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TWO-LAYERED MODEL FOR BLOOD FLOW INSIDE INCLINED TAPERED ARTERY WITH NON-SYMMETRIC STENOSIS AND VELOCITY SLIP

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ABSTRACT

The paper deals with a two-layered blood flow model in an inclined tapered artery. In the model core region consists of red blood cell suspension and plasma is in the peripheral layer of the tube. In this paper, the flow of blood is assumed to be steady, laminar, axi-symmetric and,the body fluid blood behaves as a Newtonian fluid. Analytical expressions are obtained for axial velocity, flow rate and wall shear stress at the interface of stenotic region. Their variations with different flow parameters are plotted graphically and the behavior of these flow variables in this region, has been discussed. The present analysis includes some blood flow models as it special cases. It could be noticed that an employment of slip at interface may result in accelerating the speed and flow rate and, decelerate the wall shear stresses. Biological implications of this modeling are reported in brief.

Keywords: *Newtonian fluid, Inclined tapered artery, slip velocity, Non-symmetric stenosis and blood.*

1. INTRODUCTION

In recent years, there could be noticed an increasing interest in the pressure-flow relationship which may characterize the flow of blood through tapered blood vessels. The complex geometry of arteries *viz.*,tapering, inclining, bending, bifurcation, stenosis, etc., seems to be important, as this obviously affects the local hemodynamics. As such, there remains no doubt about the fact that tapering is a significant aspect of mammalian arterial system. Many researchers have investigated the flow of body fluid blood through uniform circular tubes (Chaturani and Biswas, 1984, Biswas, 2000). Jeffords and Knisely (1956) and, Bloch (1962) have suggested that all vessels that carry blood towards the tissues, should be considered long, narrow and slowly tapering cones rather than cylinders. Many, theoretical, numerical and experimental works of flow in uniformly tapering vessels have appeared in literature. The solution for the steady creeping flow of an incompressible Newtonian fluid through conical section has been obtained by several investigators (Harrison, 1916, Vand, 1948, Oka, 1963). The pressure development in such vessels is related to the volumetric flow rate and the angle of tapering. Targ (1951) and, Oka and Nishimura (1963) extended the solution to the case wherein the inertial effects can be ignored. It is well known that in physiological systems arteries are not all horizontal but have some inclination with axis. The gravitational force is accounted due to the consideration of inclined tube.

At the high shear stress during the flow in narrow tubes, blood behaves as a Newtonian fluid. The experimental observation of Cokelet *et al*. (1972) and theoretical investigation of Haynes (1960) indicate that blood cannot be treated as an one-layered homogeneous fluid while flowing in narrow tubes i.e. diameter ≤ 1000 µm.Skalak (1972) has been reported that the accurate description of the flow of blood through narrow tubes needs the consideration of erythrocytes as discrete particles. Srivastava and Srivastava (1983) have concluded that the blood flow can be represented accurately by a macroscopic two-layered model.

Cerny and Walawender (1966) have presented several solutions to the problem of viscous flow of an incompressible Newtonian liquid in a converging tube. Their solutions include the pressure-flow relations for cases for pulsatile and steady flows in a rigid tube and, pulsatile flow in an elastic tube. Walawender *et al.* (1972) have put forward simplified solutions for limiting cases of very slow and very fast flow for a Newtonian fluid in a uniformly tapering tube. Charm and Kurland (1965) have derived a pressure-flow relationship for Newtonian flow in a tapered tube and, claimed that their result is identical with that presented by Vand (1948). Verma and Parihar (2010) have attempted to study the effect of stenosis and hematocrit on flow variables for Newtonian fluid flow through a tapered artery. Experimentally, only a limited amount of work on blood flow in tapered tubes has been reported (Merril *et al*., 1965, Walawender*et al.* 1972, Walawender and Chen, 1975). Also, the whole blood data for Newtonian fluid in 1^0 and2^0 tapered tubes, as suggested by Walawender *et al*., (1972) agree very well with their simplified pressure flow predictions, with substantial inertia loses. The aim of this paper is to study the flow of blood (Newtonian fluid) through narrow cylindrical tapered tubes.

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It is now well known that under diseased conditions, an abnormal and unnatural growth (stenosis) develops in the lumen at various locations of cardiovascular system (CVS). Arteriosclerosis or stenosis is one of the most wide spread diseases. Although, an exact cause behind such unfamiliar formation is not quite clear, however, deposition of fat, cholesterol or lipid in vessel wall may give rise to stenosis which causes blockage to normal blood supply. Recently, considerable attention is noticed to study the blood flow characteristics in stenosed arteries (Sukla *et al.* (1980). Young (1968) analyses theoretically the effects of stenosis on flow characteristics of blood and has concluded that resistance to flow (impedance) and wall shear stress increase with an increase of stenosis size. Forrester and Young (1970) have extended the theory of Young (1968) to include effects of flow separation on a mild stenosis. MacDonald (1979) has proposed a mathematical model for blood flow in stenosed tubes, by assuming that blood behaves as a Newtonian fluid. Liu *et al*. (2004) has established the flow of blood (Newtonian) in a tapered artery with an axially symmetric stenosis. In our present study, blood flow in a tapered artery with a mild stenosis at the wall, is considered.

