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EFFECT OF MAGNETIC FIELD ON THE UNSTEADY FLOW OF DUSTY INCOMPRESSIBLE SECOND ORDER OLDROYD VISCO-ELASTIC LIQUID THROUGH THE RIGHT CIRCULAR CYLINDER

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ABSTRACT

The purpose of the present paper is to analyse the effect of uniform magnetic field applied perpendicularly to the unsteady flow of dusty incompressible Oldroyd visco-elastic liquid of second order under the influence of transient pressure gradient through a long right circular cylinder. This problem has been solved in the generalized visco-elastic model and the velocity field for visco-elastic liquid and the dust particles have been derived analytically in the closed form. The particular cases corresponding to Oldroyd, Maxwell, Rivlin-Ericksen dusty liquid and ordinary viscous dusty fluid models are derived for velocity field. Also we have deduced the case when uniform magnetic field is withdrawn.

INTRODUCTION

Interest in problems of mechanics of system with more than one phase has developed rapidly in the past few years. Situations which occur frequently are concerned with the flow of a liquid or gas which contains uniformly distribution of solid particles. Such situations arise, for instance the movement of dust laden air, in fluidization, in the use of dust in gas cooling system, in hydro cyclones, in problems of pollution, in tidal waves etc.The mathematical description of such divers systems must of course vary widely. Model equations describing the motion of such mixed system have been given by Saffman (1962).

There is another class of flow problems which concerns with the study of the flow of the dusty visco-elastic liquids such as latex particles in emulsion paints, reinforcing particles in polymer melts and rock crystals in molten lava etc. However the studies of this class of problems and rheological aspects of such flow have not received much attention although this has become bearing on the problems of petroleum and chemical engineering interest. The unsteady flow of dusty visco-elastic liquids of various kinds through channels of various cross-section with time dependent pressure gradient have been studied by many researchers such as : Dube and Srivastava (1972), Bagchi and Maiti (1980), Kumar and Singh (1990); Garg, Shrivastava and Singh (1994), Johri and Gupta (1999), Kundu and Sengupta (2001); Das (2004),Singh Gupta and Varshney (2005); Varshney and Singh (2006); Singh (2010); Agrawal, Agrawal and Varshney (2012); Singh and Varshney (2012); Prasad, Nagaich and Varshney (2012); Mishra, Kumar and Singh (2013) etc. have discussed the effect of magnetic field on the flow of dusty incompressible visco-elastic second order Oldroyd fluid through a rectangular channel.

In the present problem our aim is to discuss the unsteady flow of second order Oldroyd visco-elastic liquid through a long right circular cylinder under the influence of uniform magnetic field applied perpendicularly to the flow of visco-elastic liquid with transient pressure gradient. The analytical solutions for velocity of visco-elastic liquid and the dust particles are obtained in eligent form. The particular cases for dusty visco-elastic Oldroyd (1958) model liquid, dusty visco-elastic Maxwell liquid, dusty Rivlin-Ericksen liquid, dusty viscous liquid have been derived. Also we have deduced the case when the magnetic field is withdrawn.

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BASIC THEORY FOR SECOND ORDER OLDROYD VISCO-ELASTIC LIQUID

For slow motion, the rheological equations for second order Oldroyd visco-elastic liquid are:

$$\left.\begin{aligned} \tau_{ij} &= -p\delta_{ij} + \tau_{ij}^{'} \\ \left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\tau_{ij}^{'} &= 2\mu\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)e_{ij} \\ e_{ij} &= \frac{1}{2}(v_{i,j}+v_{j,i}) \end{aligned}\right\} \tag{1}$$

where τ_{ij} is the stress tensor, $\tau_{ij}^{'}$ the deviatoric stress tensor, e_{ij} the rate of strain tensor, p the fluid pressure, λ_1 the stress relaxation time parameter, μ_1 the strain rate retardation time parameter, λ_2 the material constant, μ_2 the material constant, δ_{ij} the metric tensor, μ the coefficient of viscosity and v_i is the velocity components.

FORMULATION OF THE PROBLEM

Following assumptions have been considered for the equations of motion;

1. The interaction between particles themselves has not been considered.
2. Throughout the motion, density of the dust particles is taken to be constant.
3. The temperature within particles is considered as uniform.
4. The boundary force is neglected.
5. The dust particles are non-conducting and uniform spherical in small size.
6. Mass transfer, radiation and chemical reaction between particles and liquid are not consideration.
7. The effects of the induced magnetic field and the electric field produced by the motion of selectrically conducting visco-elastic liquid are negligible.

