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g*s-irresolute maps and g*s- homeomorphism in topological spaces

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ABSTRACT

*A.Pushpalatha and K.Anitha introduced properties of g*s-closed sets in topological space. In this paper, we introduced g*s-irresolute maps, g*s-hausdorff spaces, g*s-homeomorphism and study their basic properties in topological spaces.*

Keywords: *g*s-closed sets, g*s-open sets, g*s-continuous map, g*s-irresolute map, g*s-hausdorff and g*s-homeomorphism.*

2000 Mathematics subject classification: *54A05, 54C08, 54C05.*

1. INTRODUCTION

In 1970, Levine [8] first considered the concept of generalized closed (briefly g-closed) sets were defined and investigated. Arya and Nour [1] defined generalized semi open sets (briefly gs-open) using semi open sets. In 1987, Bhattacharyya and Lehiri [2] introduced the class of semi-generalized closed sets (briefly sg-closed). A.Pushpalatha and K.Anitha [10, 11] introduces properties of g*s-closed sets in topological space.

The notion homeomorphism plays a very important role in topology. In this paper, we introduce a new class of irresolute map called g*s-irresolute map and then we study g*s-hausdorff, g*s-homeomorphism and g*sc-homeomorphism.

2. PRELIMINARIES

Throughout this paper we shall denote by(X, τ) a topological space. For any subset A ⊆ X, int(A) and cl(A) denote the interior of A and the closure of A with respect to τ.

We shall require the following known definitions.

Definition: 2.1[9] Let(X, τ) be a topological spaces. A subset A of X is called semi-open if A⊆ cl (int (A)) and semi-closed if int (cl (A)) ⊆A. The intersection of all semi closed sets containing A is called the semi closure of A, denoted by scl (A). The union of all semi open sets contained in A is called the semi interior of A, denoted by sint (A).

Definition: 2.2[10] Let (X, τ) be a topological space. A subset A of X is called gs- closed if scl (A) ⊆U whenever A⊆U and U is open in (X, τ).

Result: 2.3[9] The complement of gs-closed set is gs-open.

Definition: 2.4[10] Let (X, τ) be a topological space. A subset A of X is called g*s-closed if scl (A) ⊆U whenever A⊆U and U is gs-open in (X, τ). The complement of g*s-closed set is g*s-open.

Result: 2.5[10] Every closed set is g*s-closed.

Definition: 2.6 [10] A map f: X→Y is called g*s-open map if f(U) is g*s-open in y for every open set U in X. Every open map is g*s-open map. A map f: X→Y is called g*s-closed map if for each closed set F in X, f(F) is a g*s- closed set in Y.

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Theorem: 2.7 [13] For any bijection f: (X, τ) →(Y, σ) the following are equivalent.
(i) f^{-1}: (Y, σ) →(X, τ) is g*s-continuous.
(ii) f is a g*s-open map
(iii) f is a g*s-closed map.

Definition: 2.8 [11] A map f: X→Y from a topological space X into a topological space Y is called g*s-continuous if the inverse image of every closed set in Y is g*s-closed in X.

Result: 2.9[11] Every continuous map is g*s-continuous and g*s- continuous map is gs-continuous.

Definition: 2.10 [11] A map f: X→Y is said to be strongly g*s-continuous if the inverse image of every g*s- open set in Y is open in X.

Definition: 2.11[11] A topological space X is g*s-compact if every g*s-open cover of X has a finite sub cover of X.

Definition: 2.12 [11] A subset B of a topological space X is called g*s-compact relative to X, if for every collection $\{A_i: i \in I\}$ of g*s-open subsets of X such that $B \subseteq \cup_{i \in I} A_i$, there exist a finite subset I_o of I such that $B \subseteq \cup_{i \in I_o} A_i$

Definition: 2.13[12] A topological space X is called a g*s-connected if X cannot be written as a disjoint union of two non-empty g*s-open sets.

Definition: 2.14 [9] A space X is said to be Hausdorff if whenever *x* and y are distinct points of X, there exist disjoint open sets U and V such that $x \in U$ and $y \in V$.

Definition: 2.15[4] A bijection f: (X, τ) →(Y, σ) is called a homeomorphism if f is both continuous and open.