A number of studies of suspensions in general and blood flow in particular, both theoretical and experimental, have reported the likely presence of slip at vessel wall (Biswas, 2000, Chaturani and Biswas, 1984). Steady flow of blood through a catheterized tapered artery with stenosis and a velocity slip at wall, is dealt by Biswas and Chakraborty (2009). Recently, blood flow through a stenosed artery with an axial velocity slip at vessel wall, is examined by Biswas and Laskar (2011). The present work is concerned with the theoretical study on two-layered blood flow with peripheral plasma layer and core region of red cell suspension in an inclined tapered tube of small angle of taper and with the formation of a mild asymmetric stenosis along the tapering wall. A slip condition at the interface of fluids is employed in this steady, laminar and Newtonian fluid flow model from the realistic point of view.

2. BASIC EQUATIONS

The basic equations of the fluid flow include the conservation equations of mass and momentum and, the constitutive equation of the fluid. The equation of continuity is given by

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.\left(\rho \vec{V}\right) = 0, \tag{1}$$

and the equation of motion is

$$\rho\left[\frac{\partial \vec{v}}{\partial t} + \left(\vec{\nabla}.\vec{V}\right)\vec{V}\right] = \rho \vec{f} - \vec{\nabla} P + \mu \nabla^2 \vec{V}, \tag{2}$$

whereρ is the density of the fluid, $\vec{\nabla}$ the vector differential operator, t the time, $\vec{V}$ the usual velocity field, μ the viscosity of the fluid and $\vec{f}$ the body force per unit mass.

For an incompressible fluid, the equation of continuity reduces to

$$\vec{\nabla}.\vec{V} = 0 \tag{3}$$

In presence of body force and the inertial term, the equation of motion (2), (using equation (3)) reduces to

$$\vec{\nabla} P = \mu \nabla^2 \vec{V} + \rho \vec{f} \tag{4}$$

For one-dimensional flow, fluid velocity has the form $\vec{V} = (0,0,u(r))$ and $\vec{f} = (0,0,f_z)$ in cylindrical co-ordinates (r,θ,z) representing radial, circumferential and axial co-ordinates; the vector equation (4) reduces to the scalar equations

$$\frac{\partial p}{\partial r} = 0, \tag{5}$$

$$\frac{\partial p}{\partial \theta} = 0, \tag{6}$$

$$\frac{\partial p}{\partial z} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \rho f_z \tag{7}$$

where, $u = u(r)$ is the axial velocity and μ is the viscosity of blood.

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It is seen from the above equations (5--7) the pressure gradient is the function of z only,

So, the equation (7) becomes

$$C+\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\rho f_z=0, \text{where}, C=-\frac{dp}{dz}. \tag{8}$$

3. FORMULATION OF THE PROBLEM

For the development of a mathematical model in the present analysis, following assumptions are made:-

(i) Blood is assumed to behave as Newtonian fluid. (ii) Motion of the fluid is laminar and steady (iii) The inertia term is neglected as the motion is unidirectional (iv) No body force acts on the body (v) The axis of symmetry of the artery is taken as z-axis (vi) Mild asymmetric stenosis is formed (vii) Cylindrical co-ordinates (r,θ,z) are taken.

Consider a two layered blood flow in a tapering constricted artery, inclined at an angle ξ to the horizontal plane. The core region is considered as red blood cell suspension in the middle layer and the peripheral region considered as plasma in the outer layer. In the model, at the innermost vessel wall ofthe artery, non-symmetric stenosis has been considered. The flow of blood is assumed to be steady, laminar and axi-symmetric and the body fluid is considered as Newtonian in nature. The model consists of Newtonian fluid of viscosities μ_1 and μ_2 in the core and plasma regions

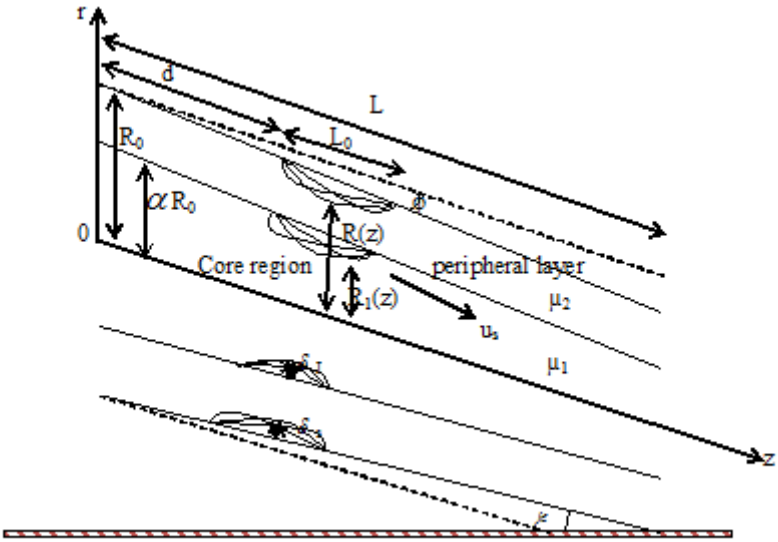

Fig1: Schematic diagram of an inclined Tapered Artery with non-symmetric stenosis.

where R_0 is the radius of non-tapering portion of the artery, R(z) and $R_1(z)$ are the critical heights of stenosis of peripheral layer and core region respectively.

