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ON SIMPLE (-1, 1) RINGS

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ABSTRACT

If R be a 2-torsion free (-1, 1) ring with the associators in the middle nucleus N_m then $(N_m, R, R)=0$ and the ring becomes associative.

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INTRODUCTION

Yen [7] considered 2-torsion free simple rings with associators in the left nucleus and showed that such type of ring is associative. Kleinfeld [2] studied the properties of the rings satisfying $(x, y, z) = (x, z, y)$ and proved that there exists simple Novikov rings which are not associative. Kleinfeld and Smith [4] generalized Novikov rings, which satisfy the condition $x(yz) = y(xz)$. Kleinfeld and Kleinfeld [3] have shown that a 2-torsion free simple ring with identity 1 must be associative. In [6] Suvarna and Subba Reddy have consider a generalization of (1, 1) rings. They proved that if R is a 2-torsion free simple ring satisfying the identities $(x, y, z) = (x, z, y)$ and $(w, (y, x, x), z) = 0$, then R is right alternative. Paul [5] studied the properties of prime rings satisfying $(x, y, z) – (x, z, y) = 0$ and $(w, [y, z], x) = 0$.

A ring R is said to be (-1, 1) ring if it satisfies the following two conditions:

$$(x, y, z) = – (x, z, y) \tag{1}$$

and $$(x, y, z) + (y, z, x) + (z, x, y) = 0 \tag{2}$$

for all $x, y, z \in R$.

In a nonassociative ring an associator is defined as $(x, y, z) = (xy)z – x(yz)$, commutator $[x, y]$ is defined as $xy – yx$ for all $x, y \in R$ and the middle nucleus is defined as $N_m = \{n \in R \,/\, (R, n, R) = 0\}$. In this paper using the results of [5 and 6] we show that a semi prime ring generated by U square to zero and hence R must be associative.

Let the associator (R, R, R) be in the middle nucleus of (-1, 1) ring that is $(R, (R, R, R), R) = 0$. (3)

Throughout this paper R represents a 2, 3-torsion free (-1, 1) ring.

We use the Teichmuller identity

$$(wx, y, z) – (w, xy, z) + (w, x, yz) = w(x, y, z) + (w, x, y)z \tag{4}$$

for all $x, y, z \in R$ which holds in any arbitrary ring.

Let $n \in N_m$ then (2) implies $(n, R, R) = 0$.

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For arbitrary $(x, y, z)\, n = (x, y, zn)$ from (4).

Again from (4) we get

$(xy, z, n) – (x, yz, n) + (x, y, zn) = x(y, z, n) + (x, y, z)n$

Implies $(x, y, zn) = (x, y, z)n$. (5)

Thus from (1) $(x, y, zn) = – (x, zn, y)$. (6)

Again (4) implies $(xz, n, y) – (x, zn, y) + (x, z, ny) = x(z, n, y) + (x, z, n)y$ which implies

$(x, zn, y) = (x, z, ny)$. (7)

From (5), (6) and (7) we see that

$(x, zn, y) = – (x, y, z)\, n$. (8)

$(x, z, ny) = – (x, y, z)\, n$. (9)

$(x, ny, z) = – (x, y, z)\, n$. (10)

Now $[n, y] \in N_m$. (11)

$S\, (x, y, z) \in N_m$.

That is $(R, (x, y, z) + (y, z, x) + (z, x, y), R) = 0$.

The following identity is valid in (-1, 1) ring [1]

$((a, x, y), b, c) = ((a, b, c), x, y) – (a, b,(c, x, y)) – (a, (b, x, y), c) + (a, b, c)[x, y] – (a, b, c[x, y]) + (a, b, [x, y])\, c = 0$

for all $a, b, c, x, y \in R$. (12)

Thus we get $0 = – (a, (b, x, y), c) = (a, (b, x, y), c)$.

With $b \in N_m$ in (12) we obtain $– (a, (b, x, y), c) = 0$.

That is $(b, x, y) \in N_m$.

Thus $(N_m, R, R) \subseteq N_m$. (13)

Lemma: 1 Let $T = \{T \in R \,/\, Rt = 0\}$ then $T = 0$.

Proof: Let $t \in R$. For every $x \in R$, $xt = 0$, so $RT \subset T$. Also $y.tx = – (y, x, t)$ using equation (1). But $(y, x, t) = 0$ as $xt = 0$ thus $TR \subset T$ consequently T *is* an ideal and $TT \subset RT = 0$, so $T = 0$. We define $a \equiv b$ if and only if $a – b \in N_m$.

Lemma: 2 $(N_m, R, R) = 0$.

Proof: Let $n \in N_m$ and $x, y, z \in R$ then (2) implies

$(zn, x, y) = – (y, zn, x) – (x, y, zn)$

$= – [(y, z, x)n + (x, y, zn)]$

$= – [(y, z, x) + (x, y, z)]n$

$= (z, x, y)n$. (14)

Using $N_m N_m \subset N_m$ and previous calculations we obtain $(zn, x, y) = (z, x, y)n$.

However (4) implies $(zn, x, y) – (z, nx, y) + (z, n, xy) = z(n, x, y) + (z, n, x)y$

Implies $(zn, x, y) = (z, nx, y) + z(n, x, y)$. (15)

Comparison of these two identities implies that $z(n, x, y) = 0$.

