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SOME RANKING METHODS FOR OCTAGONAL FUZZY NUMBERS

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ABSTRACT

In this paper, we propose some ranking methods of octagonal fuzzy numbers. Ranking based on area between centroid point of an octagonal fuzzy number and the origin, area between radius of gyration point of an octagonal fuzzy number and the origin, sign distance and deviation degree are dealt with in detail. Also a ranking algorithm to check the uniqueness of the octagonal fuzzy number is developed. All the methods are compared for few setsof octagonal fuzzy numbers.

Keywords:*Octagonal fuzzy number, ranking, centroid, radius of gyration, deviation degree.*

1. INTRODUCTION

Fuzzy systems have gained more and more attention from researchers and practitioners of various fields. In such systems, the output represented by a fuzzy set may need to be transformed into a scalar value, and this task is known as the defuzzification process. Unlike in the case of real numbers, fuzzy quantities have no natural order. As a consequence there are several ranking methods for fuzzy numbers, which suffers some defects in some form, for example, lack of uniqueness. Thus there is no single method that orders the fuzzy numbers correctly. Based on the context of the application, some methods seem to be more appropriate than others. Ranking fuzzy numbers has attracted special research attention due to their wide applications in the theory of fuzzy decision making, risk analysis, data anlysis, optimization, etc. In order to achieve greater efficiencies and accuracies in ranking results, several ranking methods have been investigated by many authors since 1976. To cite a few *Chu et. al.*[5], *Deng et. al.* [6], *Abbasbandy et. al.* [1], *Liu et. al.*[9], *Amit Kumar et. al.*[2].

The concept of octagonal fuzzy numbers was introduced in [10]. In this paper, we propose ranking methods for octagonal fuzzy numbers, which falls under three categories that are mentioned in [11]. In the first class, each index is associated with a mapping $\Re$ from the set of fuzzy quantities into real numbers. Section 3.1 introduces area between centroid point of an octagonal fuzzy number and the origin, section 3.2 deals with area between radius of gyration point of an octagonal fuzzy number and the origin and section 3.3 introduces sign distance for octagonal fuzzy numbers, which falls under first class. Section 3.4 deals with deviation degree which is in the second classification, where a reference set is set up and all the fuzzy quantities are compared with this reference set. In Section 3.5 we have proposed an algorithm which compares octagonal fuzzy numbers in pairs, which falls under third class. Section 4 compares all the methods for octagonal fuzzy numbers via numerical examples and the conclusions are presented in Section 5.

2. OCTAGONAL FUZZY NUMBERS

For the sake of completeness we recall the required definitions and results.

Definition 2.1 [8]: The characteristic function $\mu_{\tilde{A}}$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ is called a fuzzy set.

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Definition 2.2 [8]: A fuzzy set $\tilde{A}$,defined on the universal set of real number R, is said to be a fuzzy number if its membership function has the following characteristics:

i. $\tilde{A}$ is convex i.e. $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \forall x_1, x_2 \in \mathsf{R}, \forall \lambda \in [0,1]$

ii. $\tilde{A}$ is normal i.e. $\exists x_0 \in \mathbb{R}$such that $\mu_{\tilde{A}}(x_0) = 1$

iii. $\mu_{\tilde{A}}$ is piecewise continuous

Definition 2.3[10]: A fuzzy number $\tilde{A}$ is said to be a generalized octagonal fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$ where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7 \leq a_8$ are real numbers and its membership function $\mu_{\tilde{A}}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq a_1 \\ m_1(x) & a_1 \leq x \leq a_2 \\ k & a_2 \leq x \leq a_3 \\ m_3(x) & a_3 \leq x \leq a_4 \\ w & a_4 \leq x \leq a_5 \\ m_5(x) & a_5 \leq x \leq a_6 \\ k & a_6 \leq x \leq a_7 \\ m_7(x) & a_7 \leq x \leq a_8 \\ 0 & x \geq a_8 \end{cases}$$

where $0 < k < w$, $w = height(\tilde{A}), y = m_1(x)$ is the line joining the points $(a_1, 0)$ and (a_2, k), $y = m_3(x)$ is that of (a_3, k) and (a_4, w), $y = m_5(x)$ is of (a_5, w) and (a_6, k), $y = m_7(x)$ is of (a_7, k) and $(a_8, 0)$.

Fig. - 1: Graphical representation of the octgonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; 0.5, 1)$

Definition 2.4: The α -cut of an octagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$ given in Definition 2.3 is

$$\tilde{A} = [A_\alpha^L, A_\alpha^R] = \begin{cases} (A_\alpha^L)_1, (A_\alpha^R)_1] & \alpha \in [0, k] \\ (A_\alpha^L)_2, (A_\alpha^R)_2] & \alpha \in (k, 1] \end{cases} \tag{1}$$

where

$$(A_\alpha^L)_1 = a_1 + \frac{\alpha}{k}(a_2 - a_1), (A_\alpha^L)_2 = a_3 + \frac{\alpha - k}{w - k}(a_4 - a_3), (A_\alpha^R)_1 = a_8 - \frac{\alpha}{k}(a_8 - a_7), (A_\alpha^R)_2 = a_5 + \frac{\alpha - w}{k - w}(a_6 - a_5)$$

3. RANKING OF OCTAGONAL FUZZY NUMBERS

In this paper, we introduce the ranking of octagonal fuzzy numbers denoted $\Re$ based on different methods. In the first four methods the ranking function $\Re_i, i \in \{1, 2, ..., 4\}$ orders any two arbitrary octagonal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ as