The physical models of the tapered artery with non-symmetric stenosis can be written mathematically for peripheral ($R_1(z)\le r\le R(z)$) layer (Mekheimer and Kothari, 2008) as

$$\frac{R(z)}{R_0}=\begin{cases}(1-mz)\left[1-A\delta_s\left\{L_0^{\,n-1}(z-d)-(z-d)^n\right\}\right], & d\le z\le d+L_0\\ 1-mz, & otherwise\end{cases}, \tag{9}$$

and for core region ($0\le r\le R_1(z)$) is

$$\frac{R_1(z)}{R_0}=\begin{cases}(\alpha-mz)\left[1-A\delta_i\left\{L_0^{\,n-1}(z-d)-(z-d)^n\right\}\right], & d\le z\le d+L_0\\ \alpha-mz, & otherwise\end{cases}, \tag{10}$$

where $m=\frac{\tan\varphi}{R_0}$, $\delta_s=h\sec\varphi$, is the maximum height from stenosis in the peripheral layer and $\delta_i=h_i\sec\varphi$, is the maximum height from stenosis in the core region, φ is the angle of tapering and $A=\frac{n^{\frac{n}{n-1}}}{R_0L_0^{\,n}(n-1)}$.

For steady and Newtonian fluid, the governing equations (8) becomes for inclined tapering artery

$$C+\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\rho g\ \sin\xi=0, \tag{11}$$

where $C=-\dfrac{dp}{dz}$,

whose integral results as

$$u(r)=-\ \frac{r^2}{4\mu}(C+\rho g\sin\xi)+K\ \ln r+B,\quad 0\le \mathrm{r}\le \mathrm{R(z)}, \tag{12}$$

where, K and B are integrating constants. The velocity function (12) reduces to the form for core region as

$$u_1(r)=-\ \frac{r^2}{4\mu_1}(C+\rho g\ \sin\xi)+K_1\ \ln r+B_1,\quad 0\le \mathrm{r}\le \mathrm{R_1(z)}, \tag{13}$$

and for peripheral layer

$$u_2(r)=-\ \frac{r^2}{4\mu_2}(C+\rho g\ \sin\xi)+K_2\ \ln r+B_2,\quad \mathrm{R_1}\le \mathrm{r}\le \mathrm{R}, \tag{14}$$

where $K_1,\ K_2,\ B_1,\ B_2$ constants of integration. Their values can be evaluated by employing the following boundary conditions:

$$\left.\begin{array}{lll}
(i)\ \ u_2=0,\quad at\ \ r=R(z) & \text{(no slip at the boundary wall)} \\
(ii)\ u_1-u_2=u_s,\ at\ \ r=R_1(z) & \text{(slip at the interface region)} \\
(iii)\ \dfrac{\partial u_1}{\partial r}=0,\quad at\ \ r=0 & \text{(symmetry condition)} \quad . \\
(iv)\ \dfrac{\partial u_2}{\partial r}=0,\quad at\ \ r=0 & \text{(symmetry condition)}
\end{array}\right] \tag{15}$$

As a result of employing conditions (15) in the solutions (13 – 14), we obtain, the expression for axial velocities in core and ppl regions as

$$u_1(r)=u_s+\frac{C+\rho g\sin\xi}{4\mu_2}\left\{\mu_2'\left(R_1^{\,2}(z)-r^2\right)+R^2(z)-R_1^{\,2}(z)\right\},\quad 0\le \mathrm{r}\le \mathrm{R_1}(z), \tag{16}$$

and

$$u_2(r)=\frac{C+\rho g\sin\xi}{4\mu_2}\left(R^2(z)-r^2\right),\quad R_1(z)\le r\le R(z), \tag{17}$$

where,

$$\mu_2'=\frac{\mu_2}{\mu_1}$$

The total volumetric flow rate Q is defined by

$$Q=Q_1+Q_2, \tag{18}$$

where,

$$Q_1=2\pi\int_{r=0}^{R_1} r u_1(r)\ dr, \tag{19}$$

and

$$Q_2=2\pi\int_{r=R_1}^{R} r u_2(r)\ dr \tag{20}$$

These can be computed with the help of equations (16—17) in the form

$$Q = \pi R_1^{\,2}(z)u_s + \frac{\pi(C+\rho g\sin\xi)}{8\mu_2}\left\{R^4(z) - (1-\mu_2')R_1^{\,4}(z)\right\} \tag{21}$$

The expression for pressure gradient term can be obtained from equation (21) as

$$\frac{dp}{dz} = \frac{8\mu_2\left(R_1^{\,2}(z)u_s - \frac{Q}{\pi}\right)}{R^4(z) - (1-\mu_2')R_1^{\,4}(z)} + \rho g\sin\xi \tag{22}$$

