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GENERALIZED STEADY MHD POISEUILLE FLOW AND TWO INFINITE PARALLEL POROUS PLATES

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ABSTRACT

In this paper, we examine the motion of a steady flow of a viscous,electrically conducting, incompressible fluid flowing between two infinite parallel porous plates under the influence of transverse magnetic field and constant pressure gradient.The lower plate is assumed porous while the upper plate is not.The resulting coupled governing eqution of motion is solved analytically by an analytical method.Analytical expression for the fluid velocity obtained is expressed for the fluid velocity obtained is terms of Hartmann number.

***Key words**: MHD flow Navier –stokes equations, porous plate, analyticsolution.*

1. INTRODUCTION

Magnetohydrodynamics is the fluid mechanics of electrically conducting fluids. Some of these fluids include liquid metals and ironized gases known by Physicists as plasma, one example being the Solar atmosphere. The subject of MHD is largely perceived to have been initiated by Swedish electrical engineer Hannes Alfv´en [1] in 1942. If an electrically conducting fluid is placed in a constant magnetic field, the motion of the fluid induces currents which create forces on the fluid. The production of these currents has led to the design of among other devices the MHD generators for electricity production. The equations which describe MHD flow are a combination of continuity equation and Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. The governing equations are differential equations that have to be solved either analytically or numerically.

Shercliff [2] studied the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. Drake [3] considered flow in a channel due to periodic pressure gradient and solved the resulting equation by separation of variables method. Singh & Ram [4] considered laminar flow of an electrically conducting fluid through a channel in the presence of a transverse magnetic field under the influence of a periodic pressure gradient and solved the resulting differential equation.. Singh [8] considered steady magnetohydrodynamic fluid flow between two parallel plates. Kazuyuki [7] discussed inertia effects in two dimensionalMHD channel flow and Al-Hadhrami [9] considered flow of fluids through horizontal channels of porous materials and obtained velocity expressions in terms of the Reynolds number. Ganesh [10] studied unsteady MHD Stokes flow of a viscous fluid between two parallel porous plates. They considered fluid being withdrawn through both walls of the channel at the same rate.

In this paper we consider two dimensional Poiseuille flow of an electrically conducting fluid between two infinite parallel plates under the influence of transverse magnetic field under a constant pressure gradient and assess the effect to velocity if the lower plate is porous. The resulting differential equation is solved by an analytical method and the solution expressed in terms of Hartmann number.

2. MATHEMATICAL FORMULATION

Theequation of motion are the continuity equation $\nabla V = 0$. And the Navier –Stockes Eution

$$\rho\left[\left(\frac{\partial}{\partial t} + v.\nabla\right)\right]V = f_B - \nabla P + \mu\nabla^2 V \tag{2.1}$$

where ρ is the fluid density, f_B is body force per unit mass of the fluid, μ is the fluid viscosity and p is the pressure acting on the fluid. Let us examine unidirectional flow, in which we choose the axis of the channel formed by the two plates as the x- axis and assume that flow is in this direction.

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If $V = \bar{u}(\bar{x},\bar{y},\bar{z})i + \bar{v}(\bar{x},\bar{y},\bar{z})j + w(\bar{x},\bar{y},\bar{z})k$ in which $\bar{u},\bar{v}$ and $\bar{w}$ are the components of the velocity in $\bar{x}-$, $\bar{y}-$ and $\bar{z}-$ directions respectively and bars denote dimensionless quantities, then this implies $\bar{v} = \bar{w} = 0$ and $\bar{u} \neq 0$. Continuity equation yields

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{F_x}{\rho} \tag{2.3}$$

So that $\frac{\partial \bar{u}}{\partial \bar{x}} = 0$ from which we infer that u is independent of $\bar{x}$. This makes the non linear term [(V.∇)V] in the Navier-Stokes equations zero. We neglect body forces f_B which are mainly due to gravity in the Navier-Stokes equations and replace them with the Lorentz force and assuming that the flow is two dimensional i.e. that flow variables are independent of z-direction, it means that the governing equations for this flow are where $v = \frac{\mu}{\rho}$ is the kinematic viscosity and F_x is the component of the magnetic force in the direction. we note that $\bar{p}$ is a function of $\bar{x}$ only. Also assuming unidirectional flow $\bar{v} = \bar{w} = 0$ and $B_x = B_z = 0$ so that $V = \bar{u}i$ and $B = B_0 j$ where B_0 is the magnetic field strength component assumed to be applied to a direction perpendicular to fluid motion ($\bar{y}-$direction). Now

$$F_x = \sigma\left[\left(i\bar{u} \times jB_0\right)\right] \times jB_0 \tag{2.4}$$