$\tilde{A} \succ \tilde{B}$ if $\Re_i(\tilde{A}) > \Re_i(\tilde{B})$

$\tilde{A} \prec \tilde{B}$ if $\Re_i(\tilde{A}) < \Re_i(\tilde{B})$

$\tilde{A} \approx \tilde{B}$ if $\Re_i(\tilde{A}) = \Re_i(\tilde{B})$

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3.1. RANKING USING CENTROID POINT

Definition 3.1: Let $\tilde{A}$ be an octagonal fuzzy number as in Definition 2.3. The centroid point $(\bar{x}, \bar{y})$ for an octagonal fuzzy number $\tilde{A}$ is

$$\bar{x} = \frac{\int_{a_1}^{a_2} x m_1(x)dx + \int_{a_2}^{a_3} kxdx + \int_{a_3}^{a_4} x m_3(x)dx + \int_{a_4}^{a_5} wxdx + \int_{a_5}^{a_6} x m_5(x)dx + \int_{a_6}^{a_7} kxdx + \int_{a_7}^{a_8} x m_7(x)dx}{\int_{a_1}^{a_2} m_1(x)dx + \int_{a_2}^{a_3} kdx + \int_{a_3}^{a_4} m_3(x)dx + \int_{a_4}^{a_5} wdx + \int_{a_5}^{a_6} m_5(x)dx + \int_{a_6}^{a_7} kdx + \int_{a_7}^{a_8} m_7(x)dx} \tag{2}$$

$$\bar{y} = \frac{\int_0^k y m_1^{-1}(y)dy + \int_k^w y m_3^{-1}(y)dy + \int_w^k y m_5^{-1}(y)dy + \int_k^0 y m_7^{-1}(y)dy}{\int_0^k m_1^{-1}(y)dy + \int_k^w m_3^{-1}(y)dy + \int_w^k m_5^{-1}(y)dy + \int_k^0 m_7^{-1}(y)dy} \tag{3}$$

Note that the centroid point corresponds to an $\bar{x}$ value on the horizontal axis and a $\bar{y}$ value on the vertical axis and $m_1^{-1}(y), m_3^{-1}(y), m_5^{-1}(y), m_7^{-1}(y)$ are the inverse functions of $m_1(x), m_3(x), m_5(x), m_7(x)$ respectively. Since $0 < k < w$, the inverse functions exists.

Definition 3.2: Ranking using area between centroid point $(\bar{x}, \bar{y})$ of the fuzzy number $\tilde{A}$ and the origin $(0,0)$ is given by

$$\Re_1(\tilde{A}) = \bar{x}.\bar{y} \tag{4}$$

3.2. RANKING USING RADIUS OF GYRATION

Definition 3.3[7]: Radius of gyration measures the size of an object to describe the distribution of cross – sectional area in a column around its centroidal axis.

The radius of gyration of an area A with respect to the *x*-axis is defined as the quantity r_x, that satisfies the relation

$$I_x = r_x^2 A \tag{5}$$

where I_x is the moment of inertia of A with respect to the *x*-axis defined as

$$I_x = \int_A y^2 dA \tag{6}$$

Solving equation (5) for r_x, we have

$$r_x = \sqrt{\frac{I_x}{A}} \tag{7}$$

In a similar way, the radius of gyration of an area A with respect to the *y*-axis is

$$I_y = r_y^2 A \tag{8}$$

$$I_y = \int_A x^2 dA$$

$$r_y = \sqrt{\frac{I_y}{A}} \tag{9}$$

The radius of gyration of an octagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$ is denoted by $(r_x^{\tilde{A}}, r_y^{\tilde{A}})$ whose values can be obtained by equations (7) and (9)

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The moment of inertia with respect to *x*-axis and *y*-axis and the area of the octagonal fuzzy number are

$$I_x(\tilde{A}) = \int_0^k \int_{(A_\alpha^L)_1}^{(A_\alpha^R)_1} y^2 dydx + \int_k^w \int_{(A_\alpha^L)_2}^{(A_\alpha^R)_2} y^2 dydx$$

$$I_y(\tilde{A}) = \int_0^k \int_{(A_\alpha^L)_1}^{(A_\alpha^R)_1} x^2 dydx + \int_k^w \int_{(A_\alpha^L)_2}^{(A_\alpha^R)_2} x^2 dydx$$

$$Area(\tilde{A}) = \int_0^k \int_{(A_\alpha^L)_1}^{(A_\alpha^R)_1} dydx + \int_k^w \int_{(A_\alpha^L)_2}^{(A_\alpha^R)_2} dydx$$

The radius of gyration points $(r_x^{\tilde{A}}, r_y^{\tilde{A}})$ can be calculated as

$$r_x^{\tilde{A}} = \sqrt{\frac{I_x(\tilde{A})}{Area(\tilde{A})}}$$

$$r_y^{\tilde{A}} = \sqrt{\frac{I_y(\tilde{A})}{Area(\tilde{A})}}$$

Definition 3.4: Ranking using area between the radius of gyration point $(r_x^{\tilde{A}}, r_y^{\tilde{A}})$ of the fuzzy number $\tilde{A}$ and the origin $(0,0)$ is given by

$$\Re_2(\tilde{A}) = sign(\bar{x}) r_x^{\tilde{A}} r_y^{\tilde{A}}$$

where $\bar{x}$ is the *x*-coordinate of the centroid point of $\tilde{A}$ as given in equation (2).