The expressions for wall shear stresses at the vessel wall inperipheral layerand interface of fluid in core region, can computed with the help of equations (16) and (17) as

$$\tau_{R(z)} = \frac{C}{2}R(z) + \frac{\rho g\sin\xi}{2}R(z), \tag{23}$$

$$\tau_{R_1(z)} = \frac{C}{2}R_1(z) + \frac{\rho g\sin\xi}{2}R_1(z) \tag{24}$$

4. SOLUTIONS

A second representation of analytic expressions for flow variables are obtained in the following:

The expression for axial velocity for core region is obtained, using equation (16) as

$$\overline{u_1} = \overline{u}_s + \left(\overline{C} + \frac{\sin\xi}{F}\right)\left\{\mu_2'\left(\overline{R_1}^2 - \overline{r}^2\right) + \left(\overline{R}^2 - \overline{R_1}^2\right)\right\}, \tag{25}$$

and for peripheral layer as(using equation (17))

$$\overline{u_2} = \left(\overline{C} + \frac{\sin\xi}{F}\right)\left(\overline{R}^2 - \overline{r}^2\right) \tag{26}$$

The flow rate is obtained with the help of equation (21) as

$$\overline{Q} = 2\overline{R_1}^2\overline{u_s} + \left(\overline{C} + \frac{\sin\xi}{F}\right)\left\{\overline{R}^4 - R_1^{\,4}(1-\mu_2')\right\} \tag{27}$$

The expression for pressure gradient computed from equation (22) as

$$\overline{\left(\frac{\mathrm{dp}}{\mathrm{dz}}\right)} = \frac{\sin\xi}{F} + \left(\overline{Q} + 2\overline{u}_s\overline{R}_1^{\,2}\right)\left[\overline{R}^4 - (1-\mu_2')\overline{R}_1^{\,4}\right]^{-1} \tag{28}$$

The wall shear stresses for both ppland core regions, can be obtained as (using equations (23--24))

$$\overline{\tau_{R(z)}} = \left(\overline{C} + \frac{\sin\xi}{F}\right)\overline{R(z)}, \tag{29}$$

$$\overline{\tau_{R_1(z)}} = \left(\overline{C} + \frac{\sin\xi}{F}\right)\overline{R_1(z)} \tag{30}$$

The non-dimensional variables employed are

$$\overline{\mathrm{R}}(z) = \frac{R(z)}{R_0},\ \overline{\mathrm{R}}_1(z) = \frac{R_1(z)}{R_0},\ \overline{\mu}_a = \frac{\mu_a}{\mu_2},\ \mu_2' = \frac{\mu_2}{\mu_1},\ \overline{u}_1 = \frac{u_1}{u_0},\ \overline{u}_2 = \frac{u_2}{u_0},\ u_0 = \frac{CR_0^{\,2}}{4\mu_2},\ \mathrm{Q}_0 = \frac{\pi CR_0^{\,4}}{8\mu_2},\ \left(\frac{\mathrm{dp}}{\mathrm{dz}}\right)_0 = -\frac{8\mu_2 Q_0}{\pi R_0^{\,4}},\ \lambda_0 = \frac{8\mu_2 L}{\pi R_0^{\,4}} \tag{31}$$

5. RESULTS AND DISCUSSIONS

In our cardiovascular system, blood flow through different kinds of blood vessels which are uniform and non-uniform. Further, among different flow geometries, many blood vessels are tapering and inclined. Also it is reported that blood while flowing through narrow blood vessel, leaves a cell poor region near the vessel wall. Therefore a two-layered blood flow with peripheral plasma layer (ppl) and a core of red cell suspension may be realistic here. Also it may be physically important to investigate blood flow through inclined tapered artery with constriction. In view of above,

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steady laminar two-layered flow of blood (Newtonian fluid) through an inclined tapered vessel with the growth of a non-symmetrical stenosis, is considered here (Fig1). It may be noticed that the initiation and localization of stenosis, are closely related to local hemodynamic factors, like axial velocity, flow rate, shear stresses, etc. which in their dimensionless forms, are obtained in equations (25-30). It may also be observed that velocity is a function of shear viscosity μ_1 and μ_2, pressure gradientdp/dz , radius of tapered constricted arterial segment in both regions, slip velocity u_s, radial co-ordinate r and the length of the artery L. Moreover, the tapering radii R = R(z), $R_1(z)$, (in equations (9-10)) depends on the axial distance z, R_0 the radius, length of the asymmetric stenosis is L_0 at the location d with the maximum heights δ_s and δ_i in core and ppl regions respectively, n, the parameter of stenosis shape and φ, is the angle of tapering. In the present model, the following cases can be included:

(a) when $R(z) = R_1(z) = R_0$, $C = C_0$, $\tau = 0, \xi = \varphi$, it becomes one-layered flow and also it reduces to Poiseuille flow model with slip ($u_s > 0$) and zero slip ($u_s = 0$) at the wall, (b) when $\varphi > 0$ or $\varphi < 0$, yields to tapered model of diverging and converging respectively in unobstructed ($\delta = 0$) and Stenosed ($\delta > 0$) regions with slip ($u_s > 0$) at the interface of fluids and no slip ($u_s = 0$) at the outer layer wall, (c) when $R(z) = R_1(z) = R_0$, $\tau \neq 0$, $\varphi = 0$, it becomes one-layered model for one-dimensional (1-D) flow in a uniform Stenosed artery, (d) when R(z) and $R_1(z) \neq R_0$ and τ, φ are all non-zero, $u_s > 0$ at the interface, it represents the present modeling for two-layered blood flow.

