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TWO-LAYERED MATHEMATICAL MODEL FOR BLOOD FLOW INSIDE AN ASYMMETRIC STENOSED ARTERY WITH VELOCITY SLIP AT INTERFACE

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ABSTRACT

The present paper deals with a mathematical model for blood flow through an asymmetric stenosed artery with velocity slip at the interface. The model consists of a core region of red blood cell suspension in the middle layer and the peripheral plasma layer in the outer region. It is assumed that both the core and the peripheral plasma layer are represented by a Newtonian fluid with different viscosities μ_1 and μ_2 respectively. Analytical expressions are obtained for axial velocity, flow rate, wall shear stresses and pressure gradient. Their variations with different parameters are plotted in figures. The behaviour of these flow variables has been discussed here. Physiological implications of this theoretical modeling to blood flow situations, are also discussed in brief

***Keywords**: Two-layered model, blood flow, asymmetric stenosis, velocity slip, shear stress.*

1. INTRODUCTION

An abnormal growth reducing the lumen of an artery is usually called a stenosis or atherosclerosis (Young 1968, Sankar and Lee 2009, Biswas and Chakravorty 2009) which is one of the most widespread diseases that can result in serious circulatory disorders, by reducing or occluding the blood supply. As for instance, stenosis in arteries supplying blood to brain, can cause cerebral strokes, likewise its presence in coronary arteries can bring about myocardial infarction, leading to a heart failure (Sinha and Singha 1984). Bugliarello and Sevilla (1970) and Cocklet (1972) have experimentally proved that for blood flowing through small vessels, there exists a cell –poor region plasma (Newtonian fluid) and a core region of suspension of almost all the erythrocytes. Bugliarello and Sevilla (1970) presented flow of blood in small diameter tubes by a 2-layered model assuming peripheral and core fluids as Newtonian fluids of different viscosities. Two-layered blood flow models from different flow situations have been considered by Chaturani and Upadhya (1979), Chaturani and Biswas (1984), Srivastava (2002).

It is reported by many authors (Chaturani and Kaloni, 1976; Chaturani and Upadhya, 1979; Chaturani and Biswas, 1983; Philip and Chandra, 1996) who have theoretically studied the flow of blood through uniform and stenosed tubes and analyzed the influence of slip velocity or peripheral plasma layer thickness on the flow variables such as velocity, wall shear stress and flow resistance. To understand the flow patterns in stenosed arteries, Young (1968), Macdonald (1979), Shankar and Hemalatha (2006) etc. have analyzed the flow of blood through an arterial stenosis. Chaturani and Kaloni (1976), Chaturani and Ponalagusamy (1982), Sankar and Ismail (2009), Sankar and Lee (2009) and Ponalagusamy (1986)) have considered the flow of blood represented by a two-layered model. In their models, the peripheral layer thickness is assumed as priori. They have studied that velocity and flow rate increase but effective viscosity decreases due to wall slip.

Maruti Prasad and Radhakrishnamcharya (2008), Chakraborty *et al*. (2011) have proposed blood flow through an inclined tube with stenosis. Recently Biswas, D. and Paul, M. (2013) have proposed a mathematical model for blood inside an inclined non-uniform stenosed artery. However, among all the arteries present in a human body, generally many of them are uniform in shape. However, due to their non-uniformity for the presence of stenosis at the vessel wall, there arises a variation in the pressure gradient in blood flow.

It is seen that an abnormal growth of stenosis at the arterial wall is mostly non-symmetrical. With the above motivation, an attempt has been made to study the effects of slip (at the asymmetric stenosis) and the influence of flow variables (wall shear stress, velocity, flow rate and pressure gradient) for 2-layered steady laminar blood flow through an asymmetric constricted vessel with velocity slip at interface.

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2. FLOW GEOMETRY

A two- layered model for blood flow with an asymmetric stenosis at the vessel wall has been developed. The model basically consists of a core of red blood cell suspension in the middle layer and the peripheral plasma layer (PPL) in the outer layer (as shown in Fig.1). It is assumed that both the core and the peripheral plasma layer are represented by a Newtonian fluid with different viscosities μ_1 and μ_2 respectively.