Remark 3.1: The term $sign(\bar{x})$ is included in $\Re_2$ to overcome the problem of ranking the image of fuzzy numbers, i.e. to overcome $\tilde{A} \prec \tilde{B} \Rightarrow -\tilde{A} \prec -\tilde{B}$

Remark 3.2: When $k = \frac{1}{2}$ and $w = 1$

$$r_x^{\tilde{A}} = \frac{1}{12}\sqrt{6}\left[\frac{a_8 + 3a_8 + 11a_6 + 17a_5 - a_1 - 3a_2 - 11a_3 - 17a_4}{a_8 + a_7 + a_6 + a_5 - a_4 - a_3 - a_2 - a_1}\right]^{\frac{1}{2}}$$

$$r_y^{\tilde{A}} = \frac{1}{6}\sqrt{6}\left[\frac{(a_8 + a_7)(a_8^2 + a_7^2) + (a_6 + a_5)(a_6^2 + a_5^2) - (a_4 + a_3)(a_4^2 + a_3^2) - (a_2 + a_1)(a_2^2 + a_1^2)}{a_8 + a_7 + a_6 + a_5 - a_4 - a_3 - a_2 - a_1}\right]^{\frac{1}{2}}$$

3.3 RANKING USING SIGN DISTANCE

Definition 3.5: For arbitrary octagonal fuzzy numbers $\tilde{A}$ and $\tilde{B}$ with α - cut as given in equation(1), the distance between $\tilde{A}$ and $\tilde{B}$ is given by

$$D_p(\tilde{A}, \tilde{B}) = \left[\int_0^k \left|\left(A_\alpha^L\right)_1 - \left(B_\alpha^L\right)_1\right|^p d\alpha + \int_k^w \left|\left(A_\alpha^L\right)_2 - \left(B_\alpha^L\right)_2\right|^p d\alpha + \int_0^k \left|\left(A_\alpha^R\right)_1 - \left(B_\alpha^R\right)_1\right|^p d\alpha + \int_k^w \left|\left(A_\alpha^R\right)_2 - \left(B_\alpha^R\right)_2\right|^p d\alpha\right]^{\frac{1}{p}}$$

Let $\tilde{A}_0 = (0,0,0,0,0,0,0,0;0.5,1)$ be the fuzzy origin, so for any arbitrary octagonal fuzzy number $\tilde{A}$, we can define

$$D_p(\tilde{A}, \tilde{A}_0) = \left[\int_0^k \left|\left(A_\alpha^L\right)_1\right|^p d\alpha + \int_k^w \left|\left(A_\alpha^L\right)_2\right|^p d\alpha + \int_0^k \left|\left(A_\alpha^R\right)_1\right|^p d\alpha + \int_k^w \left|\left(A_\alpha^R\right)_2\right|^p d\alpha\right]^{\frac{1}{p}}, \quad p \geq 1 \tag{10}$$

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Let γ be a function from the set of all octagonal fuzzy numbers to $\{-1,1\}$ defined as

$$\gamma(\tilde{A}) = sign\left[\int_0^k \left(A_\alpha^L\right)_1 d\alpha + \int_k^w \left(A_\alpha^L\right)_2 d\alpha + \int_0^k \left(A_\alpha^R\right)_1 d\alpha + \int_k^w \left(A_\alpha^R\right)_2 d\alpha\right] \tag{11}$$

Using equations(10) and (11) we define a new ranking $\Re_3$ called the sign distance as follows

$$\Re_3(\tilde{A}) = \gamma(\tilde{A}) D_p(\tilde{A}, \tilde{A}_0)$$

Remark 3.3: When $k = \frac{1}{2}, w = 1$, suppose $a_1 > 0$ then for

$$p = 1,\ \Re_3(\tilde{A}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8)$$

$$p = 1,\ \Re_3(\tilde{A}) = \frac{1}{6}\sqrt{6}\left[a_1^2 + a_2^2 + a_1 a_2 + a_3^2 + a_4^2 + a_3 a_4 + a_5^2 + a_6^2 + a_5 a_6 + a_7^2 + a_8^2 + a_7 a_8\right]^{\frac{1}{2}}$$

3.4 RANKING USING DEVIATION DEGREE

In this section we deal with ranking of octagonal fuzzy numbers based on left and right deviation degree.

Definition 3.6: For a set of n octagonal fuzzy numbers $\{\tilde{A}^1, \tilde{A}^2, ..., \tilde{A}^n\}$ where $\tilde{A}^i = (a_1^i, a_2^i, ..., a_8^i; k, w)$, call $x_{\min} = \min\{a_1^1, a_1^2, ..., a_1^n\}$ and $x_{\max} = \max\{a_8^1, a_8^2, ..., a_8^n\}$. The left deviation degree and right deviation degree of $\tilde{A}^i (i \in \{1,2,...,n\})$ are defined as

$$d_{\tilde{A}^i}^L = \int_0^k m_{7\tilde{A}^i}^{-1}(y)dy + \int_k^w m_{5\tilde{A}^i}^{-1}(y)dy - \int_0^w x_{\min} dy$$

$$d_{\tilde{A}^i}^R = \int_0^w x_{\max} dy - \int_0^k m_{1\tilde{A}^i}^{-1}(y)dy - \int_k^w m_{3\tilde{A}^i}^{-1}(y)dy$$

where $m_{1\tilde{A}^i}^{-1}(y), m_{3\tilde{A}^i}^{-1}(y), m_{5\tilde{A}^i}^{-1}(y), m_{7\tilde{A}^i}^{-1}(y)$ are the inverse functions of $m_1(x), m_3(x), m_5(x), m_7(x)$ of the octagonal fuzzy number $\tilde{A}^i$ as given in Definition 2.3.