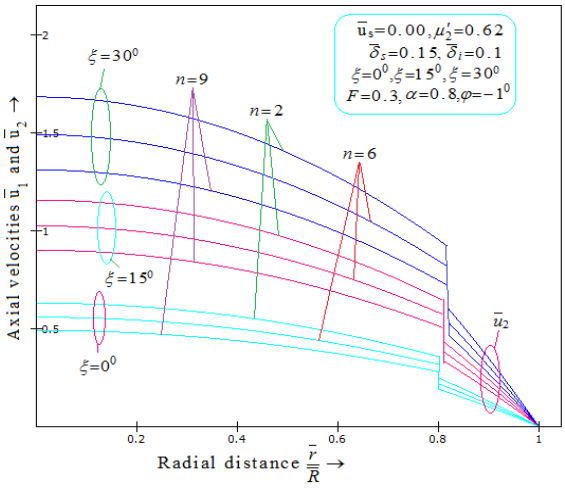

Fig 2: Variation of axial velocity against radial distance for different inclination and tapering.

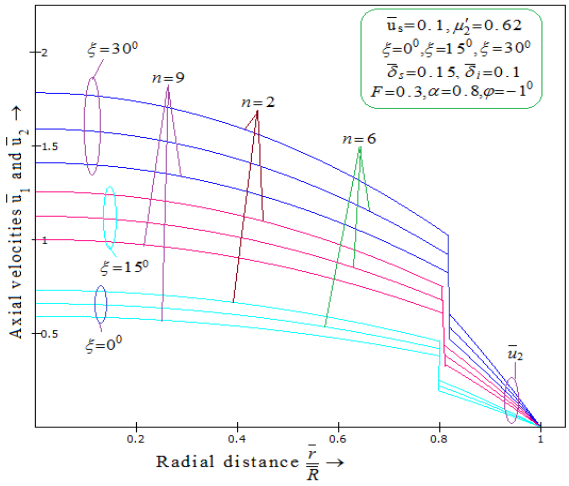

Fig 3: Variation of axial velocity against radial distance for different inclination and tapering.

In the analysis, the combined influence of some parameters has been established, in case of non-tapering, convergent tapering, divergent tapering of arterial segments. Further, analysing the quantitative effect of non-symmetric stenosis shape parameter n (≥ 2), effective radii R(z) and $R_1(z)$, maximum heights attained by stenosises δ_s and δ_i, slip velocity u_s (> 0) at the interface of fluids in the constricted region, angle of inclination ξ (> 0), tapering angle φ (> 0) and

Newtonian behaviour of blood etc., computer codes have been developed for the numerical computations of the analytic forms obtained for axial velocities u_1 and u_2 , flow rate and wall shear stresses τ_{R1} at the interface of fluids, for the parameter values α = 0.8 (core region), μ_2 = 2cp, μ_1= 1.2cp, δ_s=0.00-0.15, δ_i =0.00-0.12, Q =0.5, 1.0, 1.5, pressure gradient C =0.5, 1.0, 1.5(Verma and Parihar, 2009), m = tanφ = -1^0, 0^0, 1^0, $\xi = 0^0, 15^0, 30^0$, n =2.0, n =6.0, n =9.0, z = d to d + L_0, and $R_1(z) \leq r \leq R(z)$ for ppl and $0 \leq r \leq R_1(z)$ for core region. On the basis of abovementioned analysis, an effort has been taken to indicate the variations in flow, for such parameters.

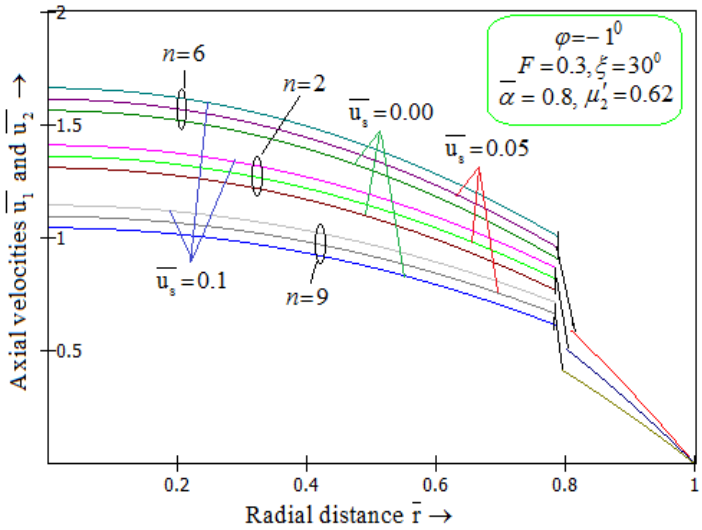

Fig 4: Variation of axial velocity against radial distance for different slip.

