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# EFFECT OF THERMAL RADIATION ON MHD BOUNDARY LAYER FLOW OVER A MOVING VERTICAL POROUS PLATE WITH SUCTION

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*ABSTRACT*

*The aim of the present work is to study the combined effects of thermal radiation, viscous dissipation and suction on the steady free convective boundary layer flow of an incompressible viscous fluid over a moving vertical porous plate. The equations governing the flow are coupled and non linear. These governing differential equations are reduced to ordinary differential equations by introducing the stream function and are solved numerically by using Runge Kutta fourth order Shooting Method. The effects of different physical parameters governing the flow such as Magnetic parameter M, Permeability parameter K, Prandtl number Pr, Thermal radiation parameter R, Thermal Grashof number Gr, Mass Grashof number Gc, Schmidt number Sc, Eckert number Ec on the velocity field, temperature distribution and concentration are discussed through graphs and tables.*

## 1. INTRODUCTION

The phenomenon of heat and mass transfer has been the object of extensive research due to its applications in Science and Technology. These types of phenomena are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi solid bodies such as earth and so on. The flow of fluids through porous medium has a wide range of applications in engineering, industrial such as extraction of crude oil, storage of nuclear wastes and heat exchangers. Further, to study underground water resources, seepage of water in riverbeds also one needs to investigate the flows through porous medium. The study of flow of electrically conducting fluid called Magneto Hydrodynamics (MHD). It has gained the attention due to its applications in the design of heat exchangers, induction pumps, and nuclear reactors, in oil exploration and in space vehicle propulsion. Thermal radiation in fluid dynamics has become a significant branch of the engineering sciences and is an essential aspect of various scenarios in mechanical, aerospace, chemical, environmental, solar power and hazards engineering.

The effect of mass transfer on free convective flow past a semi infinite plate was studied by Gebhart and Pera [1]. Soundalgekar and Ganesan [2] have analyzed transient free convective flow past a semi infinite vertical flat plate by using an implicit finite difference method. Bejan and Khair [3] analyzed natural convection in porous medium with heat and mass transfer. Jha [4] studied MHD free convective flow past a uniformly accelerated vertical plate through a porous medium when magnetic field is fixed with the moving plate. Sharma *et al.* [5] investigated steady laminar free convection flow of an electrically conducting fluid along a porous vertical infinite plate. The effect of radiation on mixed convection along a vertical plate with uniform surface temperature was analyzed by Hossain and Takhar [6]. Steady flow of a two dimensional free convection in a vertical infinite porous plate with radiation effect discussed by Rapits [7]. Chowdhury and Islam [8] investigated MHD free convection flow past a vertical surface by Laplace-transform technique. Magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux were studied by Acharya *et.al* [9]. Chamkha and Khaled [10] investigated the problem of coupled heat and mass transfer by magnetohydrodynamic free convection from inclined plate.The behaviour of steady free convective MHD flow past a vertical porous moving surface was presented by Sharma and Pareek [11]. Singh *et al.* [12] discussed the effect of heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Makinde *et al.* [13] analyzed the unsteady free convective flow

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with suction on an accelerating porous plate. Hydromagnetic free convection flow over a surface was considered by Aldoss *et al.* [14]. Nield and Bejan [15] describe the detailed research information on convection in porous media. Suneetha *et al.* [16] have investigated the radiation effects on the MHD free convection flow past an impulsively started vertical plate with variable surface temperature and concentration. Chandrakala [17] discussed the free convection flow of a viscous incompressible fluid with uniform heat flux in the presence of thermal radiation. Anki Reddy *et al.* [18] discussed the thermal radiation effects on MHD flow due to an exponentially stretching sheet.

The objectives of this paper are to investigate the combined effects of radiation parameter, magnetic parameter, Eckert number as well as suction parameter on velocity, temperature and concentration profiles. Results are discussed quantitatively through graphs for different parameters

## 2. FORMULATION OF THE PROBLEM

We consider the steady two dimensional MHD boundary layer flow of an incompressible viscous radiating fluid along porous vertical plate in the presence of magnetic field and viscous dissipation. The velocity of the fluid far away from the plate surface is assumed zero for a quiescent state fluid. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of the linear momentum equation. The x-axis is taken along the plate in the upward direction and the y-axis is taken normal to it. A transverse magnetic field of strength $B_0$ is applied parallel to the y-axis. Under these assumptions, the flow of a radiative conducting fluid, along with Boussinesq approximation, can be shown to be governed by the following system of equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\beta g(T-T_\infty)+\beta^* g\,(C-C_\infty)-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{K'}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\nu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$u = B\,x,\ v = V,\ T = T_w = T_\infty + ax,\ C = C_\infty + bx \quad \text{when } y = 0 \tag{5a}$$

$$u \to 0,\ T \to T_\infty,\ C \to C_\infty \qquad \text{as } y \to \infty \tag{5b}$$

where u and v are the velocity components along the *x* and *y* directions, $\nu$ is the kinematic viscosity, $\beta$, $\beta^*$ are the thermal and concentration expansion coefficient, $\sigma$ is the electric conductivity of the fluid, $\rho$ is the density, $K'$ is the permeability of the porous medium, T is the temperature of the fluid in the boundary layer, C is the species concentration in the boundary layer, $T_\infty$ is the temperature for away from the plate, $C_\infty$ Species concentration of the ambient fluid, $\alpha$ is the thermal diffusivity, D is the mass diffusivity, B is a constant, a and b denotes the stratification rate of the gradient of ambient temperature and concentration profiles.