In order to reflect the relative variation of L-R deviation degree of octagonal fuzzy number, we employ the expectation value of centroid of fuzzy number to construct the transfer co-efficient of $\tilde{A}^i, (i \in \{1,2,...,n\})$ given by

$$\gamma^{\tilde{A}^i} = \frac{M_i - x_{\min}}{x_{\max} - x_{\min}}$$

where M_i is the $x-$ coordinate of the centroid of the octagonal fuzzy number $\tilde{A}^i$, as given in equation(2)

The ranking index based on deviation degree is given by

$$\Re_4(\tilde{A}^i) = \frac{\gamma^{\tilde{A}^i} d_{\tilde{A}^i}^L}{\gamma^{\tilde{A}^i} d_{\tilde{A}^i}^L + (1 - \gamma^{\tilde{A}^i}) d_{\tilde{A}^i}^R}$$

Notation: In Definition 3.6, the exponent*i* denotes index and not the power to which it is raised.

Proposition 3.1: *The ranking function $\Re_4$ always satisfy the property that if the ranking result is $\tilde{A}^i \succ \tilde{A}^j$ then $-\tilde{A}^i \prec -\tilde{A}^j$.*

Proof: Let $-\tilde{A}^i$ be the image of $\tilde{A}^i$, $-\tilde{A}^i = (-a_8, -a_7, ..., -a_1; k, w)$. Using the symmetry properties, we have $d_{-\tilde{A}^i}^L = d_{\tilde{A}^i}^R, d_{-\tilde{A}^i}^R = d_{\tilde{A}^i}^L$ and $\gamma^{-\tilde{A}^i} = 1 - \gamma^{\tilde{A}^i}$. Thus

$$\Re_4(\tilde{A}^i) \succ \Re_4(\tilde{A}^j) \Leftrightarrow \frac{\gamma^{\tilde{A}^i} d^L_{\tilde{A}^i}}{\gamma^{\tilde{A}^i} d^L_{\tilde{A}^i} + (1-\gamma^{\tilde{A}^i}) d^R_{\tilde{A}^i}} > \frac{\gamma^{\tilde{A}^j} d^L_{\tilde{A}^j}}{\gamma^{\tilde{A}^j} d^L_{\tilde{A}^j} + (1-\gamma^{\tilde{A}^j}) d^R_{\tilde{A}^j}}$$

$$\Leftrightarrow 1 - \frac{\gamma^{\tilde{A}^i} d^L_{\tilde{A}^i}}{\gamma^{\tilde{A}^i} d^L_{\tilde{A}^i} + (1-\gamma^{\tilde{A}^i}) d^R_{\tilde{A}^i}} < 1 - \frac{\gamma^{\tilde{A}^j} d^L_{\tilde{A}^j}}{\gamma^{\tilde{A}^j} d^L_{\tilde{A}^j} + (1-\gamma^{\tilde{A}^j}) d^R_{\tilde{A}^j}}$$

$$\Leftrightarrow \frac{(1-\gamma^{\tilde{A}^i}) d^R_{\tilde{A}^i}}{\gamma^{\tilde{A}^i} d^L_{\tilde{A}^i} + (1-\gamma^{\tilde{A}^i}) d^R_{\tilde{A}^i}} < \frac{(1-\gamma^{\tilde{A}^j}) d^R_{\tilde{A}^j}}{\gamma^{\tilde{A}^j} d^L_{\tilde{A}^j} + (1-\gamma^{\tilde{A}^j}) d^R_{\tilde{A}^j}}$$

$$\Leftrightarrow \frac{\gamma^{-\tilde{A}^i} d^L_{-\tilde{A}^i}}{\gamma^{-\tilde{A}^i} d^L_{-\tilde{A}^i} + (1-\gamma^{-\tilde{A}^i}) d^R_{-\tilde{A}^i}} < \frac{\gamma^{-\tilde{A}^j} d^L_{-\tilde{A}^j}}{\gamma^{-\tilde{A}^j} d^L_{-\tilde{A}^j} + (1-\gamma^{-\tilde{A}^j}) d^R_{-\tilde{A}^j}}$$

$$\Leftrightarrow \Re_4(-\tilde{A}^i) \prec \Re_4(-\tilde{A}^j).$$

3.5 RANKING ALGORITHM

Definition 3.7: For any octagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k, w)$, the rank, mode, divergence & spread at *w* and *k* level are defined as :

i. $\Re_{k,w}(\tilde{A}) = \frac{w(a_3 + a_4 + a_5 + a_6)}{4}$

ii. $w - Mode(\tilde{A}) = \frac{w(a_4 + a_5)}{2}$

iii. $w - Divergence(\tilde{A}) = w(a_6 - a_3)$

iv. $LeftSpread_{k,w}(\tilde{A}) = w(a_4 - a_3)$

v. $RightSpread_{k,w}(\tilde{A}) = w(a_6 - a_5)$

vi. $\Re_{0,k}(\tilde{A}) = \frac{k(a_1 + a_2 + a_3 + a_6 + a_7 + a_8)}{6}$

vii. $k - rightmode(\tilde{A}) = \frac{k(a_6 + a_7)}{2}$

viii. $k - leftmode(\tilde{A}) = \frac{k(a_2 + a_3)}{2}$

ix. $0 - Divergence(\tilde{A}) = k(a_8 - a_1)$

x. $LeftSpread_{0,k}(\tilde{A}) = k(a_2 - a_1)$

xi. $RightSpread_{0,k}(\tilde{A}) = k(a_8 - a_7)$

Theorem 3.1: *Let* $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k_1, w_1)$ *and* $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8; k_2, w_2)$ *be two octagonal fuzzy numbers such that*, if (i) to (iii) are equal for $\tilde{A}$ and $\tilde{B}$, then

