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FINITE ELEMENT ANALYSIS OF SORET AND RADIATION EFFECTS ON TRANSIENT MHD FREE CONVECTION FROM AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE WITH HEAT ABSORPTION

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ABSTRACT

This paper investigates the effects of thermal diffusion (Soret) on an unsteady Transient MHD free convective and Heat Transfer of a viscous, incompressible, electrically conducting and heat-absorbing fluid past a suddenly started infinite vertical plate taking into account the thermal radiation is presented. Assuming the medium to be non scattered and the fluid to be non gray, emitting–absorbing, and optically thin radiation limit properties. The dimensionless governing coupled, non linear boundary layer partial differential equations are solved by an efficient finite element method. Computations are performed for a wide range of some important governing flow parameters, viz., Soret, Thermal radiation parameter, Magnetic field parameter (Hartmann number), Heat source parameter and Schmidt number. The effects of these flow parameters on the velocity, temperature and concentration fields are shown graphically. Finally, the effects of these flow parameters on the skin friction, rate of heat and mass transfer coefficients at the wall are prepared through tabular forms. These findings are in quantitative agreement with earlier reported studies.

***Keywords**: Heat and Mass transfer, MHD Flows, Thermal diffusion, Radiation and Finite element method.*

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1. INTRODUCTION

Natural or free convection is a physical process of heat and mass transfer involving fluids which originates when the temperature as well as species concentration change causes density variations inducing buoyancy forces to act on the fluid. Such flows exist abundantly in nature, and due to its applications in engineering and geophysical environments, these have been studied extensively in practice. The heating of rooms inside buildings using radiators is an example of application of heat transfer by free convection. Detailed areas of applications of free convection flow are found in Ghoshdastidar [1] and Nield and Bejan [2]. Welty *et al.* [3] defines mass transfer as the transport of one constituent from a region of higher concentration to that of a lower concentration. Mass transfer is the basis for many biological and chemical processes. Biological processes include the oxygenation of blood and the transport of ions across membranes within the kidney. Chemical processes include the chemical vapour deposition of Silane (SiH_4) onto a silicon wafer, the doping of silicon wafer to form a semiconducting thin film, the aeration of wastewater, and the purification of ores and isotopes. Mass transfer also occurs in many other processes such as absorption, adsorption, drying, precipitation, membrane filtration and distillation. Churchill [4], using an explicit finite difference method. Because the explicit finite difference scheme has its own deficiencies, a more efficient implicit finite difference scheme has been used by Soundalgekar and Ganesan [5]. A numerical solution of transient free convection flow with mass transfer on a vertical plate by employing an implicit method was obtained by Soundalgekar and Ganesan [6]. Takhar *et al.* [7] studied the transient free convection past a semi – infinite vertical plate with variable surface temperature using an implicit finite difference scheme of Crank Nicolson type. Soundalgekar *et al.* [8] analyzed the problem of free convection effects on Stokes problem for a vertical plate under the action of transversely applied magnetic field. Sacheti *et al.* [9] obtained an exact solution for the unsteady MHD free convection flow on an impulsively started vertical plate with constant heat flux. Shanker and Kishan [10] discussed the effect of mass transfer on the MHD flow past an impulsively started vertical plate with variable temperature or constant heat flux.

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Radiation effects on free convection flow have become very important due to its applications in space technology, processes having high temperature, and design of pertinent equipments. Moreover, heat transfer with thermal radiation on convective flows is very important due its significant role in the surface heat transfer. Recent developments in gas cooled nuclear reactors, nuclear power plants, gas turbines, space vehicles, and hypersonic flights have attracted research in this field. The unsteady convective flows in a moving plate with thermal radiation were examined by Cogley *et al.* [11] and Mansour [12]. The combined effects of radiation and buoyancy force past a vertical plate were analyzed by Hossain and Takhar [13]. Hossain *et al.* [14] analyzed the influence of thermal radiation on convective flows over a porous vertical plate. Seddeek [15] explained the importance of thermal radiation and variable viscosity on unsteady forced convection with an align magnetic field. Muthucumaraswamy and Senthil [16] studied the effects of thermal radiation on heat and mass transfer over a moving vertical plate. Pal [17] investigated convective heat and mass transfer in stagnation – point flow towards a stretching sheet with thermal radiation. Aydin and Kaya [18] justified the effects of thermal radiation on mixed convection flow over a permeable vertical plate with magnetic field. Chauhan and Rastogi [19] analyzed the effects of thermal radiation, porosity, and suction on unsteady convective hydromagnetic vertical rotating channel. Ibrahim and Makinde [20] investigated radiation effect on chemically reaction MHD boundary layer flow of heat and mass transfer past a porous vertical flat plate. Pal and Mondal [21] studied the effects of thermal radiation on MHD Darcy Forchheimer convective flow pasta stretching sheet in a porous medium. Palani and Kim [22] analyzed the effect of thermal radiation on convection flow past a vertical cone with surface heat flux. Recently, Mahmoud and Waheed [23] examined thermal radiation on flow over an infinite flat plate with slip velocity. N. Ahmed *et al.* [24] studied the Soret and radiation effects on transient MHD free convection from an impulsively started infinite vertical plate