Following Cogley *et al.* [19], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q_r}{\partial y}=4\alpha^2\,(T-T_\infty) \tag{6}$$

Where $\alpha^2=\int_0^\infty \delta\lambda\frac{\partial B_1}{\partial T}$ is the absorption coefficient, $\lambda$ is frequency, $B_1$ is Planck's function and $\delta$ is the radiation absorption coefficient

The following non-dimensionless parameters are introduced

$$\eta=\sqrt{\frac{B}{\nu}}y,\ F(\eta)=\frac{\psi}{x\sqrt{B\nu}},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\ f_w=\frac{V}{\sqrt{B\nu}} \tag{7}$$

where $\psi$ is the stream function with $u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$, $\eta$ is the similarity variable, $f_w$ is the non-dimensional suction velocity.

With the non-dimensional variables (7), Eqns. (2), (3) and (4) become

$$F''' + FF'' - (F')^2 + Gr\theta + Gc\phi - (M+K)F' = 0 \tag{8a}$$

$$\theta'' + Pr[F\theta' - F'\theta + R\,\theta + Ec\,(F'')^2] = 0 \tag{8b}$$

$$\phi'' + Sc[F\phi' - F'\phi] = 0 \tag{8c}$$

where the primes denote the differentiation with respect to η, $Gr = \frac{g\beta(T_w - T_\infty)}{x B^2}$ is the local temperature Grashof number, $Gc = \frac{g\beta^*(C_w - C_\infty)}{x B^2}$ is the local concentration Grashof number, $M = \frac{\sigma B_0^2}{\rho B}$ is the magnetic parameter, $K = \frac{\nu}{K' B}$ is the permeability parameter, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, R= $\frac{4\alpha^2}{\rho c_p B}$ is the thermal Radiation parameter, $Ec = \frac{B^2 x^2}{c_p (T_w - T_\infty)}$ is the Eckert number or Viscous dissipation parameter and Sc = $\frac{\nu}{D}$ is the Schmidt number.

The corresponding boundary conditions are

$$F' = 1,\ F = -f_w,\ \theta = 1,\ \phi = 1 \quad \text{when } \eta = 0 \tag{9a}$$

$$F'(\infty) \to 0,\ \theta(\infty) \to 0,\ \phi(\infty) \to 0 \quad \text{as } \eta \to \infty \tag{9b}$$

From the technological point of view, the important physical parameters for this type of boundary layer flow are the local skin-friction coefficient, the local Nusselt number and Sherwood number which are proportional to $F''(0)$, $-\theta'(0)$ and $-\phi'(0)$

## 3. RESULTS AND DISCUSSION

The set of coupled non-linear governing boundary layer equations (8a-c) together with the boundary conditions (9) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. In order to get a physical insight into the problem, a representative set of numerical results are shown graphically in figures 1-9, to illustrate the influence of physical quantities R, Gr, Gc, M, Sc, fw, K, Pr, Ec on velocity, temperature and concentration profiles. The values of Schmidt number (Sc) are chosen to be 0.24, 0.62, 0.78 and 2.62, representing diffusing chemical species of most common interest in air like $H_2$, $H_2O$, $NH_3$ and Propyl Benzene respectively.

Figure1 illustrates the effect of the magnetic parameter M on velocity, temperature, concentration profiles. We observed that increasing the magnetic field parameter reduces the velocity due to an increase in the Lorentz force which acts against the flow if the magnetic field is applied in the normal direction. This naturally leads to an increase in the temperature (and concentration) within the boundary layer. Figure 2 presents typical velocity, temperature and concentration profiles for various values of permeability parameter *K*. It is noticed that as the permeability parameter increases, the velocity decreases but a reverse trend is noticed in both temperature and concentration profiles.

Figure 3 and 4 show the influence of Grashof number (Gr, Gc) on velocity, temperature and concentration profiles. We note that when the Grashof number (Gr, Gc) increases, the velocity increases and the temperature, concentration decrease. For different values of the suction parameter $f_w$, the velocity, temperature and concentration are shown in figure 5. It is seen that the velocity, temperature and concentration increases with an increase in $f_w$.

The influence of Schmidt number Sc on the velocity, temperature and concentration are shown in figure 6. The Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The effect of radiation parameter R on the temperature variations is depicted in figure 7. The radiation parameter *R* defines the relative contribution of conduction heat transfer to thermal radiation transfer. As R increases, considerable reduction is observed in temperature profiles. The effect of the viscous dissipation parameter i.e., the Eckert number *Ec* on velocity,