Definition: 2.16[4] A bijection f: (X, τ) →(Y, σ) is called a gs- homeomorphism if f is both gs-continuous and gs-open.

3. g*s-irresolute maps in topological spaces

In this section we introduce the concepts of g*s-irresolute maps in topological spaces.

Definition: 3.1 A map f: (X, τ)→ (Y, σ) is called g*s -irresolute if the inverse image of every g*s-closed set in Y is g*s-closed in X.

Theorem: 3.2 A map f: X → Y is g*s-irresolute if and only if for every g*s-open A of Y, $f^{-1}(A)$ is g*s-open in X.

Proof: Necessity: If f: X→Y is g*s-irresolute, then for every g*s-closed B of Y, $f^{-1}(B)$ is g*s-closed in X. If A is any g*s-open subset of Y, then A^c is g*s-closed. Thus $f^{-1}(A^c)$ is g*s-closed, but $f^{-1}(A^c) = (f^{-1}(A))^c$ so that $f^{-1}(A)$ is g*s-open in X.

Sufficiency: If for all g*s-open subsets A of Y, $f^{-1}(A)$ is g*s-open in X and if B is any g*s-closed subset of Y, then B^c is g*s-open. Also $f^{-1}(B^c) = (f^{-1}(B))^c$ is g*s-open in X. Thus $f^{-1}(B)$ is g*s-closed in X. Hence f is g*s-irresolute.

Theorem: 3.3 If a map f: X→Y is g*s-irresolute, then it is g*s-continuous.

Proof: Let A be a closed set in Y. Since every closed set is g*s-closed, A is g*s-closed in Y. Since f is g*s-irresolute, $f^{-1}(A)$ is g*s-closed in X. Hence f is g*s-continuous.

Remark: 3.4 The converse need not be true as seen from the following example.

Example: 3.5 Let X =Y = {a, b, c}, τ = { ϕ, X, {a}, {c}, {a, c}} and σ = { ϕ, Y, {a}}.Let f: (X, τ) → (Y, σ) be defined by f (a) =f (c) = b and f (b) = c. Then f is g*s-continuous. However, {b} g*s-closed in Y but $f^{-1}(\{b\})$ = {a, c} is not g*s-closed in X. Therefore, f is not g*s-irresolute.

Theorem: 3.6 If f: X → Y and g: Y → Z are both g*s-irresolute, then g ∘ f: X→Z is g*s-irresolute.

Proof: Let A be a g*s-open subset of Z. Since g is g*s-irresolute, $g^{-1}(A)$ is g*s-open in Y. Since f is g*s-irresolute, $f^{-1}(g^{-1}(A))$ is g*s-open in X. Thus $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is g*s-open in X. Hence g∘f is g*s-iresolute.

Theorem: 3.7 Let X, Y and Z be any topological spaces. For any g*s-irresolute map f: X→Y and any g*s -continuous map g: Y→Z, the composition g∘f: X→ Z is g*s-continuous.

Proof: Let F be a closed set in Z. Since g is g*s-continuous, $g^{-1}(F)$ is g*s-closed in Y. Since f is g*s-irresolute, $f^{-1}(g^{-1}(F))$ is g*s-closed in X. Thus $(g\circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is g*s-closed in X. Hence g∘f is g*s-continuous.

Theorem: 3.8 If a map f: X→Y is g*s-irresolute and a subset B of X is g*s-compact relative to X, then the image f (B) is g*s-compact relative to Y.

Proof: Let $\{A_i : i \in I\}$ be any collection of g*s-open subsets of Y such that $f(B) \subset \cup\{A_i : i \in I\}$. Then $B \subset \cup\{f^{-1}(A_i) : i \in I\}$ holds. By hypothesis there exists a finite subset I_o of I such that $B \subset \cup\{f^{-1}(A_i) : i \in I_o\}$. Therefore we have $f(B) \subset \cup\{A_i : i \in I_o\}$ which shows that f(B) is g*s-compact relative to Y.

Theorem: 3.9 If f: X→Y is g*s-irresolute surjection and X is g*s-connected, then Y is g*s-connected.

Proof: Suppose Y is not g*s-connected. Let Y=A∪B where A and B are disjoint non-empty g*s-open set in Y. Since f is g*s-irresolute and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty and g*s-open in X. This contradicts the fact that X is g*s-connected. Hence Y is g*s-connected.