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic or transfer chemical reactions. M. A. Hossain *et.al.* [24] investigated the problem of natural convection flown along a vertical wavy surface with uniform surface temperature in the presence of heat generation/ absorption. In this direction M.A. Alam *et al*.[25] studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation.

The objective of the present paper is to study the effects of thermal diffusion (Soret) on an unsteady Transient MHD free convective and Heat Transfer of a viscous, incompressible, electrically conducting and heat-absorbing fluid past a suddenly started infinite vertical plate taking into account the thermal radiation presented. The non dimensional governing coupled partial differential equations are involved in the present analysis and are solved by finite element method which is more economical from computational view point and The results obtained are good agreement with the results of Ahmed *et al.* [24] in some special cases.

2. BASIC EQUATIONS

The equations governing the motion of an incompressible, viscous, electrically conducting radiating fluid past a solid surface in presence of a magnetic field are

The equation of continuity: $div\vec{q} = 0$

The momentum equation: $\rho\left[\frac{\partial \vec{q}}{\partial \vec{t}} + \left(\vec{q}.\vec{\nabla}\right)\vec{q}\right] = -\vec{\nabla}p + \vec{J} \times \vec{B} + p\vec{g} + \mu\nabla^2\vec{q}$

The energy equation: $\rho c_p\left[\frac{\partial T'}{\partial t'} + \left(\vec{q}.\vec{\nabla}\right)T'\right] = k\nabla^2 T' + \varphi + \frac{\vec{J}^2}{\sigma} - \frac{\partial q_r}{\partial \eta'} - \rho c_p Q_0\left(T' - T'_\infty\right)$

The species continuity equation: $\frac{\partial C'}{\partial t'} + \left(\vec{q}.\vec{\nabla}\right)C' = D_M \nabla^2 C' + D_T \nabla^2 T'$

The Ohm's law: $\vec{J} = \sigma\left(\vec{E} + \vec{q} \times \vec{B}\right)$

We Consider Soret effect on unsteady radiative MHD free convection and mass transfer flow of an incompressible, viscous, and electricity conducting and heat-absorbing fluid past a suddenly moving infinite vertical plate in presence of a uniform transverse magnetic field of strength B0.
Our investigation is restricted to the following assumptions:

i. All the fluid properties except the density in the buoyancy force term are constants.
ii. The viscous dissipation of energy is negligible.
iii. The Ohmic dissipation of energy is negligible.
iv. The magnetic Reynolds number is so small that the induced magnetic field can be neglected.
v. The flow is parallel to the plate.

Initially, the plate and the surrounding fluid were at rest at the same temperature T'_∞ with concentration level C'_∞ at all points in the fluid. At time $t' > 0$, the plate is suddenly moved in its own plane with velocity U_0. The plate temperature and concentration are instantly raised to $T'_w(< T'_\infty)$ and $C'_w(< C'_\infty)$, which are thereafter maintained constant.We introduce a coordinate system (x', y', z') with X'-axis along the plate in the upward vertical direction, Y'-axis normal to the plate and directed into the fluid region and the Z'-axis along the width of the plate. Let $q' = (u', 0, 0)$ be the fluid velocity and $B' = (0, B_0, 0)$ be the magnetic induction vector at a point (x', y', z') in the fluid at time $t' > 0$,Under the above foregoing assumptions and Boussinesq approximation, the equations governing the flow and transport reduce to the following equations:

Continuity Equation:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial u'}{\partial t'} = v' \frac{\partial^2 u'}{\partial y'} + g\beta(T' - T'_\infty) + g\beta'(C' - C'_\infty) - \frac{\sigma\beta_0^2}{\rho} u' \tag{2}$$

Energy Equation:

$$\rho c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} - \rho c_p Q_0 (T' - T'_\infty) \tag{3}$$

Species Diffusion Equation:

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + D_T \frac{\partial^2 T'}{\partial y'^2} \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$\forall t' \le 0 : u' = 0, T' = T'_\infty, C' == C'_\infty, \forall y' \tag{5}$$

$$\forall t' > 0 : u' = U_0, T' = T'_w, C' = C'_w, \text{ at } y' = 0;\ u' = 0, T' \to T'_\infty, C' \to C'_\infty \text{ at } y' \to \infty \tag{6}$$

As mentioned earlier, in the case of optically thin limit, the fluid cannot absorb its own emitted radiation, but it absorbs the radiation emitted by boundaries. Following Cogly et al. [11], the rate of flux of radiation in the optically thin limit for a non gray gas near equilibrium is given by

$$\frac{\partial q_r}{\partial y'} = 4I(T' - T'_\infty) \tag{7}$$

where $I = \int_0^\infty K_{\lambda w} \left(\frac{de_{b\lambda}}{dT'} \right) d\lambda$

$K_{\lambda w}$ being the absorption coefficient and $e_{b\lambda}$ is the Plank function.

We now introduce the following non dimensional variables and parameters:

$$y = \frac{U_0 y'}{v}, t = \frac{U_0^2 t'}{v}, u = \frac{u'}{U_0}, T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \Pr = \frac{\mu c_p}{k}, Sc = \frac{v}{D_M}, Q = \frac{vQ_0}{U_0^2}$$

$$Gr = vg\beta \frac{T'_w - T'_\infty}{U_0^3}, Gm = g\beta' v \frac{C'_w - C'_\infty}{U_0^3}, R = \frac{4Iv^2}{U_0^2 k}, M = \frac{\sigma B_0^2 uv}{\rho U_0^2}, Sr = \frac{D_T (T'_w - T'_\infty)}{v(C'_\infty - C'_\infty)}$$

The non dimensional form of Eqs. (2) – (6) are

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr(T) + Gm(C) - Mu \tag{8}$$

$$\Pr\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial y^2}-(R+Q)T \tag{9}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+Sr\frac{\partial^2 T}{\partial y^2} \tag{10}$$

With the conditions

$$u=0,\quad T=0,\quad C=0,\quad \forall y,\quad t\le 0.$$

$$\left.\begin{array}{l} u=1\quad T=1\quad C=1\quad at\quad y=0 \\ u=0\quad T=0\quad C=0\quad at\quad y\to\infty \end{array}\right\}\quad \forall t>0 \tag{11}$$

3. METHOD OF SOLUTION

By applying Galerkin finite element method for (8) over the element (e), $(y_j \le y \le y_k)$ is:

$$\int_{y_j}^{y_k}\left\{N^{(e)^T}\left[\frac{\partial^2 u^{(e)}}{\partial y^2}-\frac{\partial u^{(e)}}{\partial t}-Mu^{(e)}+P\right]\right\}dy=0 \tag{12}$$

Where $P=(Gr)T+(Gm)C$

Integrating the first term in Eq. (15) by parts one obtains

$$\left\{N^{(e)^T}\frac{\partial u^{(e)}}{\partial y}\right\}_{y_j}^{y_k}-\int_{y_j}^{y_k}\left\{\frac{\partial N^{(e)^T}}{\partial y}\frac{\partial u^{(e)}}{\partial y}-N^{(e)^T}\left[\frac{\partial u^{(e)}}{\partial t}+Mu^{(e)}-P\right]\right\}dy=0 \tag{13}$$

Neglecting the first term in Eq. (12), one gets:

$$\int_{y_j}^{y_k}\left\{\frac{\partial N^{(e)^T}}{\partial y}\frac{\partial u^{(e)}}{\partial y}+N^{(e)^T}\left[\frac{\partial u^{(e)}}{\partial t}+Mu^{(e)}-P\right]\right\}dy=0$$

Let $u^{(e)}=N^{(e)}\phi^{(e)}$ be the finite element approximation solution over the element $(y_j \le y \le y_k)$

Where $N^{(e)}=\begin{bmatrix}N_j & N_k\end{bmatrix},\ \phi^{(e)}=\begin{bmatrix}u_j & u_k\end{bmatrix}^T$ and $N_j=\dfrac{y_k-y}{y_k-y_j}, N_k=\dfrac{y-y_j}{y_k-y_j}$ are the basis functions,

One obtains:

$$\int_{y_j}^{y_k}\left\{\begin{bmatrix}N'_jN'_j & N'_jN'_k\\ N'_jN'_k & N'_kN'_k\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}\right\}dy+\int_{y_j}^{y_k}\left\{\begin{bmatrix}N_jN_j & N_jN_k\\ N_jN_k & N_kN_k\end{bmatrix}\begin{bmatrix}\dot{u}_j\\ \dot{u}_k\end{bmatrix}\right\}dy+M\int_{y_j}^{y_k}\left\{\begin{bmatrix}N_jN_j & N_jN_k\\ N_jN_k & N_kN_k\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}\right\}dy$$

$$=P\int_{y_j}^{y_k}\begin{bmatrix}N_j\\ N_k\end{bmatrix}dy$$

Simplifying we get

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_j\\ \dot{u}_k\end{bmatrix}+\frac{M}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 1\end{bmatrix}$$