From which we find that

$$\frac{F_x}{\rho} = -\frac{\sigma}{\rho}B_0^2\bar{u} \tag{2.5}$$

Using the above equation in equation we find

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho}B_0^2\bar{u} \tag{2.6}$$

Or

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho}B_0^2\bar{u}\sin^2(\alpha) \tag{2.7}$$

where α is the angle between V and B. Equation (2.7) is general in the sense that the two fields can be assessed at any angle α for $0 \leq \alpha \geq \pi$

$$\rho\frac{\partial \bar{u}}{\partial t} = -v_0\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{p}}{\partial \bar{x}} + \mu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho}B_0^2\bar{u}\sin^2(\alpha) \tag{2.8}$$

$$\frac{\partial T}{\partial \bar{t}} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial \bar{y}^2} \tag{2.9}$$

$$\bar{u}(y,t) = 0,\ \bar{T} = \bar{T}_\infty \text{ at } \bar{t} = 0,$$
$$\bar{u}(-L,\bar{t}) = 0,\ \bar{u}(L,\bar{t}) = \frac{v}{L},\ \bar{T} = \bar{T}_w \text{ at } \bar{t} > 0 \tag{2.10}$$

We use the quantities in the above equation into (2.7) to find

$$\bar{x} = xL,\ \bar{y} = yL,\ \bar{p} = pp\frac{v^2}{L^2},\ \bar{u} = \frac{uv}{L},\ \bar{t} = \frac{tL^2}{v},\ P_r = \frac{\mu c_p}{k},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ Ha^2 = \frac{\sigma L^2 B_0^2}{\mu} \tag{2.11}$$

Eequation 2.8 and 2.9 now becomes

$$\frac{\partial u}{\partial t} = -\frac{R_2}{\rho}\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} - M^2 u \tag{2.12}$$

where M = M* sin α and $M^* = lB_0\sqrt{\frac{\sigma}{\mu}} = Ha$ where Ha is the Hartmannn number given by $Ha^2 = \frac{\sigma B_0^2 l^2}{\mu}$.

Differentiate equation (2.12) with respect to x to find $\frac{d^2P}{dx^2} = 0$ and on integrating this we find $\frac{dp}{dx} = -c$ (a constant).

Hence on substitution, (2.12) becomes

$$P_r \frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} \tag{2.13}$$

whose solution under the boundary conditions u = 0 for y = ±1 is given by
u(y, t) = 0, θ(-1, t) = 0 at t = 0

u(-1, t) = 0, u(1, t) = 1, θ(1, t) = 1 at t > 0 (2.14)

III. SOLUTION OF THE PROBLEM

In this section we used separation of variable technique to solve equations (2.12) and (2.13) subject to the boundary condition (2.14). We assumed the solutions of equations (2.12) and (2.13) to be respectively in the form
u(y, t) = u(y)u(t) (2.15)

θ(y, t) = θ(y)θ(t) (2.16)

As a consequence of equations (2.15) and (2.16), the solutions of the velocity profile and temperature distribution are

$$u(y,t) = e^{-\lambda^2 t}\left(c_1 e^{m_1 y} + c_2 e^{m_2 y}\right) \tag{2.17}$$

$$\theta(y,t) = e^{-\frac{\lambda^2}{\Pr}t} c_3 \cos\lambda y + c_4 \sin\lambda y \tag{2.18}$$

where

$B = M^2 + \lambda^2$

$$m_1 = \frac{A + \sqrt{A^2 + 4B}}{2}$$

$$m_2 = \frac{A - \sqrt{A^2 + 4B}}{2}$$

$$c_1 = -c_2 e^{m_1 - m_2}$$

$$c_2 = \frac{-e^{\lambda^2 t - m_1}}{e^{m_1 - m_2} - e^{m_2 - m_1}}$$

$$c_3 = \frac{e^{-\frac{\lambda^2}{\Pr}t}}{2\sin\lambda}$$

$c_4 = c_3 \tan\lambda$

SKIN FRICTION τ

We considered the skin frictions τ_1 and τ_2 at y = -1 and y = 1 respectively.

$$\tau_1 = \frac{du}{dy}\Big|_{y=-1} = e^{-\lambda^2\tau}\left(c_1 m_1 e^{-m_1} + c_2 m_2 e^{-m_2}\right) \tag{2.19}$$

$$\tau_2 = \frac{du}{dy}\Big|_{y=1} = e^{-\lambda^2\tau}\left(c_1 m_1 e^{m_1} + c_2 m_2 e^{m_2}\right) \tag{2.20}$$