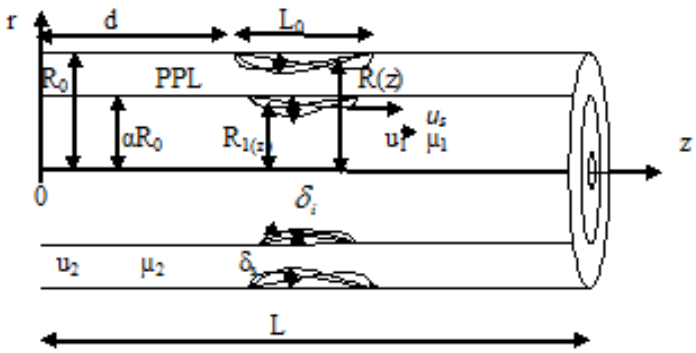

Fig.1: Flow Geometry with asymmetric shape of stenosed artery in a two- layered model.

The geometry of the stenosis which is developed in the arterial wall in an asymmetric manner, is mathematically modeled in dimensionless form (Ponalagusamy, 1968) as

For PPL

$$\left.\begin{aligned} \frac{R(z)}{R_0} &= 1 - \mathrm{A}\left[\mathrm{L}_0^{n-1}(z-d) - (z-d)^n\right], \quad \mathrm{d} \le \mathrm{z} \le \mathrm{d}\ \ \mathrm{L}_0, \\ &= 1, \qquad \text{Otherwise,} \end{aligned}\right\} \tag{1}$$

The function $R_1(z)$ which represents the shape of the central layer, has been assumed to be of the from

$$\left.\begin{aligned} \frac{R_1(z)}{R_0} &= \alpha - \mathrm{A}_1\left[\mathrm{L}_0^{n-1}(z-d) - (z-d)^n\right], \qquad \mathrm{d} \le \mathrm{z} \le \mathrm{d}\ \ \mathrm{L}_0 \\ &= \alpha, \qquad \text{Otherwise,} \end{aligned}\right\}, \tag{2}$$

where $R(z)$ is the radius of the tube with stenosis, R_0 is the constant radius of the tube, R_1 is radius of the artery in the core region such that $\alpha = \frac{R_1(z)}{R_0}$, L_0 is the length of the stenosis, L the length of the tube, d the stenosis location, δ_S and δ_i are the maximum heights of the stenosis in the PPL and that in the core region respectively at $z = d + \frac{\mathrm{L}_0}{n^{\frac{1}{n-1}}}$ such that the ratio of the stenotic height to the radius of the artery is much less than unity i.e, $\frac{\delta}{R_0} << 1$.

$$\mathrm{A} = \frac{\delta s}{R_0 L_0^n} \cdot \frac{n^{\frac{n}{(n-1)}}}{(n-1)}, \qquad \mathrm{A}_1 = \frac{\delta i}{R_0 L_0^n} \cdot \frac{n^{\frac{n}{(n-1)}}}{(n-1)} \quad .$$

Here, $n \ge 2$ is a parameter determining the shape of the stenosis.

It is of interest to note that an increase in the value of n leads to the change of stenosis shape. When n=2, the geometry of stenosis becomes symmetrical at $z = d + \frac{L_0}{2}$ and $\alpha = \frac{\delta i}{\delta s}$.

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3. MATHEMATICAL FORMULATION

Let us consider a steady, laminar flow of blood through an axially non- symmetric but radially symmetric stenosed artery - a circular tube and one-dimensional flow obeying the constitutive equation for a Newtonian fluid. Fluid velocity vector has the form $\vec{u} = (0, 0, u(r))$ in cylindrical polar system (r, θ, z) representing the radial, circumferential and axial coordinate respectively. The equations of motion governing the fluid flow in (r, θ, z) coordinate system (Schlichting, 1968) are written as

$$\frac{\partial p}{\partial r} = 0, \tag{3}$$

$$\frac{\partial p}{\partial \theta} = 0, \tag{4}$$

$$\frac{\mu}{r} . \frac{\partial}{\partial r}\left(r \frac{\partial \mu}{\partial r} \right) = \frac{\partial p}{\partial z}, \tag{5}$$

where $u = u(r)$ denotes the axial velocity, μ is the viscosity of blood and P the pressure. As a result of equations (3-5), the governing equation of one-dimensional fluid flow is given by

$$c + \frac{\mu}{r} . \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r} \right) = 0, \tag{6}$$

where $\mathrm{C} = \left(-\frac{dp}{dz} \right)$ is the pressure gradient, and its integral is

$$u(r) = -\frac{cr^2}{4\mu} + A \ln r + B, \tag{7}$$

4. BOUNDARY CONDITION

The boundary conditions for the present problem are

(i) $\mathrm{u}_2 = o,$ at $\mathrm{r} = \mathrm{R}(\mathrm{z})$ (no slip at the stenotice wall) 8(a)