Where prime and dot denotes differentiation w.r.to'y' and time't' respectively. Assembling the element Equations for two consecutive elements $(y_{i-1} \le y \le y_i)$ and $(y_i \le y \le y_{i+1})$ following is obtained

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_{i-1}\\ \dot{u}_i\\ \dot{u}_{i+1}\end{bmatrix}+\frac{M}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \tag{14}$$

Now put row corresponding to the node 'i' to zero, from Eq. (14) the difference schemes with $l^{(e)} = h$ is:

$$\frac{1}{h^2}\left[-u_{i-1}+2u_i-u_{i+1}\right]+\frac{1}{6}\left[\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1}\right]+\frac{M}{6}\left[u_{i-1}+2u_i+u_{i+1}\right]=P \tag{15}$$

Applying the trapezoidal rule, following system of equations in Crank-Nicholson method are obtained:

$$A_1u_{i-1}^{n+1}+A_2u_i^{n+1}+A_3u_{i+1}^{n+1}=A_4u_{i-1}^{n}+A_5u_i^{n}+A_6u_{i+1}^{n}+P^* \tag{16}$$

Now from Eqs. (9) and (10), following equations are obtained:

$$B_1T_{i-1}^{n+1}+B_2T_i^{n+1}+B_3T_{i+1}^{n+1}=B_4T_{i-1}^{n}+B_5T_i^{n}+B_6T_{i+1}^{n} \tag{17}$$

$$C_1C_{i-1}^{n+1}+C_2C_i^{n+1}+C_3C_{i+1}^{n+1}=C_4C_{i-1}^{n}+C_5C_i^{n}+C_6C_{i+1}^{n}+S^* \tag{18}$$

Where

$$A_1=2-6r+Mk, A_2=8+12r+4Mk, A_3=2-6r+Mk, A_4=2+6r-Mk,$$
$$A_5=8-12r-4Mk, A_6=2+6r-Mk, P^*=12kP=12k\left[(Gr)T_i^n+(Gm)C_i^n\right]$$
$$B_1=-6r+2\,\mathrm{Pr}+(R+Q)k, B_2=12r+8\,\mathrm{Pr}+4(R+Q)k, B_3=-6r+2\,\mathrm{Pr}+(R+Q)k,$$
$$B_4=6r+2\,\mathrm{Pr}-(R+Q)k, B_5=-12r+8\,\mathrm{Pr}-4(R+Q)k, B_6=6r+2\,\mathrm{Pr}-(R+Q)k,$$
$$C_1=-3r+Sc, C_2=6r+4Sc, C_3=-3r+Sc, C_4=3r+Sc, C_5=-6r+4Sc, C_6=3r+Sc$$
$$S^*=6kScSr\frac{\partial^2T}{\partial y^2}=6kScSr\frac{\partial^2T_i}{\partial y_i^{\,2}}$$

Here $r=\frac{k}{h^2}$ and h, k are mesh sizes along y –direction and time-direction respectively. Index 'i' refers to space and 'j' refers to the time. In the Eqs. (16), (17) and (18), taking i = 1to n and using boundary conditions (11), then the following system of equations are obtained:

$$A_iX_i=B_i \quad i=1 \text{ to } n \tag{19}$$

Where A_i 's are matrices of order n and X_i *'s*, B_i 's are column matrices having n-components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by *C* – programme. In order to prove the convergence and stability of Galerkin finite element method, the same *C* – programme was run with smaller values of h and k and no significant change was observed in the values of *u, T* and *C*. Hence the Galerkin finite element method is stable and convergent.