NUSSELT NUMBER Nu

We also considered the Nusselt numbers Nu_1 and Nu_2 at y = -1 and y = 1 respectively.

$$Nu_1 = \lambda e^{-\frac{\lambda^2}{\Pr}t}\left(c_3 \sin y + c_4 \cos \lambda\right) \tag{2.21}$$

$$Nu_2 = \lambda e^{-\frac{\lambda^2}{\Pr}t}\left(-c_3 \sin y + c_4 \cos \lambda\right) \tag{2.22}$$

IV. DISCUSSION OF RESULTS

The effect of inclined Hartmann in an unsteady MHD couette flow between two infinite parallel porous plates with heat transfer. The velocity profile and the temperature distribution are shown graphically against y for different values of the Hartmann number Ha and Prandlt number Pr.Figure 1, 2, 3 and 4 depicts decrease in velocity u as Ha increases with effect of increase in the angle of inclination α on velocity.Tables 1, 2, 3 and 4 show decrease in skin friction τ_1 and increase in skin friction τ_2 with increase in Hartman number Ha and angle of inclination α.Tables 5 and 6 show the variation of Nusselt numbers Nu_1 and Nu_2 with different values of Prandtl number Pr and time t. Both tables show that Nu_1 and Nu_2 decreases with increase in Prandtl number Pr and time t.

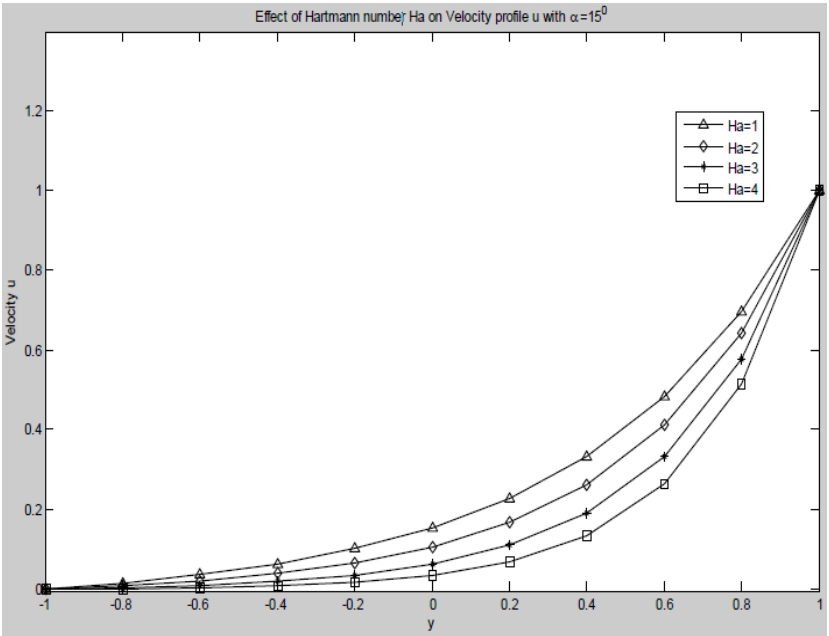

Figure - 1: Effect of Hartmann number Ha on velocity profile u with $\alpha = 15^0$, A = 1, t = 0.5, and λ = 1.