4. g*s-Hausdorff in topological spaces

In this section we have introduce the concept of g*s-Hausdorff in topological spaces.

Definition: 4.1 A space X is said to be g*s-Hausdorff if whenever x and y are distinct points of X, there exist disjoint g*s-open sets U and V such that $x \in U$ and $y \in V$.

Theorem: 4.2 Let X be a space and Y be Hausdorff. If f: X→Y is g*s-continuous injective, then X is g*s-Hausdorff.

Proof: Let x and y be any two distinct points of X. Then f(x) and f(y) are distinct points of Y, because f is injective. Since Y is Hausdorff, there are disjoint open sets U and V in Y containing f(x) and f(y), respectively. Since f is g*s-continuous and U∩V=∅, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g*s-open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is g*s-Hausdorff.

Theorem: 4.3 Let X be a space and Y be g*s- Hausdorff. If f: X→Y is g*s -irresolute injective, then X is g*s-Hausdorff.

Proof: Let x and y be any two distinct points of X. Then f(x) and f(y) are distinct points of Y, because f is injective. Since Y is g*s-Hausdorff, there are disjoint g*s-open sets U and V in Y containing f(x) and f(y), respectively. Since f is g*s-irresolute and U∩V=∅, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g*s-open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is g*s-Hausdorff.

Theorem: 4.4 Let X be a space and Y be g*s-Hausdorff. If f: X→Y is strongly g*s-continuous injective, then X is Hausdorff.

Proof: Let x and y be any two distinct points of X. Then f(x) and f(y) are distinct points of Y, because f is injective. Since Y is g*s-Hausdorff, there are disjoint g*s-open sets U and V in Y containing f(x) and f(y), respectively. Since f is strongly g*s-continuous and U∩V=∅, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is Hausdorff.

5. g*s – Homeomorphism in topological spaces

In this section, we introduce and study two new homeomorphisms namely g*s-homeomorphism, g*sc-homeomorphism and prove that the set of all g*s-homeomorphism forms a group under the operation of composition of maps.

Definition: 5.1 A bijection f: $(X, \tau) \rightarrow (Y, \sigma)$ is called a g*s-homeomorphism if f is both g*s-open and g*s-continuous. We denote the family of all g*s-homeomorphisms of a topological space (X, τ) onto itself by g*s-h(X, τ).

Theorem: 5.2 Every homeomorphism is a g*s-homeomorphism.

Proof: Let f $(X, \tau) \rightarrow (Y, \sigma)$ be a homeomorphism. To prove that f is g*s-homeomorphism. Since f is homeomorphism, f is bijection and also f is both open and continuous. Since every open map is g*s-open and every continuous map is g*s-continuous, f is bijection, g*s-open and g*s- continuous. Hence f is g*s-homeomorphism.

Remark: 5.3 The converse of the above theorem5.2 need not be true as seen from the following example.

Example: 5.4 Consider X = Y = {a, b, c}, τ = { ϕ, X, {a, b}}, σ = { ϕ, Y, {a}}. Let f: (X, τ)$\rightarrow$ (Y, σ) be a n identity map. Then f is g*s-homeomorphism but not homeomorphism. Since {a, b} is open in (X, τ) but the image is not open in (Y, σ).

Theorem: 5.5 Every g*s-homeomorphism is gs-homeomorphism.

Proof: Let f: (X, τ)$\rightarrow$ (Y, σ) be a g*s - homeomorphism. To prove that f is gs-homeomorphism. Since f is g*s-homeomorphism, f is bijection and also f is both g*s-open and g*s-continuous. Since every g*s-open map is gs-open and every g*s-continuous map is gs-continuous, we have f is gs-open, gs- continuous and bijection. Hence f is gs-homeomorphism.

Remark: 5.6 The converse of the above theorem need not be true as seen from the following example.

Example: 5.7 Consider X = Y = {a, b, c}, τ = { ϕ, X, {a}}, σ = { ϕ, Y, {a}, {c}, {a, c}}. Let f: (X, τ)$\rightarrow$ (Y, σ) be an identity map. Then f is gs-homeomorphism but not g*s-homeomorphism. Since {a, b} is closed in (Y, σ) but f^{-1}({a, b}) = {a, b} is not g*s-closed in (X, τ).