(a) Left Spread $_{k,w}(\tilde{A})$ > Left Spread $_{k,w}(\tilde{B})$ iff Right Spread $_{k,w}(\tilde{A})$ > Right Spread $_{k,w}(\tilde{B})$

(b) Left Spread $_{k,w}(\tilde{A})$ < Left Spread $_{k,w}(\tilde{B})$ iff Right Spread $_{k,w}(\tilde{A})$ < Right Spread $_{k,w}(\tilde{B})$

(c) Left Spread $_{k,w}(\tilde{A})$ = Left Spread $_{k,w}(\tilde{B})$ iff Right Spread $_{k,w}(\tilde{A})$ = Right Spread $_{k,w}(\tilde{B})$

If (i) to (ix), *w* and *k* are equal for $\tilde{A}$ and $\tilde{B}$, then

(d) Left Spread $_{0,k}(\tilde{A})$ = Left Spread $_{0,k}(\tilde{B})$ and

(e) Right Spread $_{0,k}(\tilde{A})$ = Right Spread $_{0,k}(\tilde{B})$

Proof:

$$\Re_{k,w}(\tilde{A}) = \Re_{k,w}(\tilde{B}) \Rightarrow w_1(a_3 + a_4 + a_5 + a_6) = w_2(b_3 + b_4 + b_5 + b_6) \quad (12)$$

$$w - Mode(\tilde{A}) = w - Mode(\tilde{B}) \Rightarrow w_1(a_4 + a_5) = w_2(b_4 + b_5) \quad (13)$$

$$w - Divergence(\tilde{A}) = w - Divergence(\tilde{B}) \Rightarrow w_1(a_6 - a_3) = w_2(b_6 - b_3) \quad (14)$$

Solving equations (12), (13) and(14) we get

$$w_1 a_3 = w_2 b_3 \quad (15)$$

$$w_1 a_6 = w_2 b_6 \quad (16)$$

$LeftSpread_{k,w}(\tilde{A}) > LeftSpread_{k,w}(\tilde{B})$

$\Leftrightarrow w_1(a_4 - a_3) > w_2(b_4 - b_3)$

$\Leftrightarrow w_1 a_4 > w_2 b_4$ (using Eq.(15))

$\Leftrightarrow w_1 a_5 < w_2 b_5$, since Eq. (13)

$\Leftrightarrow -w_1 a_5 > -w_2 b_5$

$\Leftrightarrow w_1(a_6 - a_5) > w_2(b_5 - b_6)$, since Eq (16)

$\Leftrightarrow RightSpread_{k,w}(\tilde{A}) > RightSpread_{k,w}(\tilde{B})$

Similarly (b) and (c)can be proved.

To prove (d):Given that $w_1 = w_2$; $k_1 = k_2$ and properties (i) to (ix) are equal for $\tilde{A}$ and $\tilde{B}$. Thus we have $a_3 = b_3$; $a_6 = b_6$ (from equations(15) and (16))

$$\Re_{0,k}(\tilde{A}) = \Re_{0,k}(\tilde{B}) \Rightarrow k_1(a_1 + a_2 + a_3 + a_6 + a_7 + a_8) = k_2(b_1 + b_2 + b_3 + b_6 + b_7 + b_8) \quad (17)$$

$$k - rightmode(\tilde{A}) = k - rightmode(\tilde{B}) \Rightarrow k_1(a_6 + a_7) = k_2(b_6 + b_7) \quad (18)$$

$$k - leftmode(\tilde{A}) = k - leftmode(\tilde{B}) \Rightarrow k_1(a_2 + a_3) = k_2(b_2 + b_3) \quad (19)$$

$$Divergence_{0,k}(\tilde{A}) = Divergence_{0,k}(\tilde{B}) \Rightarrow k_1(a_8 - a_1) = k_2(b_8 - b_1) \quad (20)$$

Solving equations(17), (18), (19) and (20) we have

$$k_1 a_1 = k_2 b_1 \quad (21)$$

$$k_1 a_8 = k_2 b_8 \quad (22)$$

Since $k_1 = k_2$, $a_6 + a_7 = b_6 + b_7$; $a_2 + a_3 = b_2 + b_3$; $a_1 = b_1$ and $a_8 = b_8$.

$a_6 = b_6$ and $a_6 + a_7 = b_6 + b_7 \Rightarrow a_7 = b_7$

$a_3 = b_3$ and $a_2 + a_3 = b_2 + b_3 \Rightarrow a_2 = b_2$

Thus $a_1 = b_1, a_2 = b_2, \ a_7 = b_7$ and $a_8 = b_8$

$$\Rightarrow LeftSpread_{0,k}(\tilde{A}) = LeftSpread_{0,k}(\tilde{B}) \text{ and } RightSpread_{0,k}(\tilde{A}) = RightSpread_{0,k}(\tilde{B})$$

Hence the proof.

Corollary 3.1: If two octagonal fuzzy numbers whose *k*, *w* and (i) to (ix) of Definition 3.7 are equal,then the two octagonal fuzzy numbers coincide.

