

LRS BIANCHI TYPE-I DISSIPATIVE FUTURE UNIVERSE WITHOUT BIG RIP

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ABSTRACT

LRS Bianchi type-I dissipative future universe without Big Rip in the context of Ecart formalism has been investigated. If cosmic dark energy behaves like a fluid with equation of state $p = \omega\rho$ (p and ρ being pressure and energy density respectively) as well as generalized Chaplygin gas simultaneously then Big Rip does not arise even for equation of state parameter $\omega < -1$ and scale factor is found to be regular for all time.

Keywords: Phantom fluid, Big rip, LRS Bianchi type-I universe.

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1. INTRODUCTION

In the last decades cosmologist have taken considerable interest in understanding the Physics behind the accelerated expansion of the universe. From the recent cosmological observations coming from CMBR data set of the three years WMAP and data from the SNe Ia supernova by the High-z supernova search team (Riess *et al.* [24]) followed by supernova cosmology project (Perlmutter *et al.* [22]; Steinhardt. *et al.*; [27]) which was further confirmed by recent observations of SNe Ia of high confidence level that the universe is accelerating (Tonry *et al.* [31]; Riess *et al.* [25]; Clocchiatti *et al.* [4]). This outstanding fact shows that the accelerated expansion is due to some mysterious form of

energy called Dark Energy (DE) with equation of state parameter (EoS) $\omega = \frac{p}{\rho}$, where p is pressure and ρ is energy density (Carroll *et al.*[5]). The best and simplest candidate for such dark energy is the so called cosmological constant which was introduced by Einstein into his gravitational field equations in an adhoc fashion. When $\omega < -1$, the dark energy model is phantom and $-1 < \omega < -\frac{1}{3}$, it is quintessence.

The dark energy model in which parameter ω cross the phantom divide $\omega = -1$ both sides then termed as quintom. Due to lack of observational evidence usually equation of state parameter ω considered as constant in making distinction between constant and variable ω (Kujat *et al.* [18]; Bartelmann, *et al.* [1]), with phase wise value -1, 0, +1/3, and +1 for vacuum fluid, dust fluid, radiation, and stiff dominated universe respectively. But in general, ω is a function of time or redshift (Jimenez [16]; Das *et al.*. [8]). Phantom cosmologies often have the property that they end in a final singularity in which the universe is destroyed in finite proper time by excessive expansion (Caldwell [6]). Phantom dark energy can lead to a singularity in which scale factor and density become infinite at a finite time called as Big Rip or Cosmic Dooms Day (Caldwell *et al.* [7]; Emilio *et al.* [9]). Cosmologist started making efforts to avoid this problem of Big Rip using $\omega < -1$. Sahni and Shtanov [28] have obtained cosmological model for the future universe without Big Rip problem with $\omega < -1$ in context of brain-world scenario. Yadav [32] have investigated FRW model for the future universe without Big Rip. Recently Ghate and Patil [10; 11] have obtained models for future universe without Big Rip in Kaluza-Klein and higher dimensional space times respectively.

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The Chaplygin Gas is also considered as a source of dark energy having negative pressure with equation of state given by

$$p = \frac{-A}{\rho}, \quad (1)$$

where p and ρ are respectively pressure and energy density with $\rho > 0$, A is positive constant (Kamenshchik, *et al.* [19]; Bento *et al.* [2]; Gorini *et al.* [12]). Bertolami *et al.* [3] have discussed that Generalized Chaplygin Gas (GCG) is better fit for latest supernova data. The equation of state for GCG is given by

$$p = \frac{-A}{\rho^{\frac{1}{\alpha}}} \quad (2)$$

with $1 \leq \alpha < \infty$.

For $\alpha = 1$, the equation (2) becomes (1).

Bianchi type models have been studied by several authors to understand better the observed small amount of anisotropy in the universe. The same models have also been used to examine the role of certain anisotropic sources during the formation of the large-scale structures, as we see in the universe today. Some Bianchi cosmologies, for example are natural hosts of large-scale magnetic fields and therefore their study can shed light on the implications of cosmic magnetism for galaxy formation. The simplest Bianchi family that contains the flat FRW universe as a special case are the type-I space-time. Ray *et al.* [26], Yadav and Yadav [33], Kumar [20], and Pradhan *et al.* [23] are some of the authors who have investigated dark energy models in general relativity with variable EoS parameters in different contexts. Yadav and Saha [34] have obtained an LRS Bianchi-I anisotropic cosmological model with dominance of dark energy. Ghate and Sontakke have investigated Bianchi type-IX universe with anisotropic dark energy in Lyra Geometry [13].

In this paper, Bianchi type-I dissipative future universe without Big rip has been studied. It is shown that if cosmic dark energy behaves like a fluid with equation of state $p = \omega\rho$ (p and ρ being pressure and energy density respectively) as well as generalized Chaplygin gas simultaneously then Big Rip does not arise even for equation of state parameter $\omega < -1$ and scale factor is found to be regular for all time.