The variations of axial velocities u_1 and u_2 with different slip at the interface of fluids and inclination are obtained from equations (25-26), exhibited in the Figs (2-4). Figures (5-8) show variation of flow rate Q (obtained from equation (27)) against axial distance z, stenosis heights, slip or no slip, three parameters n of stenosis in the constricted region of interface. Figures (9-11) show variation of shear stress τ_{R1} at the interface of fluids against axial distance obtained from equation (30) for different parameters.

It is noticed that the axial velocity profiles that have been obtained from the equations (25-26), for slip and no-slip cases with different inclinations, heights of stenosis and different axial locations is shown in the Figs2-4. The observations can be recorded for some differences in shape of profiles is obtained by employing inclination or non-inclination of tapering arteries with slip or no slip at the interface in the constricted region. In case of zero slip and slip, velocity indicates a parabolic trend from tube axis to interface and beyond the interface up to vessel wall in the outer layer, the trend in profiles remain the same. From the figure, it is revealed that all three parameters of n(=2,6,9) of asymmetric stenosis, magnitude of axial velocity increases with increase in the inclination of the artery and velocity slip at the stenotic region of the interface of fluids and It is noticed that the maximum velocity attains at the tube axis and minimum at the vessel wall and also the increasing the heights of stenosis the velocity becomes lower, for both slip and no-slip conditions at the interface. As axial co-ordinate z at the stenotic region increases from either end to the throat of asymmetric stenosis, velocity decreases from its greatest value at the maximum tubular area to a minimum at the minimum constricted area, and therfore, velocity is maximum at the beginning of the stenosis and minimum at the throat of the stenosis.

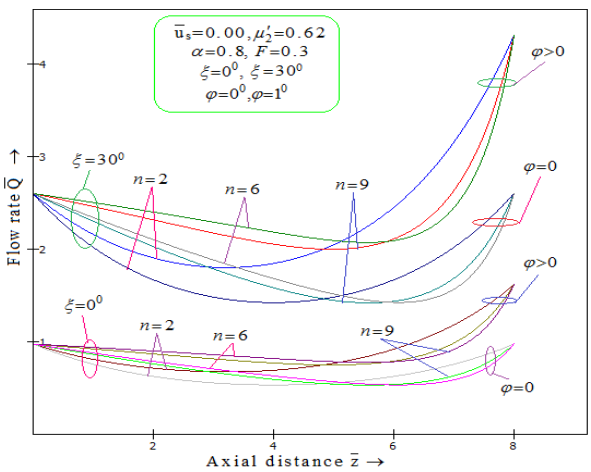

Fig 5: Variation of flow rate against axial distance for different inclination and tapering with zero slip.

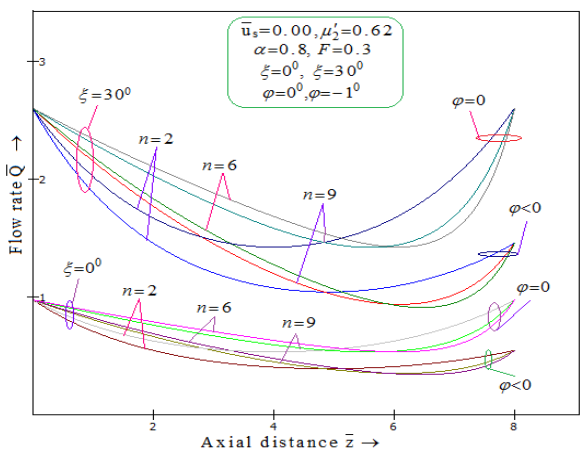

Fig 6: Variation of flow rate against axial distance for different inclination and tapering with zero slip.

It could be noticed in the profile Fig 5-8, the variation of flow rate against axial distance z, for different parameters such as stenosis heights in both ppl and core regions, slip and zero slip and inclination of the artery. It appears that the trend of profile is non-linear. Flow rate magnitude is minimum at the throat of stenosis. As slip increases, magnitude of flow rate also increases. It is also noticed that the magnitude of flow rate increases with increase in the inclination of the artery.

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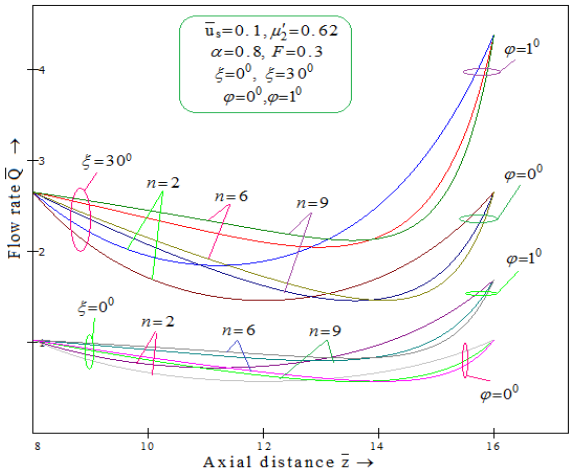

Fig 7: Variation of flow rate against axial distance for Different inclination and tapering with slip.