(ii) $\mathrm{u}_1 - u_2 = u_s$ at $\mathrm{r} = \mathrm{R}_1(\mathrm{z}),$ (slip at interface) 8(b)

(iii) $\frac{\partial u_1}{\partial r} = 0,$ at $\mathrm{r} = 0,$ (symmetric condition) 8(c)

(iv) $\frac{\partial u_2}{\partial r} = 0,$ at $\mathrm{r} = 0,$ (symmetric condition) 8(d)

where u_s is the velocity slip at the interface of the stenotic region.

5. MATHEMATICAL ANALYSIS

We get the velocity function in eq.(7) for core region as

$$u_1(r) = -\frac{cr^2}{4\mu_1} + A_1 \ln r + B_1, \; 0 \le \mathrm{r} \le \mathrm{R}_1(r), \tag{9}$$

and for the peripheral plasma layer (PPL) is

$$u_2(r) = -\frac{cr^2}{4\mu_2} + A_2 \ln r + B_2, \; \mathrm{R}_1(z) \le r \le R(z) \quad . \tag{10}$$

Using the boundary conditions (8(a)-8(d)) above in equations (9-10) we get the expressions of axial velocity in core region and PPL regions as follows

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$$u_1(r) = u_s + \frac{c}{4\mu_1}\left[R_1^2(z) - r^2\right] + \frac{c}{4\mu_2}\left[R^2(z) - R_1^2(z)\right],\ 0 \le r \le R_1(z), \tag{11}$$

$$u_2(r) = \frac{c}{4\mu_2}\left[R^2(z) - r^2\right], \quad R_1(z) \le r \le R(z) \quad . \tag{12}$$

The total flow rate Q, is obtained as

$$\mathrm{Q} = \pi R_1^2(z) u_s - \frac{\pi c}{8\mu_2}\frac{dp}{dz}\left[R^4(z) - \left(1 - \mu_2^{/}\right) R_1^4(z)\right] \quad , \tag{13}$$

where $\mathrm{Q}_1 = 2\pi \int_{r=0}^{R_1(z)} r u_1(r) dr,\ \mathrm{Q}_2 = 2\pi \int_{r=R_1(z)}^{R(z)} r u_2(r) dr, \quad \mu_2' = \frac{\mu_2}{\mu_1}$

Applying equation (13), the pressure gradient term can be expressed as

$$\frac{dp}{dz} = \frac{8\mu_2}{(1-\mu^{/}{}_2) R_1^4(z) - R^4(z)}\left[\frac{\mathrm{Q}}{\pi} - u_s R_1^2(z)\right] \tag{14}$$

Expression for wall shear stress at the stenotic wall and at the interface are obtained from the formula

$$\tau_{R(z)} = \frac{c}{2} R(z) \tag{15}$$

$$\tau_{R_1(z)} = \frac{c}{2} R_1(z) \tag{16}$$

To obtain the non-dimensional form of the flow variables, in equation. (11-16) and (1-2) we shall use the following non-dimensional variables:

$$\left.\begin{aligned}
&\bar{R}(\bar{z}) = \frac{R(z)}{R_0},\ \bar{R}_1(\bar{z}) = \frac{R_1}{R_0},\ \mathrm{u}_0 = \frac{cR_0^2}{4\mu_2},\ \mathrm{Q}_0 = \frac{\pi c R_0^4}{8\mu_2},\\
&\left(\frac{dp}{dz}\right)_0 = \left(-\frac{8\mu_2 \mathrm{Q}_0}{\pi R_0^4}\right), \quad \mu_2' = \frac{\mu_2}{\mu_1},\ \lambda_0 = \frac{8\mu L}{\pi R_0^4},\\
&\bar{\mathrm{u}}_s = \frac{u_s}{u_0}, \quad \bar{u}_1 = \frac{u_1}{u_0}, \quad \bar{u}_2 = \frac{u_2}{u_0}, \bar{z} = \frac{z}{R_0} \quad ,\\
&\bar{\mathrm{A}} = \frac{\mathrm{A}}{R_0^{n-1}}, \quad \bar{\mathrm{A}}_1 = \frac{A_1}{R_0^{n-1}}, \overline{\mu_a} = \frac{\mu_a}{\mu_2}
\end{aligned}\right) \tag{17}$$