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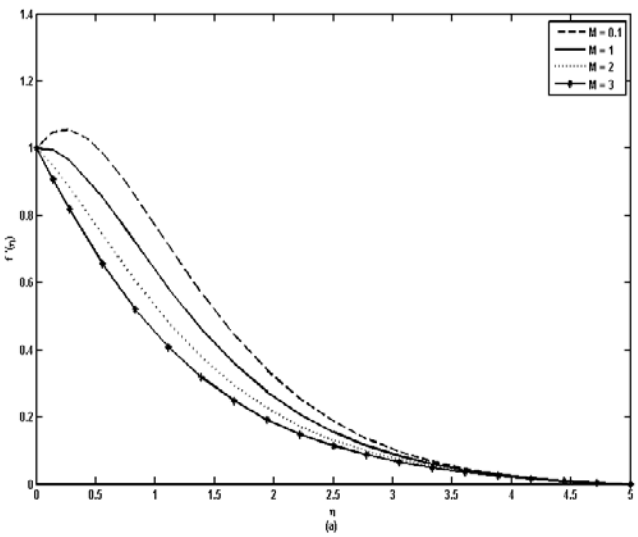
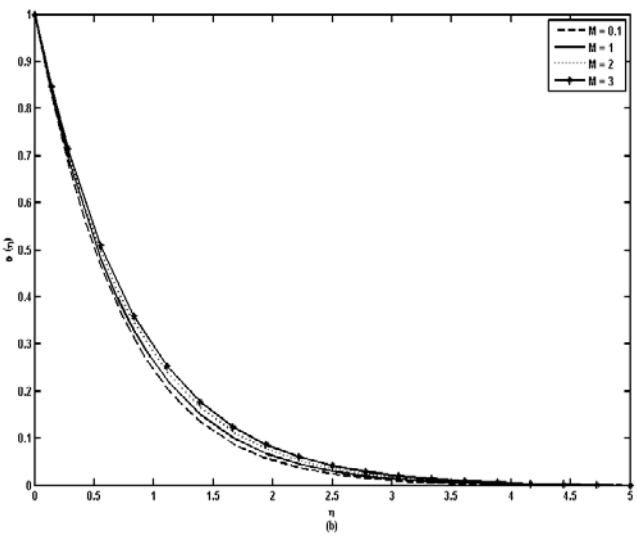
temperature and concentration are shown in figure 8. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature as well as the velocity. Figure 9 shows the variation of the temperature profile for different values of Prandtl number Pr. The effect of the Prandtl number on the temperature distribution is more pronounced. The higher values of Pr reduce the maximum value of the temperature distribution because of slow thermal diffusion.

Table 1 gives the variation of Skin friction coefficient (Cf), Nusselt number (Nu), Sherwood number (Sh) for different values of governing parameters It is found that Cf increases and Nu, Sh decreases as M, R increases

**Table – 1:** Effect of various parameters on $F''(0)$, $-\theta'(0)$, $-\phi'(0)$

| Gr | Gc | M | R | Ec | fw | Sc | K | Pr | $F''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.1 | 0.1 | 1 | 0.1 | 0.1 | 0.62 | 0.1 | 0.72 | 0.9487 | 1.1664 | 0.7128 |
| 0.5 | 0.1 | 0.1 | 1 | 0.1 | 0.1 | 0.62 | 0.1 | 0.72 | 0.7884 | 1.1850 | 0.7404 |
| 0.1 | 1.0 | 0.1 | 1 | 0.1 | 0.1 | 0.62 | 0.1 | 0.72 | 0.5159 | 1.2213 | 0.7983 |
| 0.1 | 0.1 | 1.0 | 1 | 0.1 | 0.1 | 0.62 | 0.1 | 0.72 | 1.3114 | 1.1191 | 0.6373 |
| 0.1 | 0.1 | 0.1 | 2 | 0.1 | 0.1 | 0.62 | 0.1 | 0.72 | 0.9536 | 1.4480 | 0.7109 |
| 0.1 | 0.1 | 0.1 | 1 | 1.0 | 0.1 | 0.62 | 0.1 | 0.72 | 0.9459 | 0.9734 | 0.7138 |
| 0.1 | 0.1 | 0.1 | 1 | 1.0 | 1.0 | 0.62 | 0.1 | 0.72 | 0.6203 | 0.8193 | 0.5160 |
| 0.1 | 0.1 | 0.1 | 1 | 1.0 | 0.1 | 2.62 | 0.1 | 0.72 | 1.0755 | 0.7898 | 1.5481 |
| 0.1 | 0.1 | 0.1 | 1 | 1.0 | 0.1 | 0.62 | 1.0 | 0.72 | 1.3085 | 0.8245 | 0.6384 |
| 0.1 | 0.1 | 0.1 | 1 | 1.0 | 0.1 | 0.62 | 0.1 | 2 | 0.9562 | 1.5302 | 0.7098 |