Given any two octagonal fuzzy numbers we present here an algorithm to compare them.

ALGORITHM

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k_1, w_1)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8; k_2, w_2)$ be two octagonal fuzzy numbers, then use the following steps to compare $\tilde{A}$ and $\tilde{B}$

Step - 1: Find $\Re_{k,w}(\tilde{A})$ and $\Re_{k,w}(\tilde{B})$

Case - (i): If $\Re_{k,w}(\tilde{A}) > \Re_{k,w}(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $\Re_{k,w}(\tilde{A}) < \Re_{k,w}(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $\Re_{k,w}(\tilde{A}) = \Re_{k,w}(\tilde{B})$ then go to Step 2

Step - 2: Find $w-Mode(\tilde{A})$ and $w-Mode(\tilde{B})$

Case - (i): If $w-Mode(\tilde{A}) > w-Mode(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $w-Mode(\tilde{A}) < w-Mode(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $w-Mode(\tilde{A}) = w-Mode(\tilde{B})$ then go to Step 3

Step - 3: Find $w-Divergence(\tilde{A})$ and $w-Divergence(\tilde{B})$ If $w-Mode(\tilde{A}) \geq 0$ then

Case - (i): If $w-Divergence(\tilde{A}) > w-Divergence(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $w-Divergence(\tilde{A}) < w-Divergence(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $w-Divergence(\tilde{A}) = w-Divergence(\tilde{B})$ then go to Step 4 else

Case - (i): If $w-Divergence(\tilde{A}) < w-Divergence(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $w-Divergence(\tilde{A}) > w-Divergence(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $w-Divergence(\tilde{A}) = w-Divergence(\tilde{B})$ then go to Step 4

Step - 4: Find $LeftSpread_{k,w}(\tilde{A})$ and $LeftSpread_{k,w}(\tilde{B})$ If $w-Mode(\tilde{A}) \geq 0$ then

Case - (i): If $LeftSpread_{k,w}(\tilde{A}) < LeftSpread_{k,w}(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $LeftSpread_{k,w}(\tilde{A}) > LeftSpread_{k,w}(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $LeftSpread_{k,w}(\tilde{A}) = LeftSpread_{k,w}(\tilde{B})$ then go to Step 5 else

Case - (i): If $LeftSpread_{k,w}(\tilde{A}) > LeftSpread_{k,w}(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $LeftSpread_{k,w}(\tilde{A}) < LeftSpread_{k,w}(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $LeftSpread_{k,w}(\tilde{A}) = LeftSpread_{k,w}(\tilde{B})$ then go to Step 5

Step -5: Find w_1 and w_2

Case - (i): If $w_1 > w_2$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $w_1 < w_2$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $w_1 = w_2$ then go to Step 6

Step - 6: Find $\Re_{0,k}(\tilde{A})$ and $\Re_{0,k}(\tilde{B})$

Case - (i): If $\Re_{0,k}(\tilde{A}) > \Re_{0,k}(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $\Re_{0,k}(\tilde{A}) < \Re_{0,k}(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $\Re 0,_k (\tilde{A}) = \Re_{0,k}(\tilde{B})$ then go to Step 7

Step - 7: Find $k - rightmode(\tilde{A})$ and $k - rightmode(\tilde{B})$

Case - (i): If $k - rightmode(\tilde{A}) > k - rightmode(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $k - rightmode(\tilde{A}) < k - rightmode(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $k - rightmode(\tilde{A}) = k - rightmode(\tilde{B})$ then go to Step 8

Step - 8: Find $k - leftmode(\tilde{A})$ and $k - leftmode(\tilde{B})$

Case - (i): If $k - leftmode(\tilde{A}) > k - leftmode(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $k - leftmode(\tilde{A}) < k - leftmode(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $k - leftmode(\tilde{A}) = k - leftmode(\tilde{B})$ then go to Step 9

Step - 9: Find $0 - Divergence(\tilde{A})$ and $0 - Divergence(\tilde{B})$

Case- (i): If $0 - Divergence(\tilde{A}) > 0 - Divergence(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$

Case - (ii): If $0 - Divergence(\tilde{A}) < 0 - Divergence(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $0 - Divergence(\tilde{A}) = 0 - Divergence(\tilde{B})$ then go to Step 10

Step - 10: Find k_1 and k_2

Case - (i): If $k_1 > k_2$ then $\tilde{A} \succ \tilde{B}$

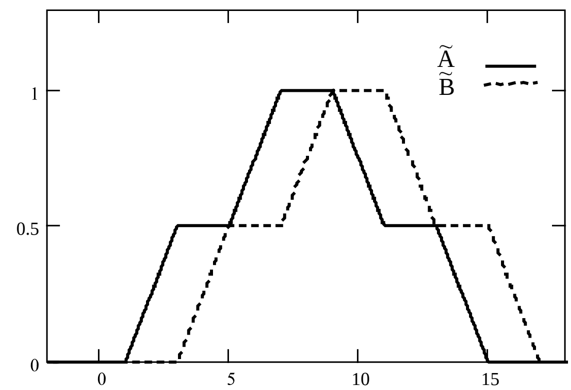
Case - (ii): If $k_1 < k_2$ then $\tilde{A} \prec \tilde{B}$

Case - (iii): If $k_1 = k_2$ then $\tilde{A} \approx \tilde{B}$

4.NUMERICAL EXAMPLES

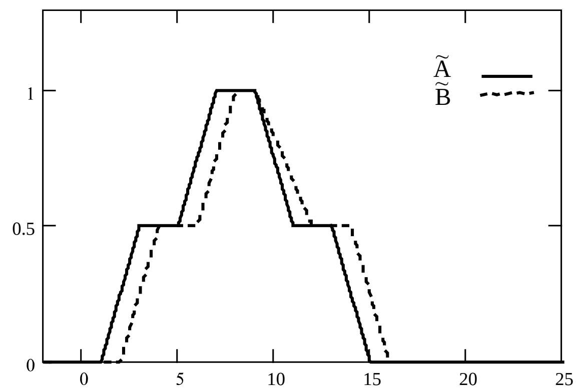
In this section, using the Mathematical software – MATHCAD 14 few sets of octagonal fuzzy numbers are represented graphically and their ranking using above mentioned methods are tabulated.