2. METRIC AND FIELD EQUATIONS

We consider LRS Bianchi type-I flat universe is given by,

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)(dy^2 + dz^2), \quad (3)$$

where $a(t)$, $b(t)$ are scale factors and t represents the cosmic time.

The universe is assumed to be filled with distribution of matter represented by energy momentum tensor for bulk viscous cosmology given by,

$$T_j^i = (\rho + \bar{p}) u^i u_j - \bar{p} g_j^i, \quad (4)$$

$$\text{with } \bar{p} = p - 3\xi H, \quad (5)$$

where ρ is an energy density, p is an isotropic pressure, \bar{p} is an effective pressure, ξ is bulk viscous coefficient, H is Hubble's Parameter. u^i is the four velocity of fluid which satisfy the condition $u^i u_i = 0$ for $i = 1, 2, 3$ and $u^0 u_0 = -1$.

Using the above equations, the matter tensor is given by,

$$T_j^i = \text{diag} . (\rho, -\bar{p}, -\bar{p}, -\bar{p}). \quad (6)$$

The Einstein's Field equations are

$$R_j^i - \frac{1}{2} g_j^i R = -T_j^i, \quad (7)$$

where R_j^i is a Ricci Tensor, R is a Ricci Scalar and T_j^i is an energy momentum tensor for bulk viscous cosmology.

Using the equations (4-6) for metric (3), the Einstein's Field equations (7) reduce to

$$\frac{\dot{b}^2}{b^2} + 2\frac{\ddot{b}}{b} = \bar{p}, \quad (8)$$

$$\frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{a}}{a} = \bar{p}, \quad (9)$$

$$\frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{a}}{a} = \bar{p}, \quad (10)$$

$$\frac{2\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} = \rho \quad (11)$$

Here $\left(\dot{}\right)$ dot represents the differentiation with respect to t .

The energy conservation equation is given as,

$$T_{;j}^{ij} = 0,$$

where $T_{;j}^{ij} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^j} [T^{ij} \sqrt{-g}] + T^{jk} \Gamma_{jk}^i.$

Using equation (3), the above equation simplifies to

$$\dot{\rho} + (\bar{p} + \rho) \left(\frac{\dot{a}}{a} + \frac{2\dot{b}}{b} \right) = 0, \quad (12)$$

where $\dot{\rho}$ is the differentiation of ρ with respect to t .

3. SOLUTION OF FIELD EQUATIONS

In order to obtain the solution, we have three linearly independent equations (8-11) and five unknowns a, b, ρ, \bar{p} and ξ . To get deterministic solution, we need two additional conditions to find the solution of equations. First we use the physical condition that expansions scalar θ is proportional to shear scalar σ , which leads to

$$a = b^n, \quad (13)$$

where $n (> 0)$ is a constant. (Luis [21]; Johari and Kalyani [17]; Singh and Beesham [29]; Singh and Kale [30]).

With the help of equation (13), the field equation (11) reduce to

$$\frac{\dot{b}^2}{b^2} = \frac{\rho}{(2n+1)}. \quad (14)$$

From equation (13), we get

$$3H = (n+2) \frac{\dot{b}}{b}. \quad (15)$$

Using equations (13), (14), (15), equation (12) gives,

$$\dot{\rho} + 3H(\bar{p} + \rho) = 0. \quad (16)$$

Using equations (4), (5), (15), equation (16) can be written as

$$\dot{\rho} + 3H [\rho + \bar{p} - 3\xi H] = 0$$

which simplifies to,

$$\dot{\rho} + 3H \left[\frac{\rho^{1+\alpha/\alpha} - A}{\rho^{1/\alpha}} - 3\xi H \right] = 0. \quad (17)$$

To obtain the solution of (17), we assume the viscosity has a power law dependence upon the density given by,

$$\xi = \xi_0 \rho^m, \quad (18)$$

where ξ_0 and m are constants.

From equations (17) and (18) we get,

$$\frac{d\rho}{dt} + 3H \left[\frac{\rho^{1+\alpha/\alpha} - A}{\rho^{1/\alpha}} \right] = \frac{(n+2)^2}{(2n+1)} \xi_0 \rho^{m+1}. \quad (19)$$

To solve (19), use the transformation $\frac{d\rho}{dt} = \dot{\rho} = \rho^{m+1}$ in (19) which reduces to,

$$\rho^{1+\alpha/\alpha} = A + \left(\rho_0^{1+\alpha/\alpha} - A \right) \left(\frac{b_0}{b} \right)^{\left[\frac{(n+2)(2n+1)}{(2n+1) - (n+2)^2 \xi_0} \right] \left[\frac{1+\alpha}{\alpha} \right]}, \quad (20)$$

where ρ_0, b_0 represent the values of $\rho(t)$ and $b(t)$ at present time t_0 respectively. It is assumed that dark energy behaves like GCG obeying equation (2) as well as fluid with equation of state.

$$p = \omega \rho, \quad (21)$$

with $\omega < -1$ simultaneously.