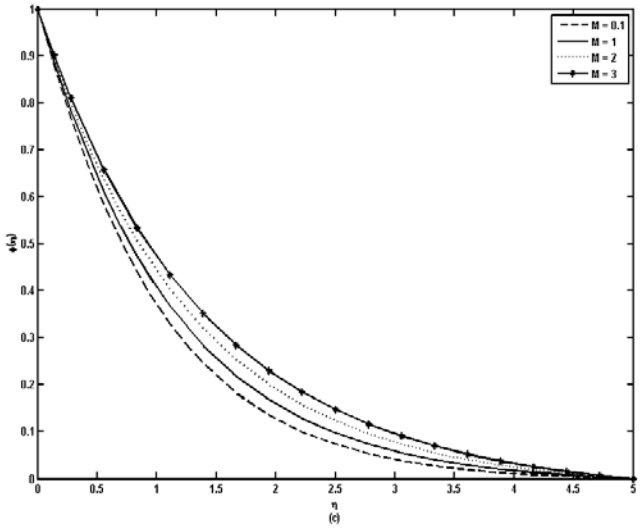
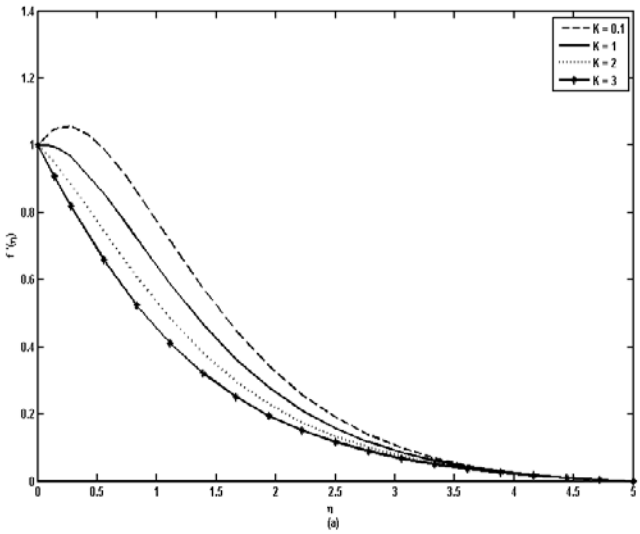
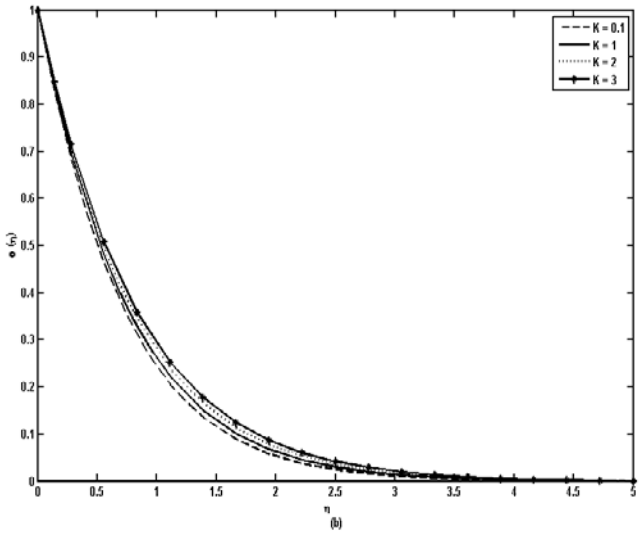



**Figure – 1:** Effect of Magnetic parameter on (a) Velocity (b) temperature (c) concentration profiles when Pr = 0.72, Sc = 0.62, Gr = Gc = 2, fw = 0.1, K = Ec = 0.1, R = 1



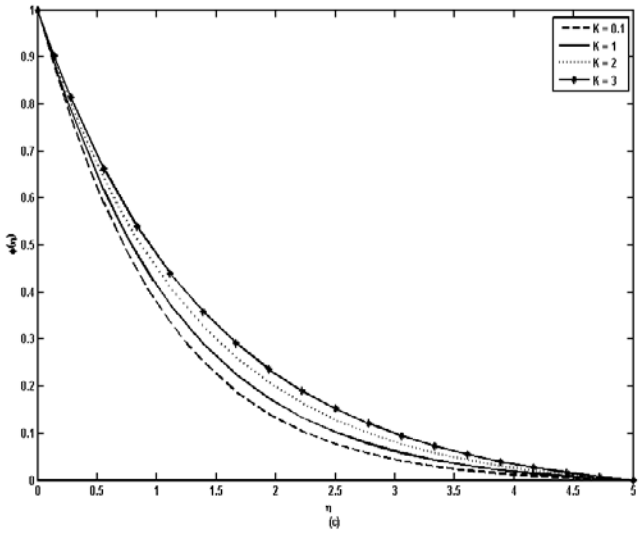
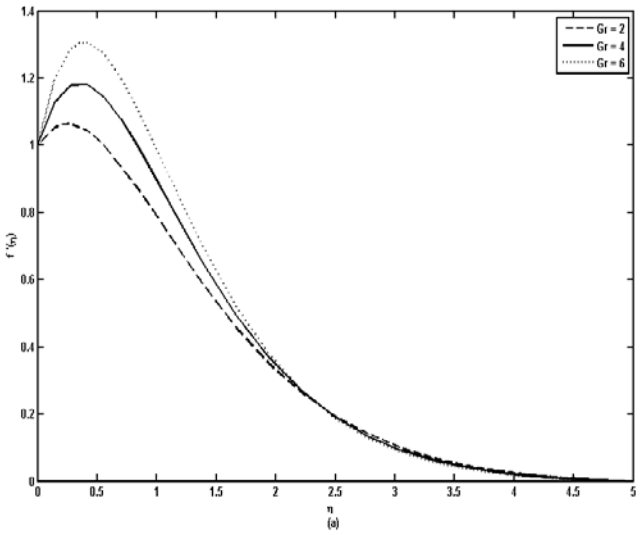
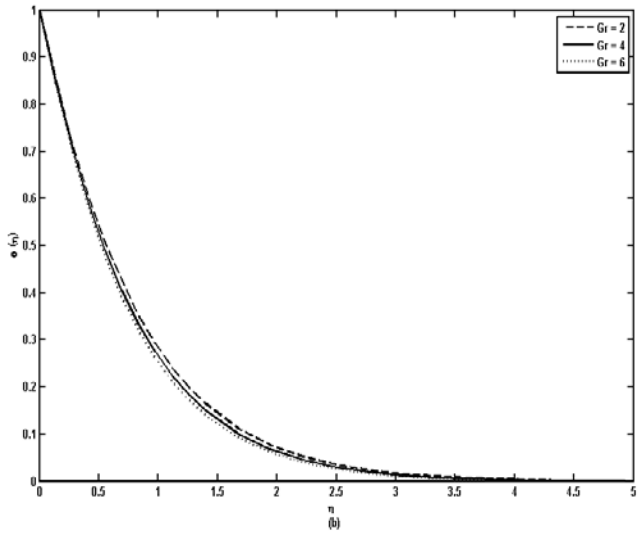