Theorem: 5.8 Let f: (X, τ) $\rightarrow$ (Y, σ) be a bijective and g*s- continuous map, then the following are equivalent.

(i) f is g*s-open map.
(ii) f is g*s-homeomorphism.
(iii) f is g*s-closed map.

Proof:
(i) $\Longrightarrow$ (ii): Suppose that f is g*s-open map. To prove that f is g*s-homeomorphism. By hypothesis, f is bijective and g*s-continuous map. By definition of g*s-homeomorphism, f is g*s-homeomorphism.

(ii) $\Longrightarrow$ (iii): Suppose that f is g*s- homeomorphism. To prove that f is g*s- closed map. Since f is g*s-homeomorphism, f is bijective and also f is g*s-open and g*s-continuous. Let F be a closed set of (X, τ). Then F^c is open set in (X, τ). Since f is g*s-open map, f (F^c) is g*s- open in (Y, σ). f (F^c) = $(f(F))^c$ is g*s-open set in (Y, σ). Thus f (F) is g*s-closed set in (Y, σ). Hence f is g*s-closed map.

(iii) $\Longrightarrow$ (i): Suppose that f is g*s- closed map. To prove that f is g*s- open map. Let A be a closed set in (X, τ). Since f is g*s-closed map, f (A) is g*s-closed set in (Y, σ). f (A) = $(f^{-1})^{-1}$(A) is g*s-closed set in (Y, σ). Which implies f^{-1} is g*s-continuous on (Y, σ).By theorem 2.7, f is g*s-open map.

Remark: 5.9 The composition of two g*s-homeomorphism need not be g*s-homeomorphism in general as seen from the following example. Consider X = Y = Z = {a, b, c}, τ = { ϕ, X, {a}, {b}, {a, b}}, σ = { ϕ, Y,{a, b}} and η = {ϕ, Z, {a}}. Let f: (X, τ) $\rightarrow$ (Y, σ) and g: (Y, σ) $\rightarrow$ (Z,η) be the identity maps. Then both f and g are g*s-homeomorphism but their composition g $\circ$ f: (X, τ) $\rightarrow$ (Z,η) is not g*s-homeomorphism. Because for the open set {b} of (X, τ), g $\circ$ f ({b}) = g (f ({b})) = g ({b}) = {b} which is not g*s-open in (Z, η).

Definition: 5.10 A bijection f: (X, τ)$\rightarrow$ (Y, σ) is said to be g*sc - homeomorphism if both f and f^{-1} are g*s-irresolute. We denote the family of all g*sc-homeomorphism of a topological space (X, τ) onto itself by g*sc-h(X, τ).

Theorem: 5.11 Every g*sc-homeomorphism is g*s-homeomorphism.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be a g*sc- homeomorphism. To prove that f is g*s-homeomorphism. That is, to prove that f is bijective and also f is both g*s-open and g*s-continuous. Since f is g*sc-homeomorphism, f and f^{-1} are g*s-irresolute and f is bijective. Now, let V be a closed set in (Y, σ).Since f is g*s-irresolute, f^{-1}(V) is g*s-closed in (X, τ). Thus f is g*s-continuous. Since f^{-1} is g*s-irresolute, $(f^{-1})^{-1}$(V) is g*s-closed in (X, τ).Therefore f^{-1} is g*s-continuous. By theorem 2.7, f is g*s-open. Hence f is g*s-homeomorphism.

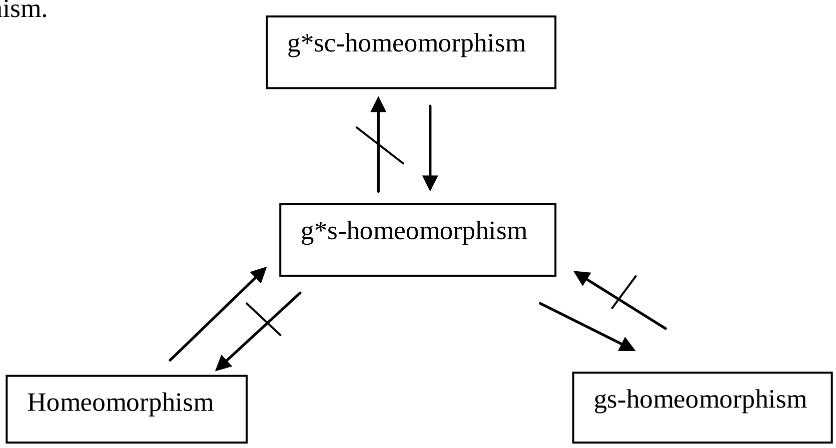
Remark: 5.12 The converse of the above theorem need not be true as seen from the following example.