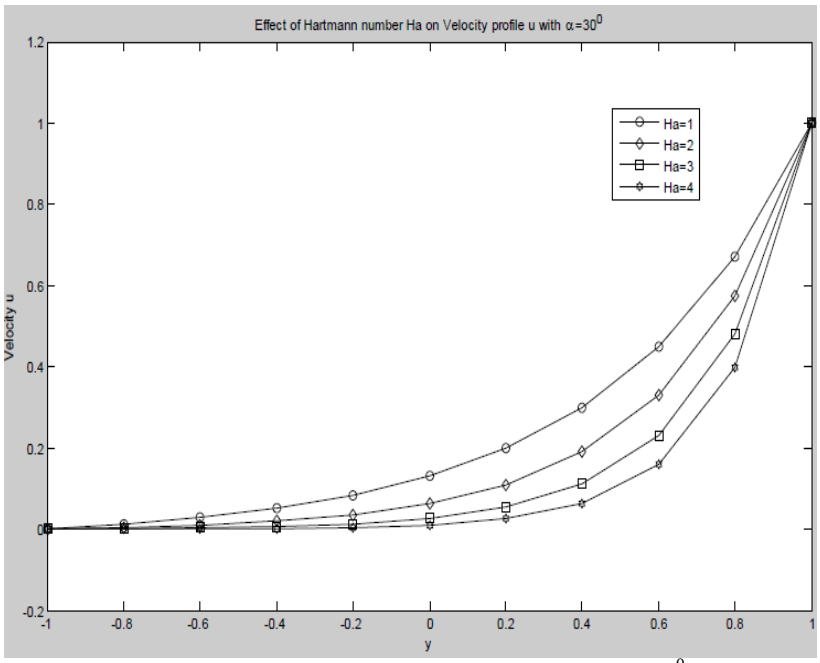

Figure - 2: Effect of Hartmann number Ha on velocity profile u with $\alpha = 30^0$, A = 1, t = 0.5 and λ = 1.

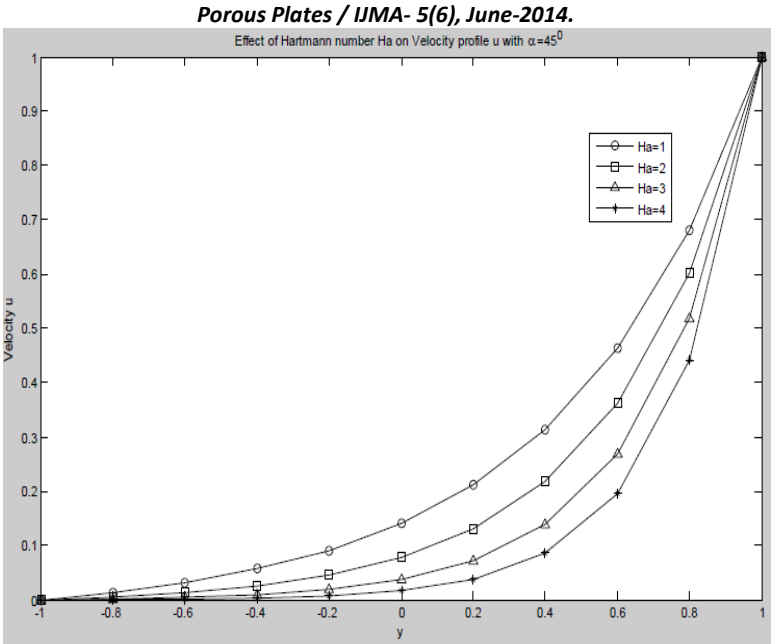

Figure - 3: Effect of Hartmann number Ha on velocity profile u with $\alpha = 45^0$, A = 1, t = 0.5 and $\lambda = 1$.

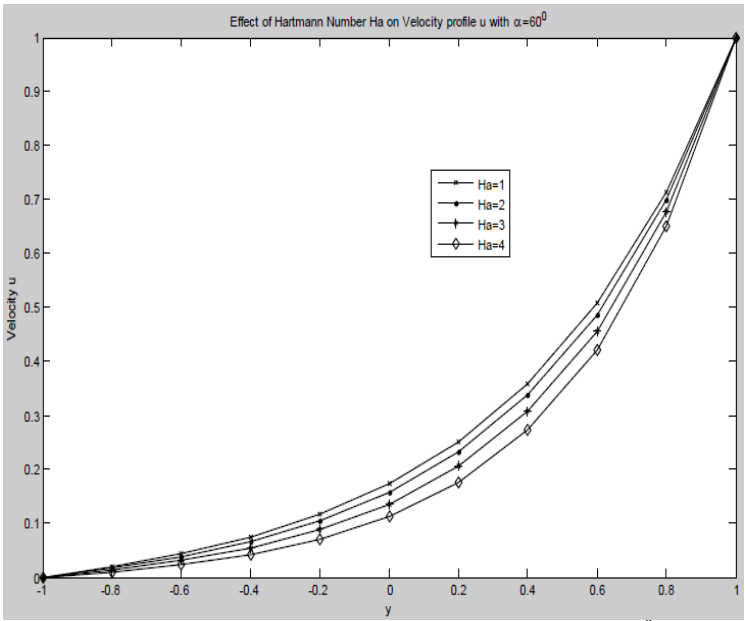

Figure - 4: Effect of Hartmann number Ha on velocity profile u with $\alpha = 60s^0$, A = 1, t = 0.5 and $\lambda = 1$.