**Figure – 2:** Effect of K on (a) Velocity (b) temperature (c) concentration profiles when Pr = 0.72, Sc = 0.62, Gr = Gc = 2, fw = M= Ec = 0.1, R = 1



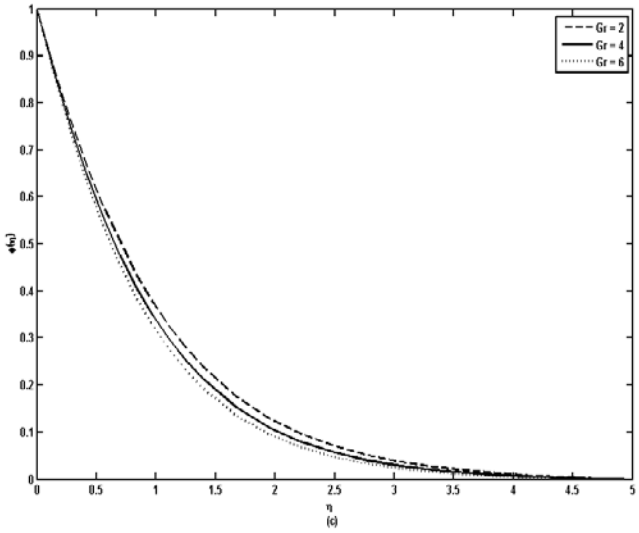
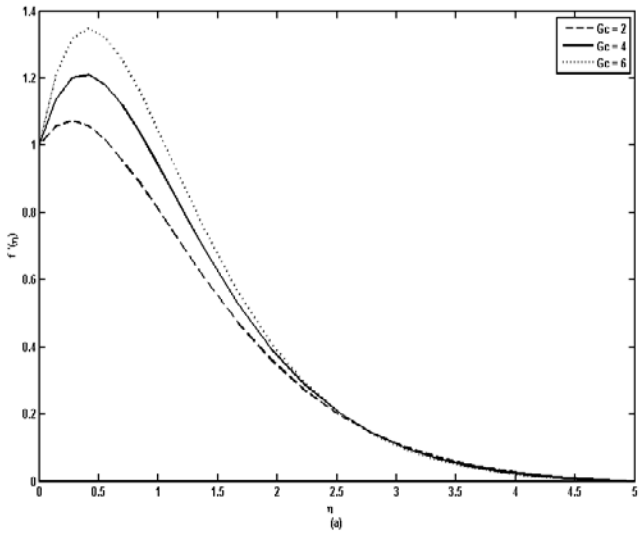
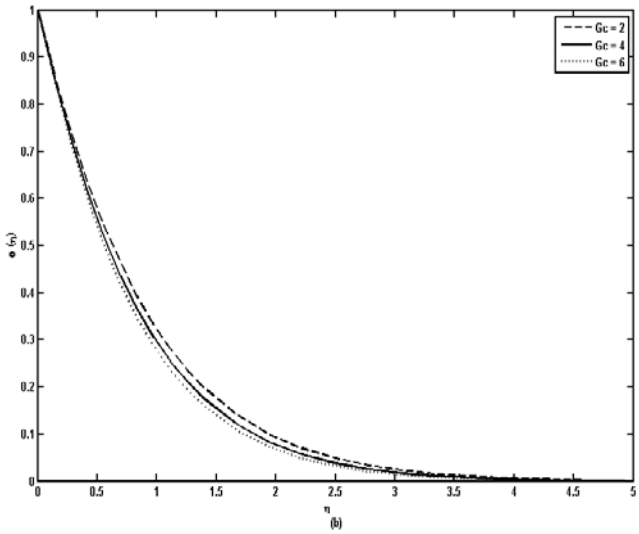



**Figure – 3:** Effect of Gr on (a) Velocity (b) temperature (c) concentration profiles when Pr = 0.72, Sc = 0.62, Gc = 2, M = fw = K = Ec = 0.1, R = 1