From (15) we see that $(zn, x, y) = (z, nx, y) + z(n, x, y)$

$= z(n, x, y) + (z, nx, y)$

$= z(n, x, y) + (z, x, y)n$ by equation (11). (16)

Therefore subtracting (14) from (16) gives $(zn, x, y)–(z, x, y)n –(zn, x, y) + z(n, x, y)–(z, x, y)n = 0$. That is $z(n, x, y) = 0$. Equivalently $z(n, x, y) \in N_m$. Thus $(r, z(n, x, y), s) = 0$ then (13) yields

$(r, z, s)(n, x, y) = 0$. (17)

The associator ideal of R may be characterized as $A = \Sigma\ (R, R, R) + R(R, R, R)$. As a result of (13) and (17) it is clear that $A(N_m ,R, R) = 0$. Since R is a simple and not associative, it follows that $A=R$, so that $R\ (N_m, R, R) = 0$ and thus $(N_m, R, R) \subset T = 0$. Hence $(N_m, R, R) = 0$.

Definition: 4 The center C is defined as $C = \{c \in N / [c, R] = 0\}$.

Lemma: 3 Middle nucleus equals the center in (-1, 1) ring.

Proof: From equation (4) with $x, y, z \in R$ and $n \in N_m$ we obtain $(xy, z, [y, n]) – (x, zy, [y, n]) + (x, z, y[y, n]) = x(z, y, [y, n]) + (x, z, y)[y, n]$ implies $(x, z, y[y, n]) = (x, z, y)\ [y, n]$. Multiply both sides by 2 we get

$2(x, z, y[y, n]) = 2(x, z, y)\ [y, n]$.

Now applying the semi Jacobi identity which is valid in any arbitrary ring we see that

$[y^2, n] = y[y, n] + [y, n]y + (y, y, n) + (n, y, y) – (y, n, y)$.

Implies $[y^2, n] = y[y, n] + [y, n]\ y + n[y, y]$.

Hence $(x, [y^2, n], z) = (x, y, [y, n], z) + (x, [y, n]\ y, z) + (x, (n, x, y), z)$.

That is $0 = (x, y, [y, n], z) + (x, [y, n]y, z)$.

Thus $2(x, y, z)\ [y, z] = 0$ and since the ring is 2- torsion free we get $(x, y, z)\ [y, z] = 0$. (18)

Now (18) and (11) shows $A\ [w, n] = 0$, so that $R[w, n] = 0$ and so $[w, n] \in T = 0$. Consequently $[R, n] = 0$ and hence $n \in C$.

Lemma: 4 If R satisfies the weak Novikov identity then $(R, R, R)^2 = 0$.

Proof: R satisfies $(w, x, yz) = y(w, x, z)$ for all $w, x, y, z \in R$. (19)

Let $u = (R, R, R)$. From (3) and Lemma (3) we see that $u \in C$. From (4) we obtain

$(wx, y, z) – (w, xy, z) + (w, x, yz) = w(x, y, z) + (w, x, y)z$.

$((wx, y, z), r, s) – ((w, xy, z), r, s) + ((w, x, yz), r, s) = (w(x, y, z), r, s) + ((w, x, y)z, r, s)$.

Now using (2) in above we obtain
$– (r, s, (wx, y, z)) – (s, (wx, y, z, r) + (r, s, (w, xy, z)) + (s, (w, xy, z), r) – (r, s, (w, x, yz)) – (s, (w, x, yz), r)$
$= (w(x, y, z), r, s) + ((w, x, y)z, r, s)$. Applying (13) we get $(w(x, y, z), r, s) + ((w, x, y)z, r, s) = 0$.

Thus $((w, x, y)\ z, r, s) = – (w(x, y, z), r, s)$. (20)

Now a repeated applications of equations (19), (1), $[R, R] \subseteq N_m$ and (20) gives

$((x, y, z)w, r, s) = (w(x, y, z), r, s)$. (21)

Then from (20) we obtain

$((w, x, y)z, r, s) = -(w(x, y, z), r, s)$

$= -((x, y, wz), r, s)$

$= -((x, y, zw), r, s)$

$= -(z(x, y, w), r, s)$ from (19)

$= (z(x, w, y), r, s)$ from (1)

$= -((z, x, w)y, r, s)$

$= ((z, w, x)y, r, s)$

$= -(z(w, x, y), r, s)$ from (4)

$= -((w, x, y)z, r, s))$. from (21)

That is $((w, x, y)z, r, s) + ((w, x, y)z, r, s) = 0$.

Thus $0 = 2((w, x, y)z, r, s)$

$= ((w, x, y)z, r, s)$

$= (w, x, y)(z, r, s)$

$= (R, R, R)(R, R, R)$

$= (R, R, R)^2$

$= U^2$.

Theorem: 1 Let R be a 2-torsion free (-1, 1) ring if the associator is in the middle nucleus then of R must be a associative.

Proof: Since $U = (R, R, R) \in C$ and $U^2 = 0$.Since the ideal generated by U square to zero we get $U = 0$. Thus the ring must be associative.

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