4. SKIN – FRICTION, RATE OF HEAT AND MASS TRANSFER

The skin friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. The skin friction at the plate, which in the non – dimensional form is given by $\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}$ (20)

The rate of heat transfer coefficient, which in the non – dimensional form in terms of the Nusselt number is given by

$$N_u=-\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{21}$$

The rate of mass transfer coefficient, which in the non – dimensional form in terms of the Sherwood number, is given by $S_b=-\left(\frac{\partial C}{\partial y}\right)_{y=0}$ (22)

5. RESULTS AND DISCUSSION

In the preceding sections, the problem on the effects of thermal diffusion (Soret) on an unsteady Transient MHD free convective and Heat Transfer of a viscous, incompressible, and electrically conducting fluid past a suddenly started infinite vertical plate taking into account the thermal radiation as well as heat source or sink was formulated and solved by finite element technique. The expressions for the velocity, temperature and concentration were obtained. To illustrate the behavior of these physical quantities, numeric values were computed with respect to the variations in the governing parameters viz., the thermal Grashof number, Modified Grashof number, radiation parameter, Prandtl number, Schmidt number, Soret number and Heat source parameter *Gr, Gm, R, Pr, Sc, Sr, Q.*

The effect of the thermal Grashof number *Gr* on the velocity is presented in Fig.1(a). The thermal Grashof number *Gr* signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. It is observed that an increase in *Gr* causes a significant increase in the velocity. Therefore, with an increase in buoyancy force due to temperature differences. Here the positive values of *Gr* correspond to cooling of the plate by free convection. The influence of the modified Grashof number *Gm* on the velocity is presented in Fig.1 (b). The modified Grashof number *Gm* defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is found that the velocity increases considerably with a rise in *Gm*. Fig. 2(a) and Fig. 2(b) show the velocity and temperature profiles for different values of the Radiation parameter, It is found that the temperature profiles *T*, being as a decreasing function of *R*, decelerate the flow and reduce the fluid velocity. Such an effect may also be expected, as increasing radiation parameter *R* makes the fluid thick and ultimately causes the temperature and the thermal boundary layer thickness to decrease. Effects of magnetic parameter M on velocity are shown in Fig. 3. It can be observed that, with the increase in M, velocity decrease. Physically, it meets the logic that the magnetic field extorts a retarding force on free convection flow which retards the flow. It can be also noticed that, near the plate in the vicinity of the boundary layer, velocity was considerably high and gradually and uniformly decreased thereafter. Velocity, temperature, and concentration profiles for some realistic values of Prandtl number *Pr* = 0.015, 0.71, 1.0, 7.0, which are important in the sense that they physically correspond to mercury, air, electrolytic solution and water are shown in Figures 4(a),(b),(c) respectively. From Fig. 4(a), it is found that the momentum boundary layer thickness increases for the fluids with *Pr* <1 and decreases for *Pr* > 1. The Prandtl number actually describes the relationship between momentum diffusivity and thermal diffusivity and hence controls the relative thickness of the momentum and thermal boundary layers. When *Pr* is small, that is, *Pr* =0.015, it is noticed that the heat diffuses very quickly compared to the velocity (momentum).This means that for liquid metals the thickness of the thermal boundary layer is much bigger than the velocity boundary layer. In Figure 4(b), we observe that the temperature decreases with increasing values of Pr. It is also observed that the thermal boundary layer thickness is maximum near the plate and decreases with increasing distances from the leading edge approaches to zero. And finally Furthermore, it is noticed that the thermal boundary layer for mercury which corresponds to Pr =0.015 is greater than those for air, electrolytic solution and water. It is justified due to the fact that thermal conductivity of fluid decreases with increasing Prandtl number Pr and hence decreases the thermal boundary layer thickness and the temperature profiles. We observed from Figure 4(c) that the Concentration of the fluid increases for large values of Prandtl number Pr. Fig.5(a). Reveals that velocity profiles decrease with the increase of Schmidt number Sc, while an opposite phenomenon is observed in case of Soret number Sr as shown in Fig. 6(a). It is observed in Fig. 5(b). that the flow is accelerated under the influence of the Schmidt number near the plate but this effect takes reverse turn at large distance from the plate. And it is clearly show that the fluid concentration quickly increases up to some thin layer of the fluid adjacent to the plate and, after this fluid layer, the concentration asymptotically decreases to its zero value as y tends to infinite.