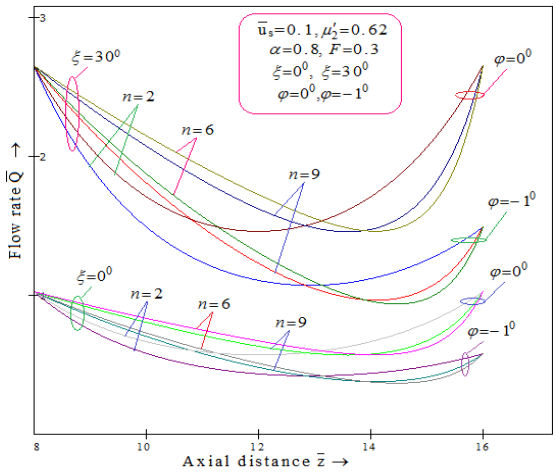

Fig 8: Variation of flow rate against axial distance for different inclination and tapering with slip.

However, magnitude of flow rate with velocity slip is greater than that of zero slip at the interface for all cases of symmetric and non-symmetric stenosis as well as for inclination of the artery also. It is also noticed that the flow rate is more influenced by tapering, the flow rate increases with divergent tapering ($\varphi > 0$), it is worth to mentioned here that the flow rate is lower at the initial of stenosis (at location d) than that of final end of stenosis (at location d+ L_0) for divergent tapering. Further, for uniform artery both end point of stenosis (at z =d and z =d + L_0) is constant where as it varies with axial co-ordinate in constricted region. But for convergent tapering ($\varphi < 0$), the flow rate decreases, the flow rate is higher at the initial of stenosis (at location d) than that of final end of stenosis (at location d+ L_0).

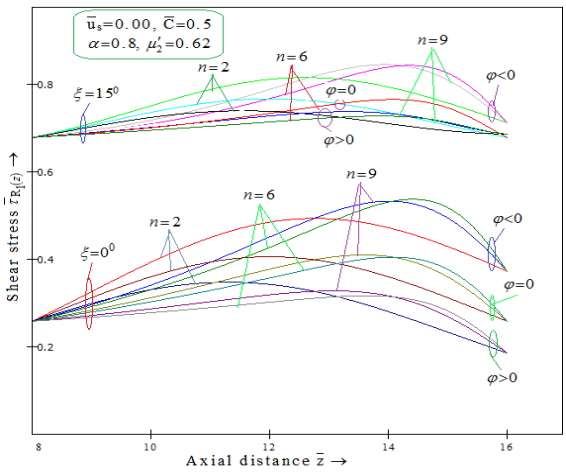

Fig 9: Variation of shear stress at the interface against axial axial distance fordifferent inclination and tapering angles.

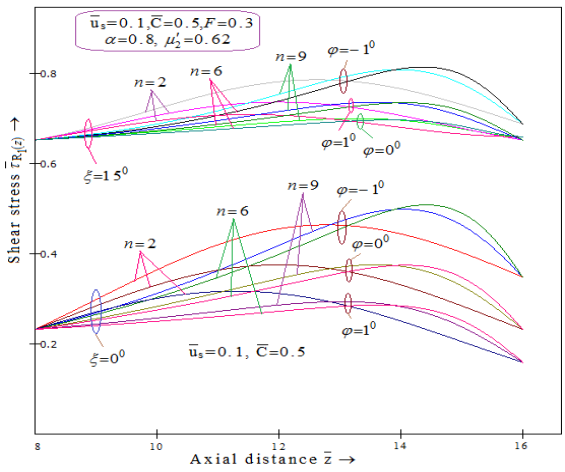

Fig 10: Variation of shear stress at the interface against distance for different inclination and tapering angles.

The variation of wall shear stress plotted against axial distance z (d$\leq$z $\leq$ d+L_0) for different parameters in tapering region as computed from equation (30) are exhibited in Figs (9-11). It could be noticed in profile (Figs 9-11), for fixed values δ_s (= 0.15), δ_i (=0.12), C (= 0.5) and C (= 0.1) that as n (= 2,6,9) increases, wall shear stress decreases in full scale of z (d =8 to d +L_0/2), these profiles exhibit a non-linear trend all throughout the constricted region, maximum magnitude is seen to attain at the throat of the constriction for symmetric stenosis (at n =2) and at very near the termination of constriction for asymmetric stenoses (n =6,9). As slip increases, the shear stress decreases, with increase in inclination, wall shear stressincreases, however, in all variation considered, it is found that τ_{R1} (at divergent tapering) < τ_{R1} (at non-tapering) < τ_{R1} (convergent tapering). It is also found at the throat the $\tau_{R1}|_{n=2}<\tau_{R1}|_{n=9}<\tau_{R1}|_{n=6}$.