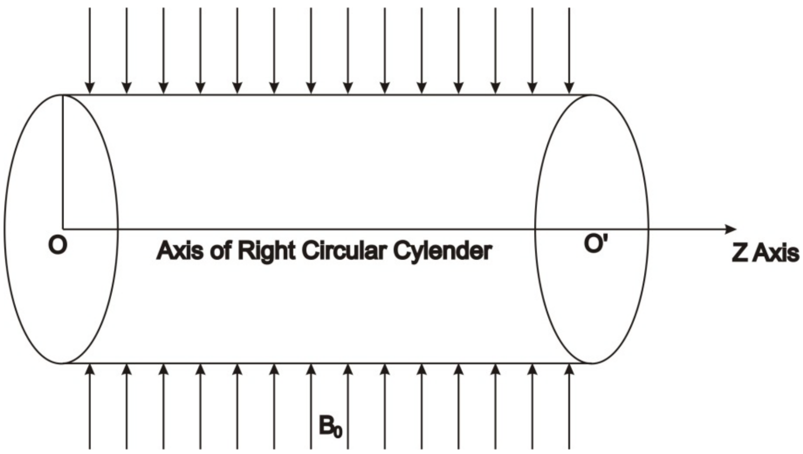

Fig: Schematic diagram of dusty visco-elastic fluid flow in a right circular cylinder

Let $P(r,\theta,z)$ be the cylindrical polar coordinates in a right circular cylinder of radius a and if (u_r,u_θ,u_z) and (v_r,v_θ,v_z) are the velocity components of the liquid and dust particles respectively at point P. Consider the flow of dusty visco-elastic liquid through a long right circular cylinder of radius a in the direction of z-axis i.e. along the axis of the channel, therefore

$$\left.\begin{aligned} &u_r=0,\ \ u_\theta=0,\ \ \ u_z=u_z(r,t) \\ &v_r=0,\ \ v_\theta=0,\ \ \ v_z=v_z(r,t) \end{aligned}\right\} \tag{2}$$

Following Saffman (1962), the equations of motion for a dusty second order visco-elastic Oldroyd liquid under the influence of transverse magnetic field are:

$$\begin{aligned} \left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial u_z}{\partial t} &= -\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial p}{\partial z}+\upsilon\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}\right) \\ &\quad +\frac{kN_0}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)(v_z-u_z)-\frac{\sigma B_0^2}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)u_z \end{aligned} \tag{3}$$

$$m\frac{\partial v_z}{\partial t} = k(u_z - v_z) \tag{4}$$

where ρ is the density of liquid, m the mass of particle, k the stokes resistance coefficient, N_0 the number of density of particles, σ the conductivity of the liquid and B_0 is the intensity of magnetic field.

The boundary conditions for liquid and dust particles are:

$$\left.\begin{aligned} &u_z = 0, \quad v_z = 0, \quad at\ r = a \\ &u_z = finite, \quad v_z = finite, \quad at\ r = 0 \end{aligned}\right\} \tag{5}$$

Introducing the following non-dimensional quantities:

$$\left.\begin{aligned} &u^* = \frac{a}{\nu}u_z\,,\ \ v^* = \frac{a}{\nu}v_z\,,\quad p^* = \frac{a^2}{\rho\nu^2}p\,,\ \ t^* = \frac{\nu}{a^2}t\,,\ \ r^* = \frac{r}{a}\,, \\ &z^* = \frac{z}{a}\,,\ \ \lambda_1^* = \frac{\nu}{a^2}\lambda_1\,,\ \ \mu_1^* = \frac{\nu}{a^2}\mu_1\,,\ \ \lambda_2^* = \frac{\nu^2}{a^4}\lambda_2\,,\ \ \mu_2^* = \frac{\nu^2}{a^4}\mu_2 \end{aligned}\right\}$$

In the equations (3), (4) and boundary conditions (5), we get (after dropping the stars)

$$\left(1 + \lambda_1\frac{\partial}{\partial t} + \lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial t} = -\left(1 + \lambda_1\frac{\partial}{\partial t} + \lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial p}{\partial z} + \left(1 + \mu_1\frac{\partial}{\partial t} + \mu_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$
$$+\beta\left(1 + \lambda_1\frac{\partial}{\partial t} + \lambda_2\frac{\partial^2}{\partial t^2}\right)(v - u) - H^2\left(1 + \lambda_1\frac{\partial}{\partial t} + \lambda_2\frac{\partial^2}{\partial t^2}\right)u \tag{6}$$

$$\frac{\partial v}{\partial t} = \frac{1}{\gamma}(u - v) \tag{7}$$

and boundary conditions are:

$$\left.\begin{aligned} &u = 0, \quad v = 0, \quad at\ r = 1 \\ &u = finite, \quad v = finite, \quad at\ r = 0 \end{aligned}\right\} \tag{8}$$

where

$$\beta = \frac{f_0}{\gamma} = \frac{N_0 k a^2}{\rho\nu}, \qquad f_0 = \frac{mN_0}{\rho}, \qquad \gamma = \frac{m\nu}{ka^2}\ \ and\ \ H = aB_0\sqrt{\frac{\sigma}{\mu}} \quad (Hartmann\ number)$$

