4.11 If the rg*b -open sets C and D form a separation of X and if Y is rg*b -connected subspace of X, then Y lies entirely within C or D.

Proof: Since C and D are both rg*b -open in X the sets C ∩ Y and D ∩ Y are rg*b -open in Y. These two sets are disjoint and their union is Y. If they were both non-empty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or in D.

Theorem: 4.12 Let A be a rg*b -connected subspace of X. If A ⊂ B ⊂ rg*b Cl(A) then B is also rg*b -connected.

Proof: Let A be rg*b -connected and let A ⊂ B ⊂ rg*b Cl(A). Suppose that B = C∪D is a separation of B by rg*b-open sets. Then by Theorem 4.11 above A must lie entirely in C or in D. Suppose that A ⊂ C, then rg*b Cl(A) ⊆ rg*b Cl(C). Since rg*b Cl(C) and D are disjoint, B cannot intersect D. This contradicts the fact that D is non-empty subset of B. So D = φ which implies B is rg*b connected.

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