Example: 5.13 Let X = Y = {a, b, c}, τ = { ϕ, X, {a}}, σ = { ϕ, Y, {a, b}}. Let f: (X, τ)$\rightarrow$ (Y, σ) be an identity map. Then f is g*s-homeomorphism but not g*sc-homeomorphism. Since f is not g*s-irresolute, the inverse image of {a, c} is not g*s-closed set in (X, τ).

Theorem: 5.14 Let f: (X, τ)$\rightarrow$ (Y, σ) and g: (Y, σ) $\rightarrow$ (Z, η) are g*sc- homeomorphism. Then their composition g $\circ$ f: (X, τ) $\rightarrow$ (Z,η) is also g*sc-homeomorphism.

Proof: Let U be a g*s-open set in (Z,η). Since g is g*s-irresolute, $g^{-1}(U)$ is g*s-open in (Y, σ). Since f is g*s-irresolute, $f^{-1}(g^{-1}(U)) = (g\circ f)^{-1}(U)$ is g*s-open in (X, τ).Therefore, $g\circ f$ is g*s-irresolute. Also for a g*s-open set G in (X, τ). We have $(g\circ f)(G) = g(f(G)) = g(W)$, where $W = f(G)$. By hypothesis, f (G) is g*s-open in (Y, σ) and so again by hypothesis, g (f(G)) is g*s-open set in (Z,η). $(g\circ f)(G)$ is g*s-open set in (Z,η). $(g\circ f)^{-1}$ is g*s-irresolute and also $g\circ f$ is bijective. Hence $g\circ f$ is g*sc-homeomorphism.

Remark: 5.15 The following diagram shows that the relationships between g*s-homeomorphism and other homeomorphism.



Note: 5.16 Let Γ be a collection of all topological spaces. We introduce a relation, say "$\equiv_{g*sc}$", into the family Γ as follows: for two elements (X, τ) and (Y, σ) of Γ, (X, τ) is g*sc- homeomorphic to (Y, σ) say $(X, \tau)\equiv_{g*sc}(Y, \sigma)$, if there exists a g*sc- homeomorphism $f: (X, \tau) \to (Y, \sigma)$.Then we have the following theorem on the relation "$\equiv_{g*sc}$".

Theorem: 5.17 The relation $\equiv_{g*sc}$ above is an equivalence relation in the collection of all topological spaces Γ.

Proof:

(i) For any element $(X, \tau)\in \Gamma$, $(X, \tau) \equiv_{g*sc}(X, \tau)$ holds. Indeed the identity function $I_x: (X, \tau) \to (X, \tau)$ is a g*sc-homeomorphism.

(ii) Suppose $(X, \tau) \equiv_{g*sc}(Y, \sigma)$, where (X, τ) and $(Y, \sigma) \in \Gamma$. Then, there exists a g*sc-homeomorphism $f: (X, \tau) \to (Y, \sigma)$. By definition it is seen that $f^{-1}: (Y, \sigma) \to (X, \tau)$ is a g*sc-homeomorphism and $(Y, \sigma) \equiv_{g*sc}(X, \tau)$.

(iii) Suppose that $(X, \tau) \equiv_{g*sc}(Y, \sigma)$ and $(Y, \sigma)\equiv_{g*sc}(Z, \eta)$,where (X, τ), (Y, σ) and $(Z, \eta) \in \Gamma$. By theorem 5.13, it is shown that $(X, \tau) \equiv_{g*sc}(Z, \eta)$.

Theorem: 5.18 The set g*sc-h(X, τ) is a group under the composition of maps.