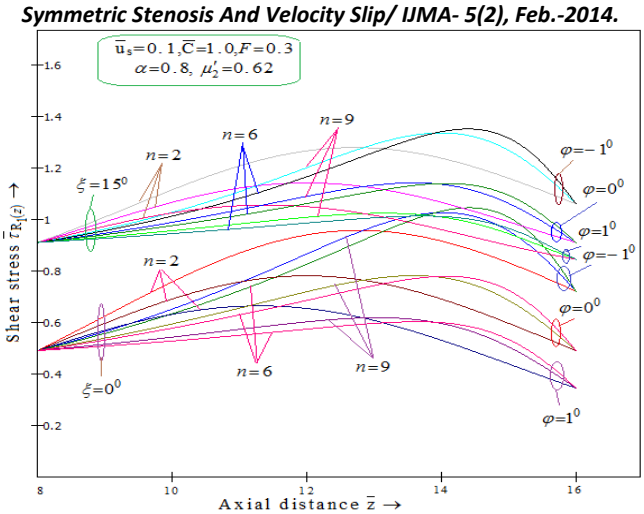

Fig 11: Variation of shear stress at the interface against axial distance for different slip

6. CONCLUSION

In the present investigation, steady and laminar flow of blood (Newtonian fluid) in an inclined tapered artery, in the presence of an axially symmetric but radially non-symmetric mild asymmetric stenosis, is considered (Fig 1). Analytical expressions for flow variables are obtained and the variations of axial velocities of both regions, rate of flow and shear stresses at the interface of fluids in the constricted region etc. have been presented graphically (Fig 2-11). The present analysis contains Poiseuille flow of blood, one dimensional blood flow models through uniform or tapered arteries with or without constriction, with slip or no slip cases, for straight or inclined geometries,are considered as its special cases.

In figs (2-4), the variation of axial velocity vs. radial distance in both ppl and core regions for different axis locations are exhibited. It is revealed that in the ppl region, profile is parabolic in nature, but in core region, deviations from parabolic trend are noticed. As z increases in stenotic region, velocity decreases from higher to lower values in both ppl and core regions. As slip velocity and angle of inclination increase, magnitude of axial velocity also increases at the core of the stenotic region. But in outer layer, velocity increaseswith increase in angle of inclination and velocity slip has no effect. It is noticed that the maximum velocity attains at the tube axis and minimum at the vessel wall.

It is reflected from Figs (5-8), the variation of flow rate against axial distance z, trend of profiles are non-linear in nature in constricted region. As z increases from initiation of stenosis to its throat, flow rate decreases from a higher magnitude to a lower value, however its magnitude at the initiation of stenosis is always higher than that of its termination, flow rate is minimum at the stenotic throat. As slip velocity increases flow rate also increases accordingly for fixed value of n (=2,6,9). Also inclination and tapering is an influential factor of flow rate, as for divergent tapering, the magnitude of flow rate higher than that of non-tapering and, convergent tapering and the magnitude of flow rate of an inclined artery is higher than that of non-inclined artery.

From Figs (6-7), It is revealed that wall shear stress at the interface of fluids in the stenotic region increases rapidly from its approached value of z =d (at initiation) and attains its maximum at the throat of stenosis (at z =d + $L_0/2$) for symmetric stenosis n =2 and for asymmetric constriction (at n =6, 9), the maximum value of τ_{R1} is reached near the end (z =d +L_0) of stenosis. There form, it decreases rapidly in the downstream, attaining minimum value at the termination (z =d +L_0) of constriction. Further, wall shear stress decreases with increasing shape parameter n in the upstream of the throat of constriction but this property reverses in the downstream. Also, τ_{R1} increases for convergent tapering ($\varphi < 0$), τ_{R1} attains maximum as compared to diverging tapering tube ($\varphi > 0$) and without tapering tube (φ =0). It may be accounted also, the τ_{R1} increases with increase in inclination of artery. Wall shear stress at the interface increases with an increase in taper angle which is in agreement with Chaturani and Prahlad's model (1985). Also, τ_{R1} increases with stenosis size which is in agreement with Young's model (Young, 1968).

In this investigation, it is easily seen that the consideration of inclined tapered artery with asymmetric stenosis, could provide better results than that of symmetric stenosed non-inclined tapered artery. Further it is found that employment of velocity slip at the interface of fluids in the constricted tapering region, both axial velocity and flow rate increases but τ_{R1} reduces. But, for increase in inclination, axial velocities, flow rate and shear stress increases also. Therefore, blood flow can be accelerated due to slip and inclination and resistance to flow could be lowered. So slip and inclination might play a significant role in blood flow modeling. Apart from this, damage to the vessel wall could be lowered at one hand and normal blood flow can be restored on the other. So that fatalityor deaths due to stenosis formation and its gradual development, could be lowered. Further, in the blood flow modeling, an accurate and exact measure in accordance with physiological situations is quite important. The present theoretical analysis may be used as a device or tool for such cases.

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