VELOCITY FUNCTION

$$\bar{u}_1(\bar{r}) = \bar{u}_s + \mu_2'\left[\bar{R}_1^2(\bar{z}) - \left(\bar{R}(\bar{z})\right)^2\left(\frac{r}{R(z)}\right)^2\right] + \left[\left(\bar{R}(\bar{z})\right)^2 - \left(\bar{R}_1(\bar{z})\right)^2\right],\ 0 \le \frac{r}{R(z)} \le \frac{R_1}{R(z)} \tag{18}$$

$$\bar{u}_2(\bar{r}) = \left(\bar{R}(\bar{z})\right)^2\left\{1 - \left(\frac{r}{R(z)}\right)^2\right\}, \quad \frac{\mathrm{R}_1(z)}{\mathrm{R}(z)} \le \frac{r}{R(z)} \le 1 \tag{19}$$

RATE OF FLOW

$$\bar{Q} = 2\left(\bar{R}_1\right)^2 \bar{u}_s + \left[\left(\bar{R}(\bar{z})\right)^4 - \left(1 - \mu_2'\right)\left(\bar{R}_1(\bar{z})\right)^4\right] \tag{20}$$

SHEAR TRESSES AT THE WALL AND INTERFACE

$$\bar{\tau}_{R(z)} = \left(\frac{\overline{dp}}{dz}\right)\bar{R}(z) \tag{21}$$

$$\bar{\tau}_{R_1(z)} = \left(\frac{\overline{dp}}{dz}\right)\bar{R}_1(z) \tag{22}$$

FLOW GEOMETRY

$$\left.\begin{aligned} \bar{R}(\bar{z}) &= 1-\bar{A}\left[\bar{L}_0^{n-1}\left(\bar{z}-\bar{d}\right)-\left(\bar{z}-\bar{d}\right)^n\right], \qquad \bar{d}\le\bar{z}\le\bar{d}+\bar{L}_0 \\ &= 1, \qquad \text{otherwise} \end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned} \bar{R}_1(\bar{z}) &= \alpha-\bar{A}_1\left[\bar{L}_0^{n-1}\left(\bar{z}-\bar{d}\right)-\left(\bar{z}-\bar{d}\right)^n\right], \qquad \bar{d}\le\bar{z}\le\bar{d}+\bar{L}_0 \\ &= \alpha, \qquad \text{otherwise} \end{aligned}\right\} \tag{24}$$

PRESSURE GRADIENT

$$\frac{\overline{dp}}{dz} = \left[\left(\overline{\mathrm{R}(z)}\right)^4-\left(1-\mu_2'\right)\left(\overline{\mathrm{R}_1(z)}\right)^4\right]^{-1}\left(\bar{Q}-2\bar{u}_s\left(\overline{\mathrm{R}_1(z)}\right)^2\right) \tag{25}$$

6. RESULTS AND DISCUSSIONS

It is already reported that knowledge of rheological and fluid dynamic properties of blood and its flow, like velocity, pressure, shear stress at wall, flow rate etc., might play an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular (cvs) ,renal and arterial diseases (Dintenfass,1981;Punder and Punder,2006;Cokelet,1972). In view of this, analytical expressions of such flow variables and their variations are obtained and shown in fig. (2-8).It may be noticed that velocity is a function of shear viscosities (μ_1,μ_2), pressure gradient $\frac{dp}{dz}$, radii R(z), R_1(z) of uniform constricted arterial segments at wall and interface, radial r and axial z coordinates, slip velocity u_s and L, the artery length. Also, the non- uniform radii R(z) and R_1(z) in eqs. (1-2) depends on axial distance z, R_0 normal artery radius in an unobstructed tube.

The present model includes the following cases:

a) When R(z) =R_1(z), δ_i =0, the present analysis results in one-layered model for blood flow through a uniform stenosed artery with cases of slip or no-slip $u_s \geq 0$,.
b) when R(z)=R_1(z) =R_0, $\delta_i = 0 = \delta_s$ and α =1, it reduces to Poiseuille flow with slip ($u_s \geq$ or zero-slip($\geq$) at blood vessel wall.
c) when R(z)=R_1(z) $\neq$ R_0, $\delta_i \neq 0 \neq \delta_s$ and $\alpha \neq 1$,($u_s \geq 0$), it represents the present modeling for blood flow.