The various sets of octagonal fuzzy numbers considered, though not exhaustive, are such that they have equal spread but different central value, same central value but different divergence at *w*-level, symmetric over zero with different spreads, spreads over negative and positive sides, same mode and mean and divergence with different spreads, different heights and different *k*-levels to be compared and the results are tabulated in Table 4.1.



Set 1: $\tilde{A} = \left(1,3,5,7,9,11,13,15;\frac{1}{2},1\right)$

$\tilde{B} = \left(3,5,7,9,11,13,15,17;\frac{1}{2},1\right)$



Set -2: $\tilde{A} = \left(1,3,5,7,9,11,13,15;\frac{1}{2},1\right)$

$\tilde{B} = \left(2,4,6,8,9,12,14,16;\frac{1}{2},1\right)$

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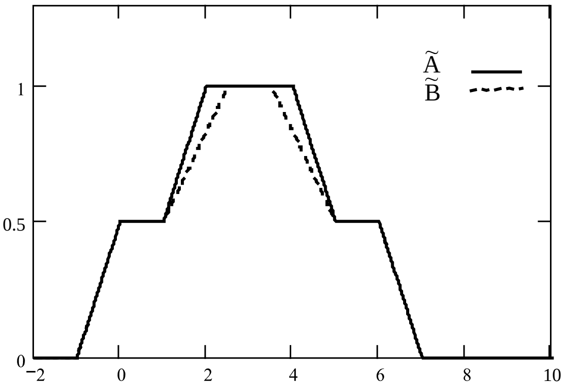
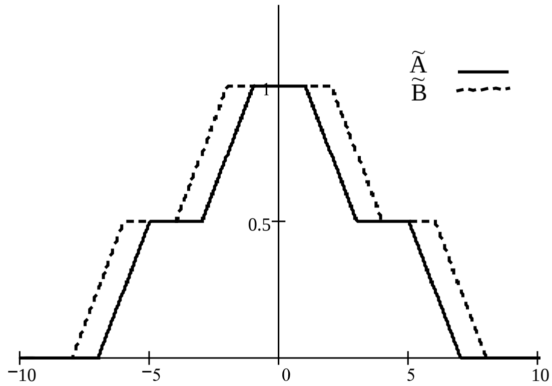


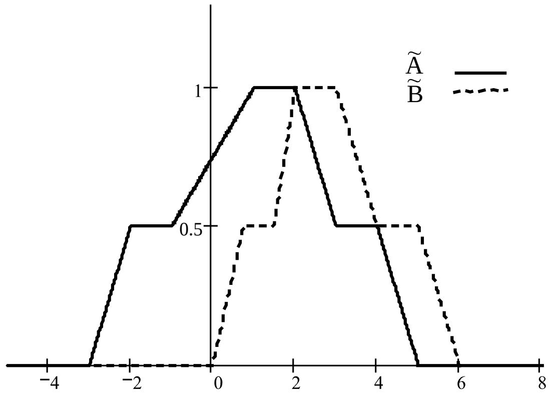
$$\text{Set }-3:\ \tilde{A}=\left(-1,0,1,2,3,5,6,7;\frac{1}{2},1\right)$$

$$\tilde{B}=\left(-1,0,1,2.5,3.5,5,6,7;\frac{1}{2},1\right)$$



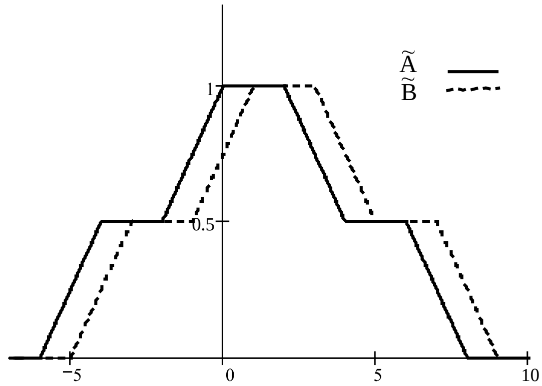
$$\text{Set }-4:\ \tilde{A}=\left(-7\ -,5,-3,-1,1,3,5,7;\frac{1}{2},1\right)$$

$$\tilde{B}=\left(-8,-6,-4,-2,2,4,6,8;\frac{1}{2},1\right)$$



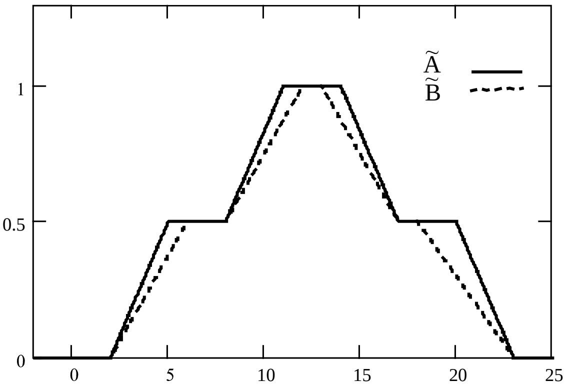
$$\text{Set }-5:\ \tilde{A}=\left(-3,-2,-1,1,2,3,4,5;\frac{1}{2},1\right)$$