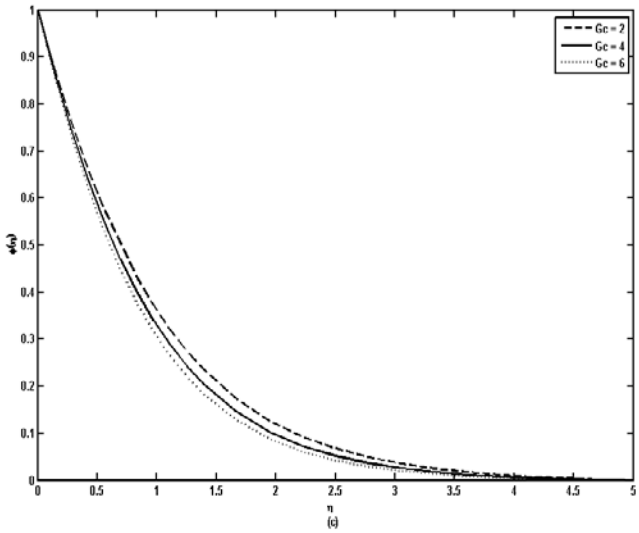
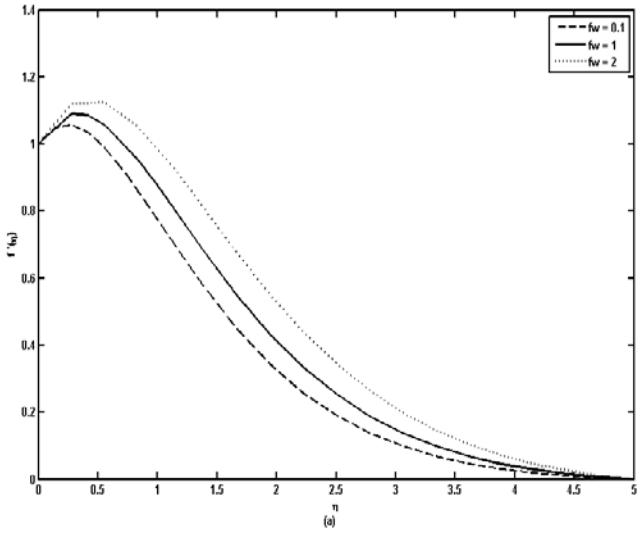
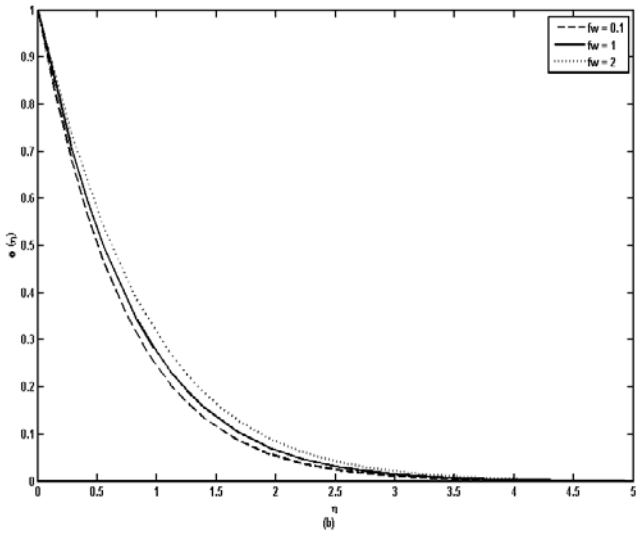



**Figure – 4:** Effect of Gc on (a) Velocity (b) temperature (c) concentration profiles when Pr = 0.72, Sc = 0.62, Gr = 2, M = fw = K = Ec = 0.1, R = 1



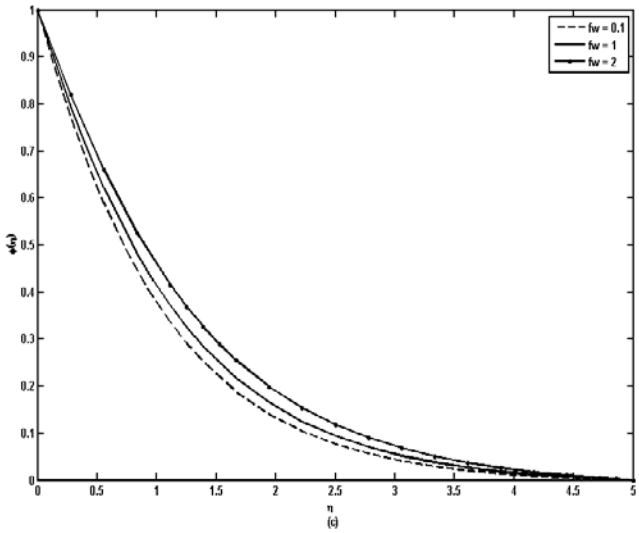
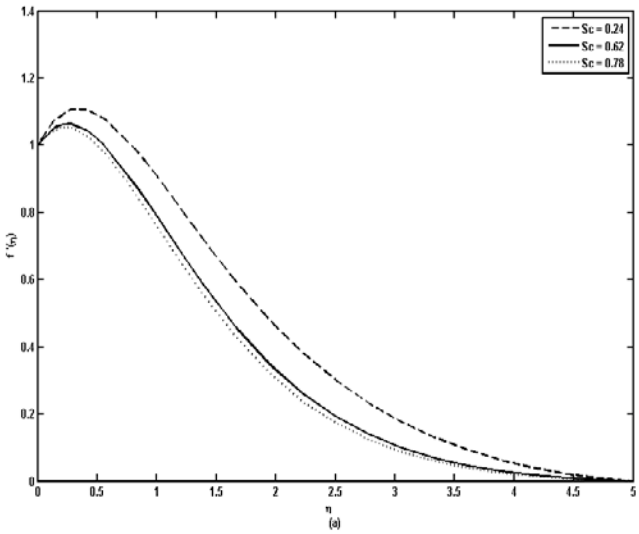
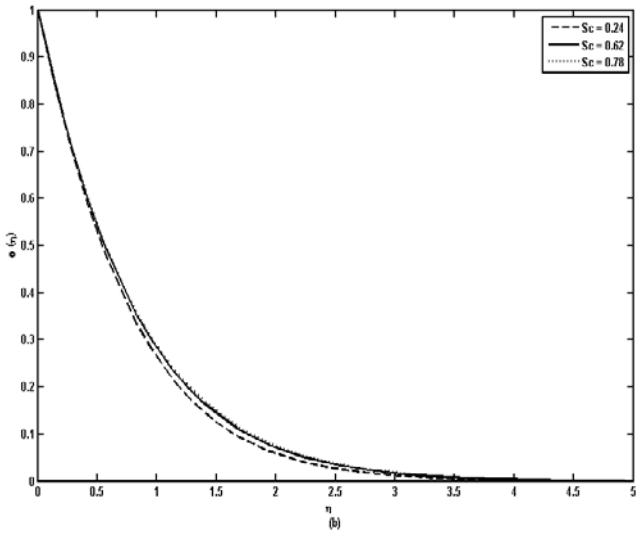