In the analysis, the combined influence of several parameters rave been developed, in cases of uniform region. To analyze the quantitative effect of uniform artery, maximum height of stenosis δ_i , δ_s, slip velocities ($u_s \geq 0$) at the interface, Newtonian behaviour of blood, two-layered flow etc., computer codes have been developed for the numerical evaluations of the analytic results obtained for velocity, flow rate, wall shear stress and pressure gradient for parameter values δ_s =.15, δ_i =.12, u_s =.00,.05,.1, Q=.5, 1, 1.5 (Verma and Parihar2009,2010) and viscosities μ_1 =1.2 cp, μ_2 = 2 cp, $\mu_2' = .62$,α = .82 (< 1) for a full scale location from z=d to d+L_0 ,and $0 \le r \le R_1(z)$, $R_1(z) \le r \le R(z)$ for PPL and core regions have been used. In the forgoing analysis, an attempt is taken up to address the variations of velocity, flow rate characteristics etc., due to such parameters.

A comparison of velocity profiles that have been obtained from eqs. (18-19), for slip and no-slip cases, maximum heights of stenosis and different axial locations for z=d+L_0/2,d+L_0/4,d+3L_0/4 for shape parameter n=2,6,9 and for other parameter values, is shown in Figs. (2-3).As tube radius r/R ranges from o (at tube axis) to 1 (at wall) on either side of axis, velocity decreases from a greater value at axis to a smaller one slip velocity at interface and, then to a minimum magnitude zero-slip velocity at boundary. As expected, velocity increases with slip at interface. Its values are higher for

flows with slip (u_s>0) than those with no-slip (u_s=0). Also it is observed from fig.(2-3) that $\overline{u_1}\,|_{n=9} < \overline{u_1}\,|_{n=2} < \overline{u_1}\,|_{n=6}$. As slip velocity increases, axial velocity increases.

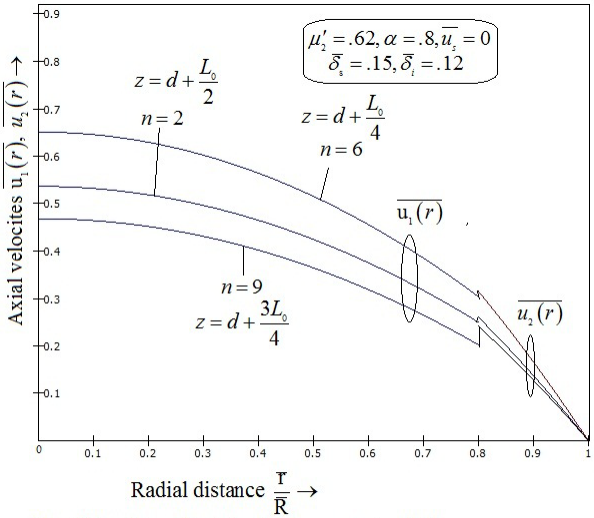

Fig.2 Variation of axial velocity against radial distance for different values of n

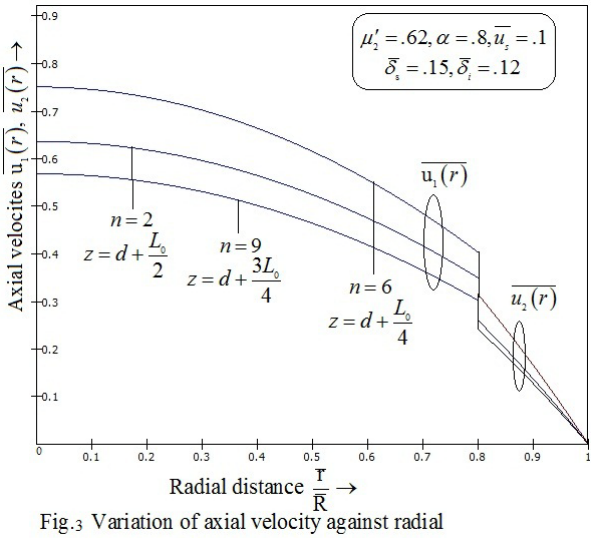

Fig.3 Variation of axial velocity against radial distance for slip velocity $\overline{u_s}$=.1