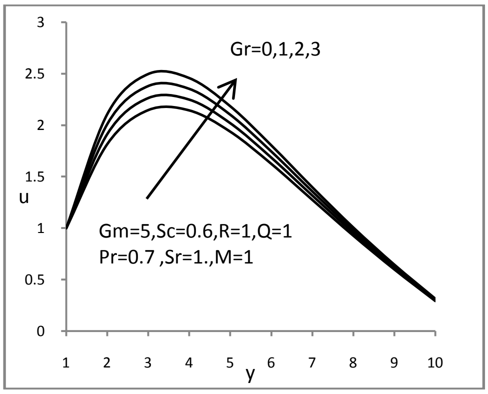

Fig.1(a). Effect of Grashof Number 'Gr' on velocity field

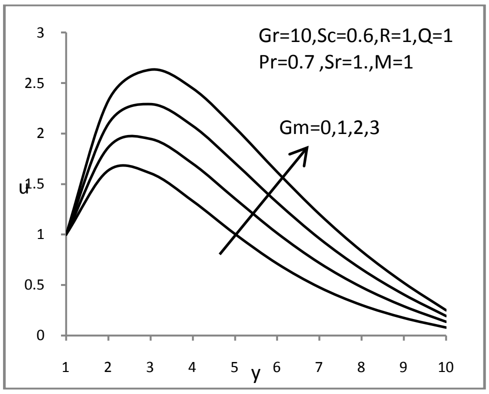

Fig.1.(b) Effect of modified Grashof Number 'Gm' on velocity field

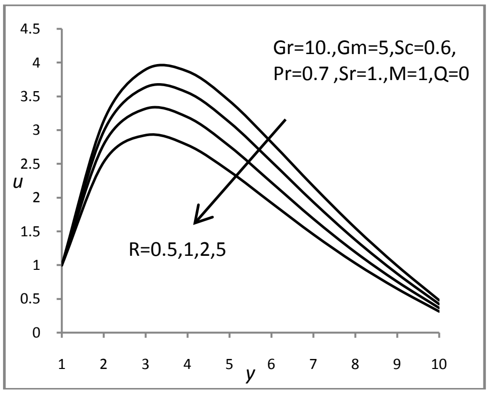

Fig.2.(a). Effect of Radiation Parameter 'R' on velocity field

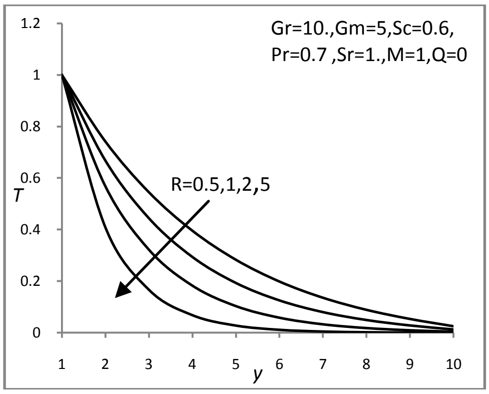

Fig.2(b). Effect Radiation parameter 'R' on Temperature field

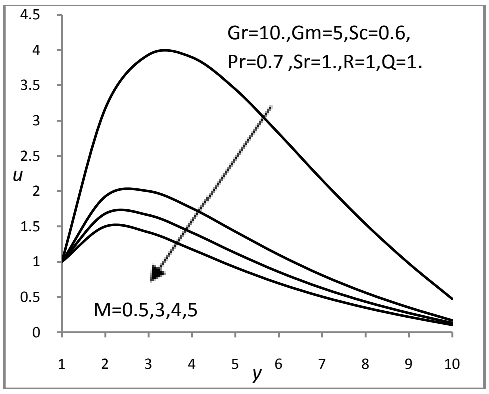

Fig.3. Effect of Hartmann number 'M' on velocity field

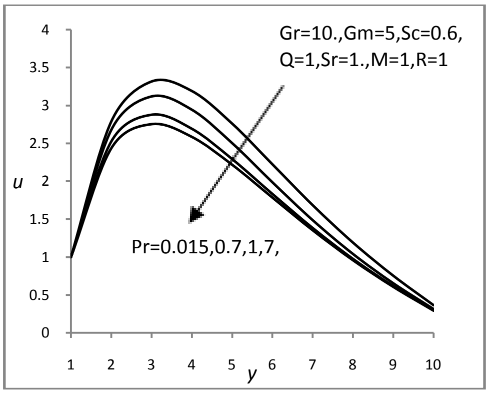

Fig.4.(a). Effect Prandlt number 'Pr' on velocity field

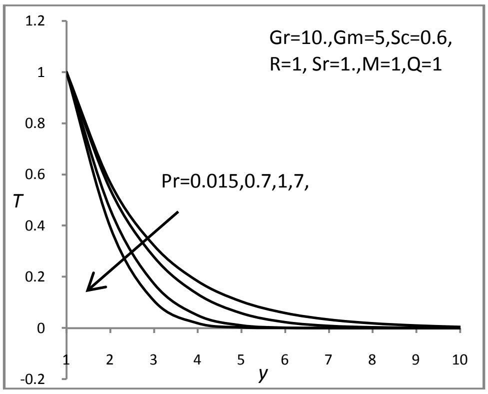

Fig.4.(b). Effect Prandtl number 'Pr' on Temperature field

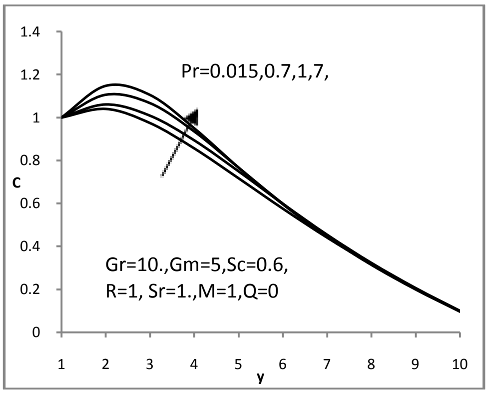

Fig.4.(c). Effect Prandtl number 'Pr' on Concentration field

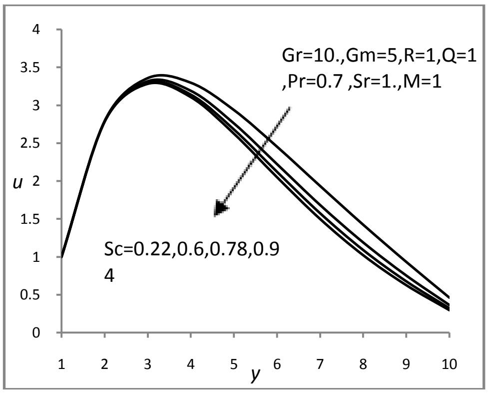

Fig.5(a). Effect Schmidt number 'Sc' on velocity field

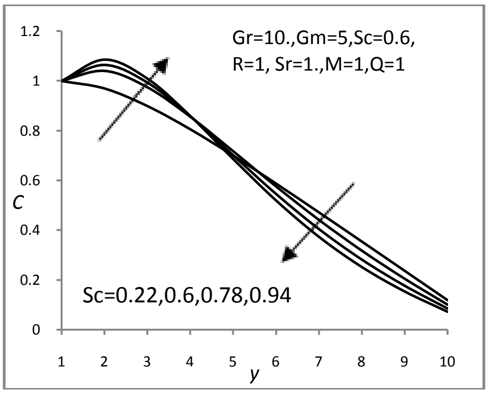

Fig.5.(b). Effect of Schmidt number 'Sc' on concentration field

Table - 1. Grashof number, Modified Grashof number, Radiation effects on skin friction with some fixed constant parameters.

Gr	τ	Gm	τ	R	τ	M	τ
0	2.769855	0	2.989721	0.5	7.739193	0.5	7.889568
1	3.17145	1	3.747527	1	7.321362	3	4.144252
2	3.570799	2	4.50344	2	6.782664	4	3.32994
3	3.97406	3	5.267562	5	6.050464	5	2.681346

The Soret number (*Sr*) does not enter directly into the momentum and energy equations. Thus the effect of Soret number on velocity and temperature profiles is not apparent. Fig. 6(a), 6(b) shows the variation of velocity and concentration profiles for different values of *Sr*. The parameter *Sr* has marked effects on the velocity and concentration profiles. It is observed that the velocity and concentration profiles increase with the increasing values of *Sr* . It is also observed from the fig. 6(b) that when *Sr* = 0.5, that is, when the ratio between concentration and temperature gradient is very small the concentration profile shows its usual trend of gradual decay. As Soret number *Sr* becomes large the profiles overshoot the uniform concentration close to the boundary. Ahmed [24] solved by Exact method, In order to ascertain the accuracy of the numerical results, the Heat transfer coefficient (N_u) results are compared with results of Ahmed [24] in table – (4). They are found to be in an excellent agreement.