$$\tilde{B}=\left(0,0.75,1.5,2,3,4,5,6;\frac{1}{2},1\right)$$



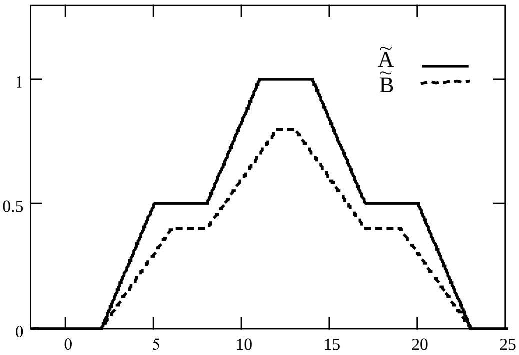
$$\text{Set }-6:\ \tilde{A}=\left(-6,-4,-2,0,0,4,6,8;\frac{1}{2},1\right)$$

$$\tilde{B}=\left(-5,-3,-1,1,3,5,7,9;\frac{1}{2},1\right)$$



$$\text{Set }-7:\ \tilde{A}=\left(2,5,8,11,14,17,20,23;\frac{1}{2},1\right)$$

$$\tilde{B}=\left(2,6,8,12,13,17,18,23;\frac{1}{2},1\right)$$



$$\text{Set }-8:\ \tilde{A}=\left(2,5,8,11,14,17,20,23;\frac{1}{2},1\right)$$

$$\tilde{B}=\left(2,6,8,12,13,17,18,23,-5,-3,-1,1,3,5,7,9;\frac{2}{5},\frac{4}{5}\right)$$

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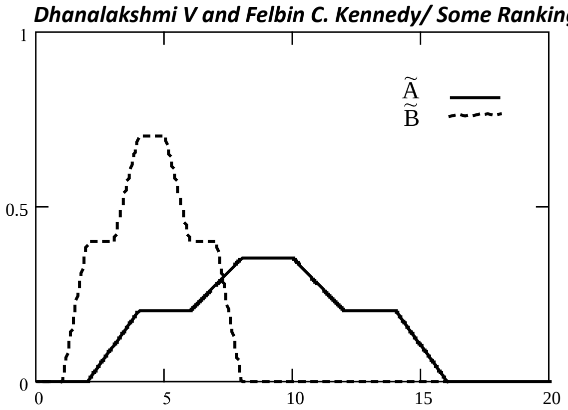

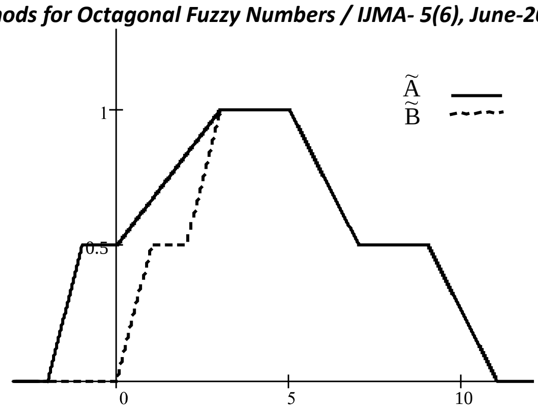

$$\text{Set}-9:\ \tilde{A}=\left(2,4,6,8,10,12,14,16;\frac{1}{5}\frac{7}{20}\right)$$

$$\tilde{B}=\left(1,2,3,4,5,6,7,8;\frac{2}{5},\frac{7}{10}\right)$$

$$\text{Set}\ -10:\ \tilde{A}=\left(-2,-1,0,3,5,7,9,11;\frac{1}{2},1\right)$$

$$\tilde{B}=\left(0,1,2,3,5,7,9,11;\frac{1}{2},1\right)$$

The above ten sets of octagonal fuzzy numbers are considered for comparison using the ranking techniques defined above.

Table - 4.1: Ranking of the above ten sets of octagonal fuzzy numbers based on the methods discussed

	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10
Centroid -Area	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{B}\succ\tilde{A}$
ROG -Area	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{B}\succ\tilde{A}$
Sign Distance $p=1$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\approx\tilde{B}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}\approx\tilde{B}$
Sign Distance $p=2$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\succ\tilde{B}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{B}\succ\tilde{A}$
Deviation Degree	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\approx\tilde{B}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{B}\succ\tilde{A}$
Ranking Algorith m	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\succ\tilde{B}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{B}\succ\tilde{A}$	$\tilde{B}\succ\tilde{A}$

Remark 4.1: In all the ranking methods considered we have ensured that the octagonal fuzzy numbers satisfy the condition $\tilde{A}\prec\tilde{B}\Rightarrow-\tilde{A}\succ-\tilde{B}.$

5.CONCLUSION

Uniqueness of octagonal fuzzy numbers is established (Corollary 3.1). In fact the algorithm proposed in section 3.5 compares any two octagonal fuzzy numbers and also affirms the corollary. We have proposed few ranking methods for octagonal fuzzy numbers, who may find their applications in decision making, optimization, risk analysis etc. which is visible through the numerical examples cited in Section 4.

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