**Figure – 5:** Effect of suction parameter fw on (a) Velocity (b) temperature (c) concentration profiles when Pr = 0.72, Sc = 0.62, Gr = Gc = 2, M = K = Ec = 0.1, R = 1



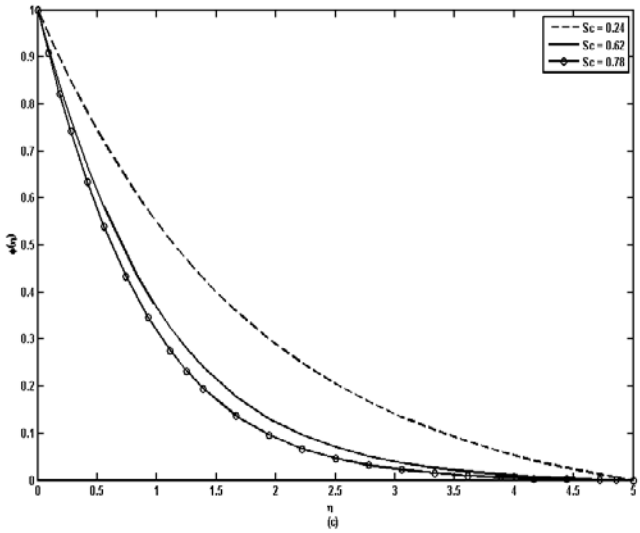
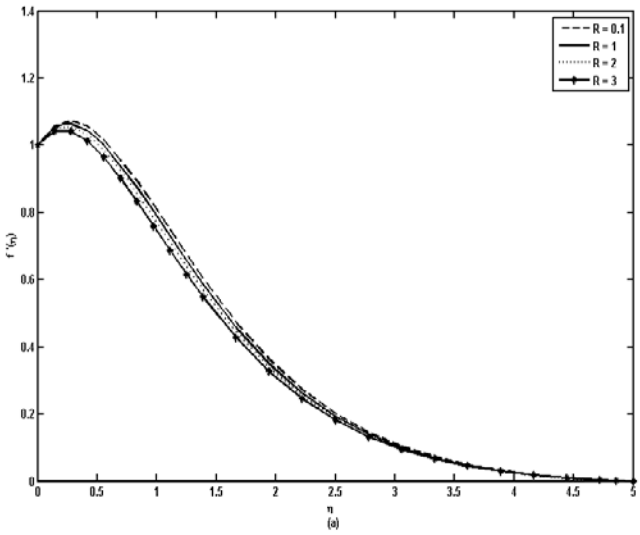
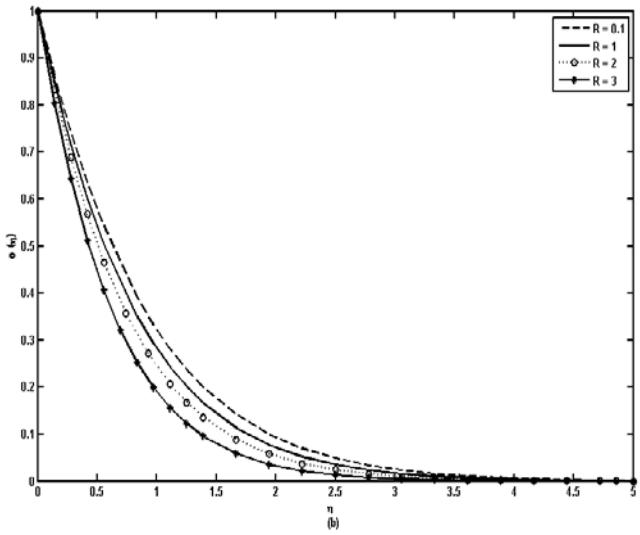



**Figure – 6:** Effect of Sc on (a) Velocity (b) temperature (c) concentration profiles when Pr = 0.72, Gr = Gc = 2, M = fw = K = Ec = 0.1, R = 1