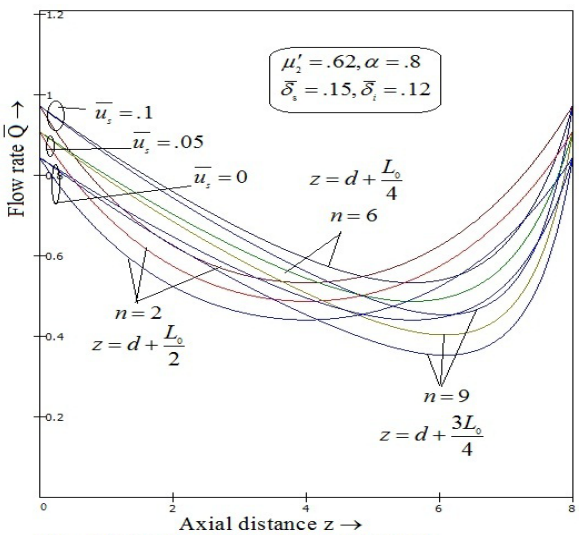

Fig.4 Variation of flow rate against axial distance for different slip velocities

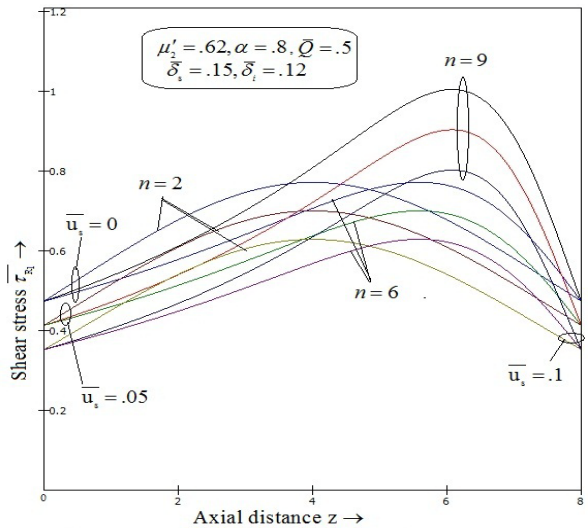

Fig.5 Variation of shear stress against axial distance for different slip velocities $\overline{u_s}$

In fig.(4), we notice that the flow $\overline{Q}$ is minimum at minimum tubular area(at throat of stenosis) for symmetric and asymmetric stenoses and maximum at either end (at the beginning or at the end of a constricted region). As $\overline{z}$ increases in the constricted region z=d to d+L_0, flow rate decreases to a maximum height of stenosis. As slip velocity increases, flow rate $\overline{Q}$ increases.

In the presence of stenosis it is seen from fig. (7) that pressure gradient is lower with an axial slip at interface than that with no-slip. As slip increases in magnitude, it decreases further. As the value of $\overline{Q}$ increases keeping other variables fixed, pressure gradient increases. The Fig. (5) shows that as axial distance $\overline{z}$ increases, wall shear stress decreases. Also as shape parameter values *n* increases, $\overline{\tau_{R_1(z)}}\,|_{n=2} < \overline{\tau_{R_1(z)}}\,|_{n=6} < \overline{\tau_{R_1(z)}}\,|_{n=9}$. As slip velocity ($u_s$) increases, $\overline{\tau_{R(z)}}$ decreases. As flow rate increases, shear stress increases.

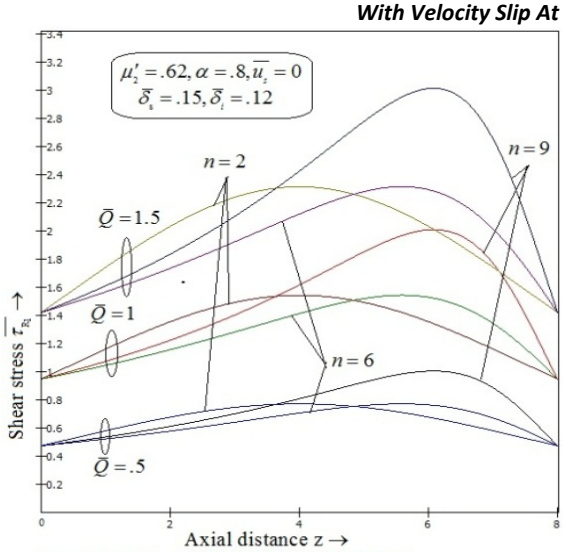

Fig. 6 Variation of shear stress against axial distance for different values of $\overline{Q}$