Proof: Define a binary operation $*$: g*sc-h(X, τ) × g*sc-h(X, τ) → g*sc-h(X, τ) by $f * g = g \circ f$ for all f, g $\in$ g*sc-h(X, τ) and $\circ$ is the usual operation of composition of maps. Then by theorem 5.13, $g \circ f \in$ g*sc-h(X, τ). We know that the composition of maps is associative and the identity map I: $(X, \tau)\to(X, \tau)$ belonging to g*sc -h(X, τ) serves as the identity element. If $f \in$ g*sc-h(X, τ), then $f^{-1} \in$ g*sc-h(X, τ) such that $f \circ f^{-1} = f^{-1}\circ f = I$ and so inverse exists for each element of g*sc-h(X, τ). Therefore (g*sc-h(X, τ), $\circ$) is a group under the operation of composition of maps.

Theorem: 5.19 Let $f: (X, \tau) \to (Y, \sigma)$ be a g*sc-homeomorphism. Then f induces an isomorphism from the group g*sc-h(X, τ) onto the group g*sc-h(Y, σ).

Proof: Let $f \in$ g*sc-h(X, τ). Then define a map ψ_f: g*sc-h$(X, \tau) \to$ g*sc-h(Y, σ) by $\psi_f(h)=f\circ h\circ f^{-1}$ for every $h \in$ g*sc-h(X, τ). By theorem 5.13, ψ_f is well defined in general, because $f\circ h \circ f^{-1}$ is a g*sc-homeomorphism for every g*sc-homeomorphism $h: (X, \tau) \to (Y, \sigma)$. Let $h_1, h_2 \in$ g*sc-h(X, τ).Then $\psi_f(h_1 \circ h_2) = f \circ (h_1\circ h_2) \circ f^{-1} = (f \circ h_1\circ f^{-1}) \circ (f \circ h_2\circ f^{-1})$ Therefore $\psi_f(h_1 \circ h_2) = \psi_f(h_1) \circ \psi_f(h_2)$. Since $\psi_f(f^{-1}\circ h \circ f) =h$, ψ_f is onto. Now, $\psi_f(h) =I$ implies $f\circ h\circ f^{-1} =I$. That implies $h =I$. This proves that ψ_f is one-one. This shows that ψ_f is an isomorphism induced by f.

Theorem: 5.20 If $f: (X, \tau) \to (Y, \sigma)$ is a g*sc-homeomorphism, then g*s-cl$(f^{-1}(B))$ = f^{-1}(g*s-cl(B)) for every $B\subseteq Y$.

Proof: Let $f: (X, \tau)\to(Y, \sigma)$ be a g*sc -homeomorphism. By definition 5.9, both f and f^{-1} are g*s-irresolute and f is bijective. Let $B\subseteq Y$. Since g*s-cl(B) is a g*s-closed set in (Y, σ), using definition of g*s-irresolute, f^{-1}(g*s-cl(B)) is g*s-closed in (X, τ). But g*s-cl$(f^{-1}(B))$ is the smallest g*s-closed set containing $f^{-1}(B)$.Therefore g*s-cl$(f^{-1}(B))\subseteq f^{-1}$(g*s-cl(B))→(1). Again, g*s-cl$(f^{-1}(B))$ is g*s-closed in (X, τ). Since f^{-1} is g*s-irresolute, f (g*s-cl$(f^{-1}(B))$) is g*s-closed in (Y, σ). Now, $B= f(f^{-1}(B)) \subseteq$ f (g*S-cl $(f^{-1}(B))$). Since f (g*s-cl$(f^{-1}(B))$) is g*s-closed and g*s-cl(B) is the smallest g*s-closed set containing B, g*s-cl(B) $\subseteq$ f(g*s-cl$(f^{-1}(B))$) that implies f^{-1}(g*s-cl(B)) $\subseteq$g*s-cl$(f^{-1}(B))$. That is, f^{-1}(g*s-cl(B))$\subseteq$g*s-cl$(f^{-1}(B))$→(2). From (1) and (2), g*s-cl$(f^{-1}(B))$=f^{-1}(g*s-cl(B)).

Corollary: 5.21 If f: (X, τ) →(Y, σ) is a g*sc-homeomorphism, then g*s-cl(f(B)) = f(g*s-cl(B)) for every B⊆X.