From equations (2) and (21) we get,

$$\omega(t) = -\frac{A}{\rho^{1+\alpha/\alpha}}, \quad (22)$$

at $t = t_0$ equation (22) gives

$$A = -\omega_0 \rho_0^{1+\alpha/\alpha}, \quad (23)$$

where ω_0 is value of $\omega = \omega(t)$ at $t = t_0$.

Put the value of A from equation (23) in equation (15) we get,

$$\rho = \rho_0 \left[-\omega_0 + (1 + \omega_0) \left(\frac{b_0}{b} \right)^{\left[\frac{(n+2)(2n+1)}{(2n+1) - (n+2)^2 \xi_0} \right] \left[\frac{1+\alpha}{\alpha} \right]} \right]^{\alpha/(1+\alpha)}. \quad (24)$$

In homogeneous model of universe, scalar field $\phi(t)$ with potential $V(\phi)$ has energy density ρ_ϕ and pressure p_ϕ respectively are

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (25)$$

$$\text{and } p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi), \quad (26)$$

where $\dot{\phi}$ is differentiation of ϕ with respect to t .

Adding equations (25) and (26), we get

$$\dot{\phi}^2 = \rho_\phi + p_\phi. \quad (27)$$

With the help of equations (2), (23), Equation (27) gives

$$\dot{\phi}^2 = \frac{\rho^{1+\alpha/\alpha} + \rho_0^{1+\alpha/\alpha} \omega_0}{\rho^{1/\alpha}}. \quad (28)$$

Substitute the value of ρ from equation (24) in equation (28) we get,

$$\dot{\phi}^2 = \frac{(1 + \omega_0) \rho_0 \left(\frac{b_0}{b} \right)^{\left[\frac{(n+2)(2n+1)}{(2n+1) - (n+2)^2 \xi_0} \right] \left[\frac{1+\alpha}{\alpha} \right]}}{\left[-\omega_0 + (1 + \omega_0) \left(\frac{b_0}{b} \right)^{\left[\frac{(n+2)(2n+1)}{(2n+1) - (n+2)^2 \xi_0} \right] \left[\frac{1+\alpha}{\alpha} \right]} \right]^{1/1+\alpha}}. \quad (29)$$

From equation (29), it is observed that when $\dot{\phi}^2 > 0$, we get positive kinetic energy for $(1 + \omega_0) > 0$ and when $\dot{\phi}^2 < 0$, we get negative kinetic energy for $(1 + \omega_0) < 0$.

Thus $(1 + \omega_0) > 0$ represents a case of quintessence and $(1 + \omega_0) < 0$ represents phantom fluid dominated universe which matches with the results obtained by Hoyle and Narlikar in C-field with negative kinetic energy for steady state theory of universe (Hoyle and Narlikar [14] ;[15]).

Now from equation (14) and (24) we get,

$$\left(\frac{\dot{b}}{b} \right)^2 = \left(\frac{3}{2n+1} \right) \Omega_0 H_0^2 \left[|\omega_0| + (1 - |\omega_0|) \left(\frac{b_0}{b} \right)^{\left[\frac{(n+2)(2n+1)}{(2n+1) - (n+2)^2 \xi_0} \right] \left[\frac{1+\alpha}{\alpha} \right]} \right]^{\frac{\alpha}{1+\alpha}}, \quad (30)$$

where $|\omega_0| = -\omega_0$, $H_0 = 100h \text{ km/s mpc}$ present value of Hubble parameter and

$$\Omega_0 = \frac{\rho_0}{\rho_{cr,0}} \text{ with } \rho_{cr,0} = \frac{3H_0^2}{8\pi G}.$$

Taking square root to both sides of equation (30) gives,

$$\frac{\dot{b}}{b} = \sqrt{\frac{3\Omega_0}{2n+1}} H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}} \left[1 + \frac{(1 - |\omega_0|)}{|\omega_0|} \left(\frac{b_0}{b} \right)^{\left[\frac{(n+2)(2n+1)}{(2n+1) - (n+2)^2 \xi_0} \right] \left[\frac{1+\alpha}{\alpha} \right]} \right]^{\frac{\alpha}{2(1+\alpha)}}. \quad (31)$$