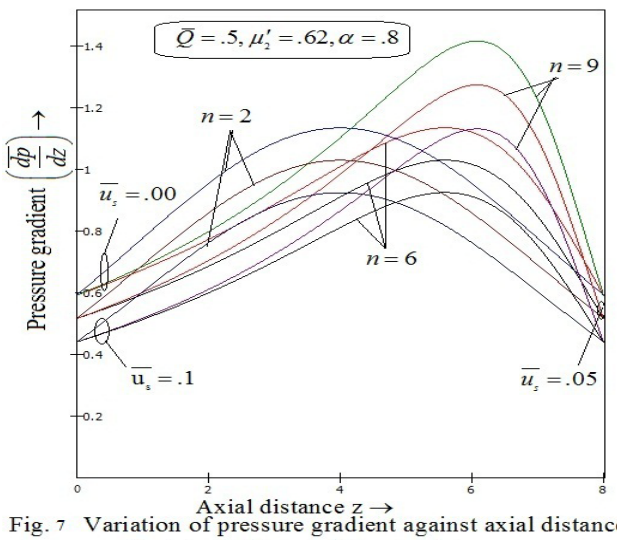

Fig. 7 Variation of pressure gradient against axial distance for $\overline{u_s} = 0, .05, .1$ and $\overline{\delta_s} = .15, \overline{\delta_i} = .12$

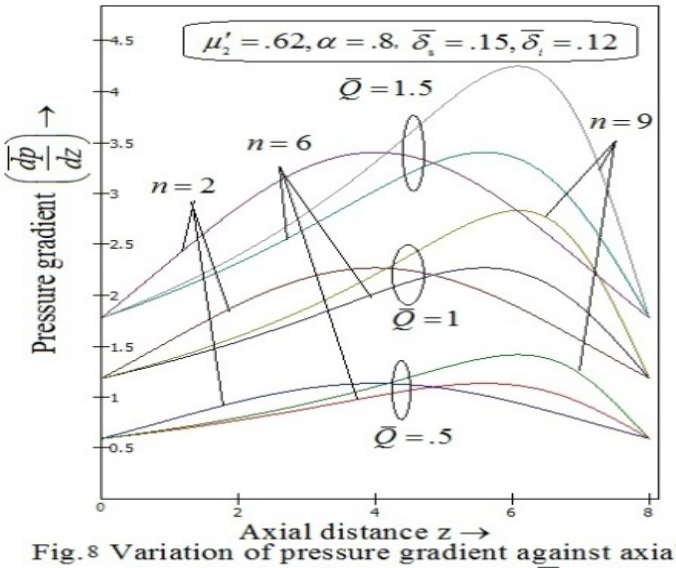

Fig. 8 Variation of pressure gradient against axial distance for different flow rate $\overline{Q} = .5, 1, 1.5$

7. CONCLUSION

In the present investigation, steady flow of blood (Newtonian fluid) through a uniform artery with the presence of an axially non-symmetrical but radially symmetric mild stenosis is considered (Fig.1). Analytic expressions of flow variables are obtained and variations of velocity, shear stress ,flow rate and pressure gradient are shown graphically (Figs.2-8).The present analysis includes Poiseuille flow of blood, 1-dimentional blood flow models through uniform vessel, with or without constriction, with slip or zero-slip cases as its special cases.

In Fig. (5) it is revealed that wall shear stress in the stenotic region increases rapidly from its approached value at z=d (at initiation) and attains its maximum at the throat of stenosis (at z=d+L_0) for symmetric stenosis at n=2 and for asymmetric constriction (at n=6, 9), the maximum value of $\overline{\tau_{R(z)}}$ is reached near the end (z=d+L_0) of stenosis. There from, it decreases rapidly in the downstream, attaining the minimum value at the termination (z=d+L_o) of constriction. As expected, it will reduce due to an employment of velocity slip at interface.

In this investigation, it is easily noticed that consideration of a 2-layered blood flow with asymmetric stenosis in uniform blood vessel, would provide better results than 1-layered axi-symmetric stenosed uniform tubes. Further it is noticed that employment of velocity slip at interface, could reduce wall shear stresses. It is seen that speed and rate of flow can be accelerated and wall shear stress can be reduced due to an employment of velocity slip. Apart from this , damage to the vessel wall could be lowered at one hand and normal blood flow can be restored on the other.Further,as slip might play a significant role in blood flow modeling, its accurate and exact measure in accordance with physiological situations, is quite important. The present mathematical model may be used as a device or tool in such cases.

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