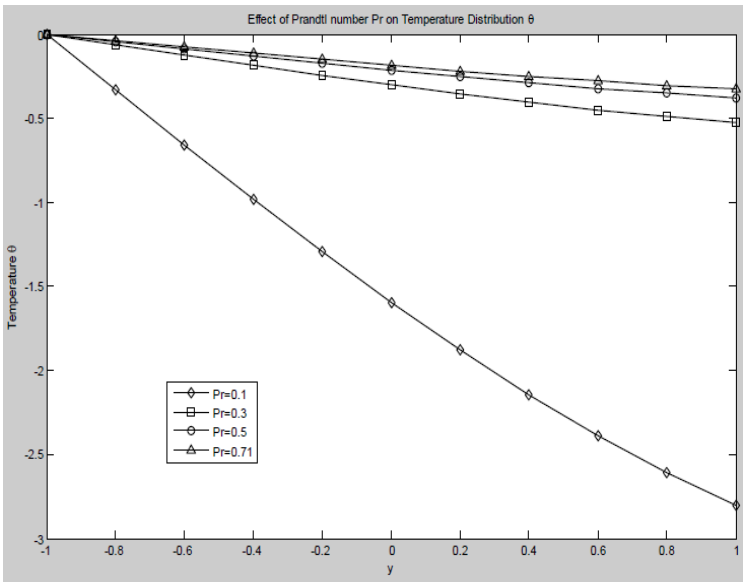

Figure - 5: Effect of Prandtl number Pr on temperature distribution θ with $\lambda = 0.5$, t=0.5

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Table - 1: Variation of skin friction τ_1 and τ_2 with different values of Hartmann number Ha at the inclination $\alpha = 15^0$.

Ha	τ_1	τ_2
1.0	0.5058	1.5028
2.0	0.0566	1.9883
3.0	0.0241	2.5761
4.0	0.0090	3.1957

Table - 2: Variation of skin friction τ_1 and τ_2 with different values of Hartmann number Ha at the inclination $\alpha = 30^0$.

Ha	τ_1	τ_2
1.0	0.0793	1.7340
2.0	0.0232	2.5997
3.0	0.0051	3.5473
4.0	3.6189×10^{-4}	4.5149

Table - 3: Variation of skin friction τ_1 and τ_2 with different values of Hartmann number Ha at the inclination $\alpha = 45^0$.

Ha	τ_1	τ_2
1.0	0.0901	1.6332
2.0	0.0340	2.3452
3.0	0.0098	3.1490
4.0	0.0024	3.9763

Table - 4: Variation of skin friction τ_1 and τ_2 with different values of Hartmann number Ha at the inclination $\alpha = 60^0$.

Ha	τ_1	τ_2
1.0	0.1273	1.3442
2.0	0.1088	1.4793
3.0	0.0851	1.6789
4.0	0.0621	1.9195

Table - 5: Variation of Nusselt number Nu_1 with different values of Prandtl number Pr and time t.

t	$Pr = 0.1$	$Pr = 0.3$	$Pr = 0.5$	$Pr = 0.71$
0.0	-0.1366	-0.1366	-0.1366	-0.1366
0.5	-1.6638	-0.3143	-0.2252	-0.1942
1.0	-20.2696	-0.7231	-0.3713	-0.2762
1.5	-246.9345	-1.6638	-0.6121	-0.3928
2.0	-3.0083×10^3	-3.8284×10^3	-1.0092	-0.5585

Table - 6: Variation of Nusselt number Nu_2 with different values of Prandtl number Pr and time t.

t	$Pr = 0.1$	$Pr = 0.3$	$Pr = 0.5$	$Pr = 0.71$
0.0	-0.0477	-0.0477	-0.0477	-0.0477
0.5	-0.5814	-0.1098	-0.0787	-0.0679
1.0	-7.0825	-0.2527	-0.1297	-0.0965
1.5	-86.228	-0.5814	-0.2139	-0.1372
2.0	-1.0511×10^3	-1.3377	-0.3526	-0.1952

V. SUMMARY AND CONCLUSION

we studied the effect of inclined Hartmann in an unsteady MHD couette flow between two infinite parallel porous plates with heat transfer. The momentum and energy equations are written in a dimensionless form using the dimensionless parameters. Variable separable technique was employed to solved the velocity profile and temperature distribution. However, at high Hartmann number, the velocity as well as the skin friction decreases. When the magnetic field is high, it reduces the energy loss through the plates. But large Nusselt number corresponds to more active convection. Also, when the Prandlt number increases, the temperature distribution decreases. Thus, it shows that magnetic field has significant effect to the flow of an unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer. This work can be applied in electric power generator, extrusion of plastics in the manufacture of Rayon and Nylon etc.

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