Expanding the R. H. S. of (31) and neglecting the higher powers of $\frac{(1 - |\omega_0|)}{|\omega_0|} \left(\frac{b_0}{b} \right)^{\left[\frac{(n+2)(2n+1)}{(2n+1) - (n+2)^2 \xi_0} \right] \left[\frac{1+\alpha}{\alpha} \right]}$ we get,

$$\frac{\dot{b}}{b} = \sqrt{\frac{3\Omega_0}{2n+1}} H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}} \left[1 + \frac{\alpha(1 - |\omega_0|)}{2(1 + \alpha)|\omega_0|} \left(\frac{b_0}{b} \right)^{\left[\frac{(n+2)(2n+1)}{(2n+1) - (n+2)^2 \xi_0} \right] \left[\frac{1+\alpha}{\alpha} \right]} \right]. \quad (32)$$

Integrating (32) we get,

$$b(t) = \frac{b_0}{\left[2(1 + \alpha)|\omega_0| \right]^{\left[\frac{(2n+1) - (n+2)^2 \xi_0}{(2n+1)(n+2)} \right] \left[\frac{\alpha}{1+\alpha} \right]}} \times \left[\left(\alpha + 2(1 + \alpha)|\omega_0| \right) e^{(n+2)(2n+1)H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}} \sqrt{\frac{3\Omega_0}{2nm+1}} (t-t_0)} - \alpha(1 - |\omega_0|) \right]^{\left[\frac{(2n+1) - (n+2)^2 \xi_0}{(n+2)(2n+1)} \right] \left[\frac{\alpha}{1+\alpha} \right]}. \quad (33)$$

From equation (33), it is clear that as $t \rightarrow \infty$, $b(t) \rightarrow \infty$, therefore the present model is free from finite time future singularity.

In this case, the Hubble distance is given by,

$$H^{-1} = \frac{\sqrt{3(2n+1)/(n+2)^2}}{\sqrt{\Omega_0} H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}}} \left[1 - \frac{\alpha(1-|\omega_0|)}{2(1+\alpha)|\omega_0|} \left(\frac{b_0}{b} \right)^{\left[\frac{(n+2)(2n+1)}{(2n+1)-(n+2)^2 \xi_0} \right] \frac{(1+\alpha)}{\alpha}} \right]. \quad (34)$$

Equation (34) shows the growth of Hubble distance H^{-1} with time such that $H^{-1} \rightarrow \frac{\sqrt{3(2n+1)/(n+2)^2}}{\sqrt{\Omega_0} H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}}} \neq 0$ as

$t \rightarrow \infty$. Thus in present Bianchi type I universe, the galaxies will not disappear as $t \rightarrow \infty$, avoiding big rip singularity. Therefore, one can conclude that if Phantom fluid behaves like GCG and fluid with $p = \omega\rho$ simultaneously then the future accelerated expansion of the universe will free from catastrophic situation like Big Rip in Bianchi type-I universe.

Equation (24) can be written as

$$\rho = \rho_0 \left[|\omega_0| + (1-|\omega_0|) \left(\frac{a_0}{a(t)} \right)^{\frac{(n+2)(2n+1)(1+\alpha)}{\alpha[(2n+1)-(n+2)^2 \xi_0]}} \right]^{\frac{\alpha}{1+\alpha}}. \quad (35)$$

From equation (35), it is clear that as $t \rightarrow \infty$, $\rho \rightarrow \rho_0 |\omega_0|^{\frac{\alpha}{1+\alpha}} > \rho_0$.

Thus one can conclude that energy density increases with time, contrary to other Phantom models having future singularity at $t = t_s$ in Bianchi type I universe.

4. CONCLUSION

LRS Bianchi type-I universe is studied when cosmic dark energy behaves simultaneously like a fluid with equation of state $p = \omega\rho$; $\omega < -1$ as well as generalized Chaplygin gas with equation of state $p = -\frac{A}{\rho^{1/\alpha}}$.

The concluding remarks are as follows:

- (i) $\dot{\phi}^2 > 0$, we get positive kinetic energy when $(1 + \omega_0) > 0$ re-presenting the case of quintessence and for $\dot{\phi}^2 < 0$, we get negative kinetic energy $(1 + \omega_0) < 0$ when representing phantom field dominated universe. These results match with the results obtained by Hoyle-Narlikar in C-field with negative kinetic energy for steady state of universe.
- (ii) As $t \rightarrow \infty$, $b(t) \rightarrow \infty$ indicating that the model is free from finite future singularity.
- (iii) As $t \rightarrow \infty$, $H^{-1} \neq 0$ indicating that at present time the galaxies will not disappear as $t \rightarrow \infty$ avoiding Big Rip singularities.
- (iv) As $t \rightarrow \infty$, $\rho \rightarrow \rho_0 |\omega_0|^{\frac{\alpha}{1+\alpha}} > \rho_0$ concluding that energy density increases with time contrary to other phantom models having future singularity at $t = t_s$.

Our model behaves analogues with the model obtained by Yadav [32].

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