Proof: Let f: (X, τ) → (Y, σ) be a g*sc -homeomorphism. Since f is g*sc-homeomorphism, f^{-1} is also a g*sc-homeomorphism. Therefore by theorem 5.20, it follows that g*s-cl (f (B)) = f(g*s-cl(B)) for every B⊆X.

Corollary: 5.22 If f: (X, τ) →(Y, σ) is a g*sc-homeomorphism, then f (g*s-int (B)) = g*s-int (f(B)) for every B⊆X.

Proof: Let f: (X, τ) →(Y, σ) be a g*sc-homeomorphism. For any set B⊆X, g*s-int(B)=(g*s-cl(B^c))^c. This implies that f (g*s-int (B)) = f (g*s-cl (B^c))^c = (f (g*s-cl (B^c))^c. Then using corollary 5.22, we get that f (g*s-int (B) = (g*s-cl (f(B^c))^c = g*s-int(f(B)).

Corollary: 5.23 If f: (X, τ) →(Y, σ) is a g*sc-homeomorphism, then for every B⊆Y, f^{-1}(g*s-int(B))=g*s-int(f^{-1}(B)).

Proof: Let f: (X, τ) → (Y, σ) be a g*sc -homeomorphism. Since f is g*sc-homeomorphism, f^{-1} is also a g*sc-homeomorphism. Therefore by corollary 5.22, it follows that f^{-1}(g*s-int (B)) = g*s-int (f^{-1}(B)) for every B⊆Y.

REFERENCES

1. Arya.S.P and Nour.T, characterizations of s-normal spaces, Indian J.Pure Appl.Math., 21, pp 717-719(1990).
2. Bhattacharya.P and Lahiri.B.K, semi generalized closed sets in topology, Indian J. Math., 29 (1987), no3, 375-378.
3. Devamanoharan.C and Pious Missier.S on ρ-homeomorphisms in topological spaces, Italian journal of pure and applied mathematics- N 30- 2013(195-214)
4. Devi.R, Maki.H and K.Balachandran (1995), semi generalized homeomorphisms and generalized semi homeomorphisms in topological spaces, Indian J.Pure Appl.Math., vol 26, no 3, pp 271-284(march).
5. Devi.R; studies on generalizations of closed maps and homeomorphisms in topological spaces.
6. Jeyanthi.M.J, Pious Missier.S and P.Thangavelu $\wedge_r$ –homeomorphisms and $\wedge_r^*$-homeomorphisms, Journal of Math science and computer Applications 1(2): 48-55, 2011.
7. Krishna.P, Dr.J.Antony Rex Rodrigo, On R-closed Maps and R-homeomorphisms in topological spaces. IOSR Journal of math vol 4, Issue 1(Nov-Dec 2012) pp 13-19.
8. Levine,N; Generalized closed sets in topology, Rend.circ.Mat.Palermo, 19(1970), 89-96.
9. Navalagi, characterizations of gs-hausdorff and sg-hausdorff spaces, Americal journal of applied mathematics and mathematical sciences vol 2, No 1,Jan-June 2013,pp 13-17.
10. Pushpalatha.A and Anitha.K. g*s-closed sets in topological spaces,Int.J.contemp.Math.Sciences,Vol.6;March-2011,no 19,917-929.
11. Pushpalatha.A and Anitha.K Strongly g*s-continuous maps and perfectly g*s-continuous in topological spaces, International Mathematical Forum, vol.7, 2012, no.24, 1201-1207.
12. Pushpalatha.A and Anitha.K, g*s-connectedness and g*s-locally closed sets in topological spaces, International Journal of Mathematical Archive-3(7), 2012, 2815-2818.
13. Thivagar.M.L, Rajesh.N and Ekicie, On $gs^{\sim}$ -homeomorphisms in topological spaces, Annals of university of Craiova Math. Com.sci, .vol 33, 2006, pa 208-215.
14. Subramonia Pilliai,V and Parimelazhagan,R, topological spaces, International Journal of Recent scientific research vol.v,Issue 1,pp 005-007,Jan 2013.
15. Vadivel,A and Vairamanickam, rgα-homeomorphism in topological spaces, Int. Journal of Math. Analysis, Vol4, 2010, no 18,881-890.

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