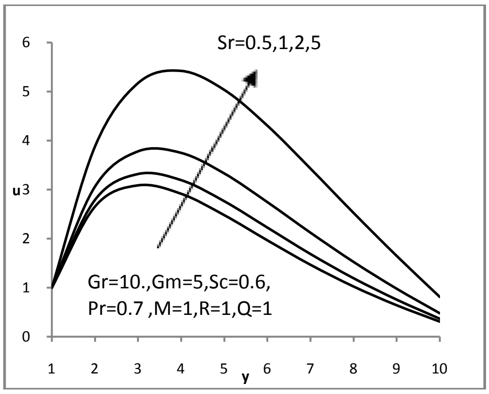

Fig.6(a). Effect of Soret number '*Sr*' on velocity field

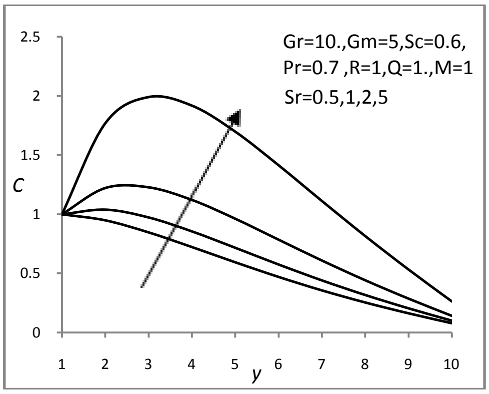

Fig.6(b). Effect of Soret number '*Sr*' on concentration field

Table - 2. Prandtl number effect on skin friction, Heat transfer, Mass transfer coefficient with fixed parameters *Gr*=10, *Gm*=5, *Sc*=0.6, *R=1*,*Q=1*, *Sr* =1 and *M*=1.

Pr	τ	N_u	S_b
0.71	6.782664	1.40947	-0.2915
3	6.489805	1.48654	-0.3534
7	6.081552	1.69932	-0.4805
11	5.849016	1.83862	-0.5597

Table - 3. Schmidt and Soret number effect on skin friction, Mass transfer coefficient with fixed parameters *Gr*=10, *Gm*=5, *Sc*=0.6,*Pr*=0.7 , *M*=1.

Sc	τ	S_b	*Sr*	τ	S_b
0.22	6.776735	-0.0018	0.5	6.418193	0.02415
0.6	6.782664	-0.2915	1	6.782664	-0.2915
0.7	6.790514	-0.4127	2	7.516401	-0.9569
0.98	6.799995	-0.5237	5	9.69737	-2.9175

Table - 4: Comparisons of Nu with Ahmed [24] had shown by the superscript star, (Q is replaced by R in [24]) with different Radiation parameter values

R	Pr=0.7* Exact Method	Pr=7* Exact Method	Pr=0.7 (Present results) Finite Element Method	Pr=7 (Present results) Finite Element Method
1	1.02215	1.70101	1.02325	1.70241
2	1.41753	1.89999	1.41923	1.89059
3	1.73263	2.09038	1.73625	2.09316
4	2.00011	2.27286	2.00183	2.27581
5	2.23609	2.44805	2.23590	2.44818

6. CONCLUSIONS

We have formulated the problem of two-dimensional fluid flow in the presence of radiative heat transfer, and thermal diffusion. A finite element technique is employed to solve the resulting coupled partial differential equations. The following conclusions are drawn from the study.

1. The velocity increases with the increase in thermal Grashof, Modified Grashof, and Soret numbers, Concentration increases with the increase Prandtl and Soret numbers.
2. Concentration increases suddenly decreases with the increasing of Schmidt number.
3. The velocity as well as temperature decreases with an increase in the Radiation parameter and Prandtl number, The velocity decreases with an increase of Harman number.
4. Skin friction increases with the increase in thermal Grashof, Modified Grashof, and Soret numbers, decreases with increasing of Radiation parameter, Prandtl number and Harman number.
5. Heat transfer coefficient increases with increasing of Prandtl number and Radiation parameter, Mass transfer coefficient decreasing with increasing of Prandtl , Schmidt and Soret number
6. On comparing the heat transform coefficient results with heat transform coefficient (Nu) results of Ahmed [24], it can be seen that they agree very well.

NOMENCLATURE

$\overline{B}$	Magnetic induction vector
B_0	Strength of the applied magnetic field
c_p	Specific heat at constant pressure
c'	Species concentration
c'_∞	Species concentration of the fluid far away from the plate
c'_w	Species concentration at the plate
D_M	Molecular mass diffusivity
D_T	Thermal diffusivity
E	Electric field
Gm	Grashof number for mass transfer
Gr	Grashof number for heat transfer
$\overline{g}$	acceleration due to gravity
g	acceleration due to gravity in magnitude
$\overline{J}$	Electric current density
$\frac{\overline{J}^2}{\sigma}$	Joule heat per unit volume
k	Thermal conductivity
M	Hartmann number
$\Pr$	Prandtl number
p	Fluid pressure
q_r	radiative flux
R	Radiation parameter
Q	Heat source parameter
Sr	Soret number
Sc	Schmidt *Number*
T'_w	Temperature at the plate
T'_∞	Temperature of the fluid far away from the plate
t'	Time
T'	Temperature
U_0	plate velocity
T	non dimensional temperature
C	non dimensional species concentration
μ	Coefficient of the viscosity
ρ	Fluid density
φ	Dissipation of energy per unit volume due to viscosity
δn	an element of the normal to the surface
σ	Electrical conductivity
ν	kinematic viscosity
β	Coefficient of volume expansion for heat transfer
β'	Coefficient of volume expansion for mass transfer

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