Since a transient pressure gradient $-Pe^{-\omega t}$ varying with time t is applied to the dusty visco-elastic Oldroyd liquid of second order, therefore we may choose the solution of equations (4) and (5) as

$$\left.\begin{aligned} &u = U(r)e^{-\omega t} \\ &v = V(r)e^{-\omega t} \end{aligned}\right\} \tag{9}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &U = 0, \quad V = 0, \quad at\ r = 1 \\ &U = finite, \quad V = finite, \quad at\ r = 0 \end{aligned}\right\} \tag{10}$$

Putting u and v in eqn. (7), we get

$$V = \frac{U}{1 - \gamma\omega} \tag{11}$$

From eqn. (6) with the help of (9) and (11), we get

$$\frac{d^2U}{dr^2} + \frac{1}{r}\frac{dU}{dR} + M^2U = C \tag{12}$$

where

$$M = \left[\frac{\{\omega(1 - \gamma\omega + \beta\gamma) - H^2(1 - \gamma\omega)\}(1 - \lambda_1\omega + \lambda_2\omega^2)}{(1 - \gamma\omega)(1 - \mu_1\omega + \mu_2\omega^2)}\right]^{\frac{1}{2}} \tag{13}$$

and

$$C = \left(\frac{1 - \lambda_1\omega + \lambda_2\omega^2}{1 - \mu_1\omega + \mu_2\omega^2}\right)\mathrm{P} \tag{14}$$

Now, by solving eqn. (12) with the help of boundary conditions (10), we obtain

$$U(r) = \frac{C}{M^2}\left(\frac{J_0(Mr)}{J_0(M)} - 1\right) \tag{15}$$

where J_0 is the Bessel's function of zeroth order.

From (9) and (15), we obtain the velocity of second order Oldroyd visco-elastic liquid

$$u = \frac{C}{M^2}\left(\frac{J_0(Mr)}{J_0(M)} - 1\right)e^{-\omega t} \tag{16}$$

and from (9), (11) and (15), we obtain the velocity of dust particles

$$v = \frac{C}{(1-\gamma\omega)M^2}\left(\frac{J_0(Mr)}{J_0(M)} - 1\right)e^{-\omega t} \tag{17}$$

PARTICULAR CASES

CASE I: If material constants λ_2 and μ_2 both are zero i.e.$\lambda_2 = 0$ and $\mu_2 = 0$, then from (16) and (17), we obtain velocities of Oldroyd visco-elastic liquid and the dust particles respectively, where M will be

$$M = \left[\frac{\{\omega(1-\gamma\omega+\beta\gamma) - H^2(1-\gamma\omega)\}(1-\lambda_1\omega)}{(1-\gamma\omega)(1-\mu_1\omega)}\right]^{\frac{1}{2}} \tag{18}$$

CASE II: If $\mu_1 = 0, \mu_2 = 0$, then from (16) and (17), we obtain velocities of Maxwell visco-elastic liquid and the dust particles respectively, where M will be

$$M = \left[\frac{\{\omega(1-\gamma\omega+\beta\gamma) - H^2(1-\gamma\omega)\}(1-\lambda_1\omega+\lambda_2\omega^2)}{(1-\gamma\omega)}\right]^{\frac{1}{2}} \tag{19}$$

Case III: If $\lambda_1 = 0,\ \lambda_2 = 0,\ \mu_2 = 0$, then from (16) and (17), we obtain velocities of Rivlien-Ericksen visco-elastic liquid and the dust particles respectively, where M will be

$$M = \left[\frac{\{\omega(1-\gamma\omega+\beta\gamma) - H^2(1-\gamma\omega)\}}{(1-\gamma\omega)(1-\mu_1\omega)}\right]^{\frac{1}{2}} \tag{20}$$

Case IV: If $\lambda_1 = 0,\ \lambda_2 = 0,\ \mu_1 = 0,\ \mu_2 = 0$, then from (16) and (17) we obtain the velocities of viscous liquid and the dust particles respectively under the influence of magnetic field, where M will be

$$M = \left[\frac{\{\omega(1-\gamma\omega+\beta\gamma) - H^2(1-\gamma\omega)\}}{(1-\gamma\omega)}\right]^{\frac{1}{2}} \tag{21}$$

DEDUCTION

If magnetic field is withdrawn i.e. $B_0 = 0$, then all the above results in absence of magnetic field can be obtained with slight change of notations.

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