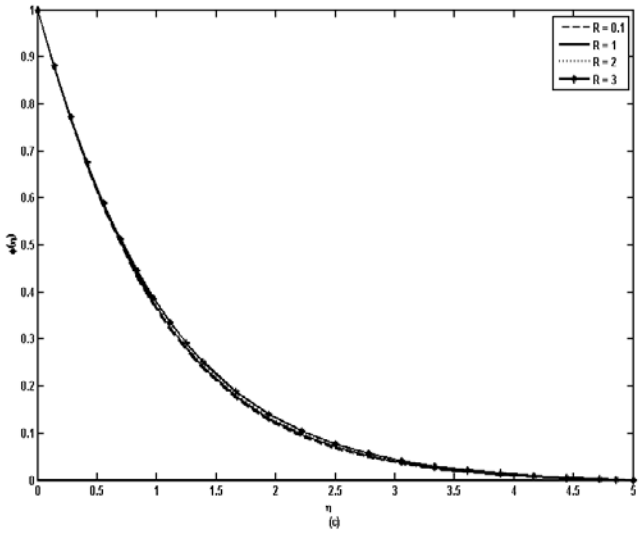
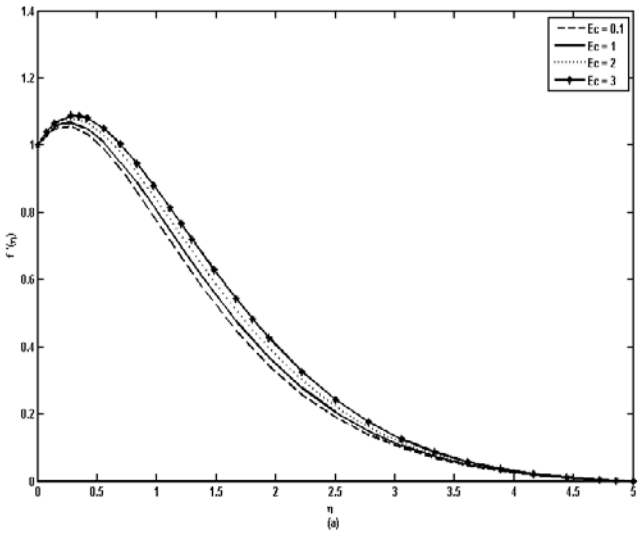
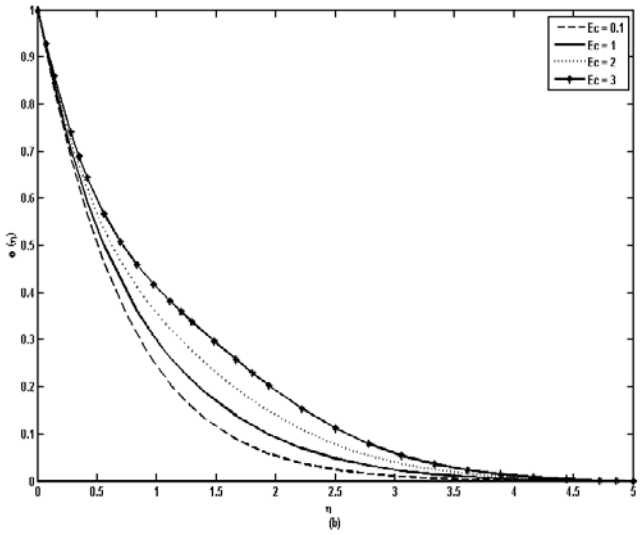



**Figure – 7:** Effect of R on (a) Velocity (b) temperature (c) concentration profiles when Pr = 0.72, Sc = 0.62, Gr = Gc = 2, M = fw = K = Ec = 0.1



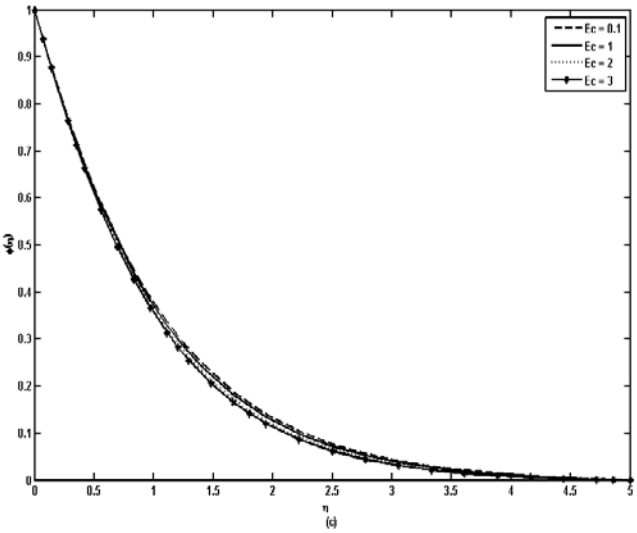



**Figure – 8:** Effect of Ec on (a) Velocity (b) temperature (c) concentration profiles when Pr = 0.72, Sc = 0.62, Gr = Gc = 2, M = fw = K = 0.1, R = 1

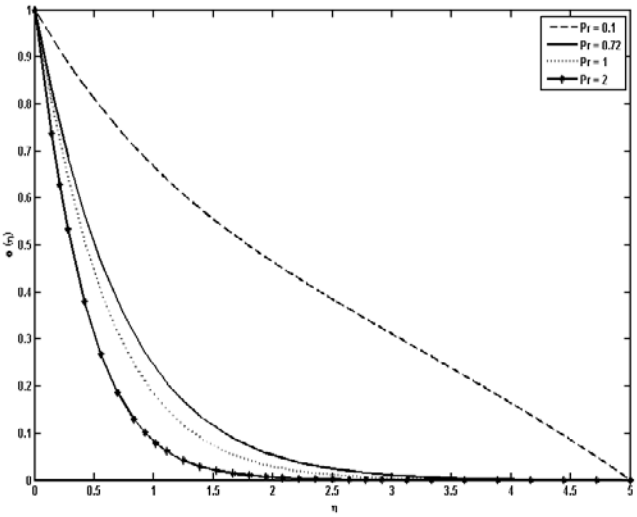



**Figure - 9:** Effect of Pr on temperature profile when Sc = 0.62, Gr = Gc = 2, M = fw = K = 0.1, Ec = 0.1, R = 1

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