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NEAR COMMUTATIVE NEAR–IDEMPOTENT SEMIGROUP

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ABSTRACT

We have studied in our previous papers ([2], [6] and [7]) special semigroups which we call near idempotent semigroups, rectangular near- idempotent semigroups, left (right) regular near idempotent semigroups and left (right) normal near idempotent semigroups. In this paper we introduce a near commutative near – idempotent semigroup and obtain its decomposition into near – null semigroups.

Key words: *near idempotent semigroup, near commutative near idempotent semigroup, near – null semigroup, left (right) regular near- idempotent semigroup, left (right) normal – near idempotent semigroup.*

1. INTRODUCTION

A semigroup S is called near idempotent semigroup if $xy^2z = xyz$ for all x, y, z ∈ S. [2]. In this paper we deal with near commutative property for a near idempotent semigroup. We study its structure through the relations λ, ρ, δ, ξ we introduced in a previous paper.[2]. In a near – commutative near – idempotent semigroup all the above relations coincide. In other words δ = ρ = λ = ξ.

2. DEFINITION

Let S be a near – idempotent semigroup, we call S a near–commutative near– idempotent semigroup if
xyzw = xzyw for all x, y, z, w in S.

Lemma 2.1: In a near – commutative near – idempotent semigroup S, δ = ξ.

Proof: Let S be a near – commutatve near – idempotent semigroup then

xyzw = xzyw for all x, y, z, w on S.

$\xi = \rho \cap \lambda$

Hence $\xi \subset \rho \subset \delta$ and $\xi \subset \lambda \subset \delta$

Hence $\xi \subset \delta$

Conversely, let a δ b in S.

Then xabay = xay and xbaby = xby, by near – commutativity,
,
$xay = xabay = xa^2by = xaby$ (1)
and
$xby = xbaby = xab^2y = xaby$ (2)

xay = xby form (1) and (2)

Hence a ξ b

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Then $\delta \subset \xi$ so that $\delta = \xi$.

Conversely,

Lemma 2.2: In a near – idempotent semigroup S, $\delta = \xi$ implies that S is near – commutative.

Proof: Let a, b $\in$S

In any near – idempotent semigroup ab δ ba

But $\delta = \xi$

Hence ab ξ ba

Thus xaby = xbay for all x, y in S.

Hence S is near – commutative.

Thus we have

Theorem 2.3: A near idempotent semigroup S is near – commutative if and only if $\delta = \xi$ on S.

Proof: It follows that every δ – class in S degenerate into a near – null semigroup [6],

Hence we get

Theorem 2.4: A near – commutative near – idempotent semigroup is a semilattice of near – null semigroup.

Theorem 2.5: In a near – commutative near – idempotent semigroup
$\delta = \rho = \lambda = \xi$
$\xi \subset \rho \subset \delta \subset \xi$ and $\xi \subset \lambda \subset \delta \subset \xi$

Hence we get $\delta = \rho = \lambda = \xi$.

Theorem 2.6: A near – idempotent semigroup S is a near – commutative near – idempotent semigroup if and only if it is both a left regular and a right regular near – idempotent semigroup.

Proof: Suppose S is a near – commutative near – idempotent semigroup.

Then xyzw = xzyw for all x, y, z, w $\in$ S.

$$\begin{aligned} xyzyw &= x.(yz).y.w \\ &= x.y.yz.w \\ &= xy^2zw \\ &= xyzw \end{aligned}$$
so that S is a left regular near – idempotent semigroup.

Conversely,

Let S is both left regular near – idempotent semigroup and right regular near – idempotent semigroup
$\delta = \lambda$ in S by left regularity
$\delta = \rho$ in S by right regularity

Hence $\delta = \lambda = \rho = \lambda \cap \rho = \xi$.

$\delta = \xi$.

Hence S is a near – commutative near – idempotent semigroup.

Theorem 2.7: A near – idempotent semigroup S is a near – commutative near idempotent semigroup if and only if S is both a left – normal near – idempotent semigroup and a right normal – near idempotent semigroup.

Proof: Let S be a near – commutative near – idempotent semigroup.

Thus xuvy = xvuy for all x, u, v, y∈ S

Hence x uv wy = x vu wy for all x, u, v, w, y∈ S.

Thus xuvwy = xvuwy

Therefore S is a right normal near – idempotent semigroup.

xuvwy = xu. vwy
= xu wvy

So that S is a left normal near – idempotent semigroup.

Hence S is both a left normal near – idempotent semigroup and a right normal near – idempotent semigroup.

Conversely,

Suppose that S is both left normal near – idempotent and right normal near – idempotent semigroup.

Left normality in S implies left regularity and right normality in S implies right regularity.

Thus S is both a left regular near – idempotent semigroup and a right regular near – idempotent semigroup.

Hence S is a near commutative near – idempotent semigroup by the theorem 2.6.

3. λ, ρ, δ and ξ IN COMPARISON WITH GREEN'S RELTIONS

J.A.Green(1951), has defined relations $\mathscr{L}$, $\mathscr{R}$, $\mathfrak{D}$, H and J on a semigroup to study its structure. But the relation λ, ρ, δ and ξ are respectively different form $\mathscr{L}$, $\mathscr{R}$, $\mathfrak{D}$ and H.

The following example shows that λ is different from $\mathscr{L}$ of Green.

Example 3.1: Consider $1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $4 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, elements of $M_2(Z_2)$ under matrix multiplication modulo 2. These elements form a semigroup with the following multiplication table

X	1	2	3	4
1	1	1	1	1
2	1	2	3	4
3	1	1	1	1
4	1	2	3	4

It is easy to see that 1, 2 and 4 are idempotent elements of the semigroup but 3 is not.

For all x, y in S.
x. 3. y = x. 1 (since 3y = 1 for all y∈ S)
= 1 (since x.1= 1 for all x∈ S)

And
x. 3^2.y = x.1.y = 1.y = 1 for all x, y∈ S.

Thus, x.3.y = x.3^2.y for all x, y ∈ S, so that 3 is a near idempotent element of the semigroup S.

Here in this example,

x.3.1.y = x.1.y = 1.y = 1

x.3.y = x.1 = 1

therefore x.3.1.y = x.3.y

x.1.3.y = x.1.y so 1 λ 3.

S.1 = {1}

S.3 = {1, 3}

1 ∪ S.1 = {3} ∪ S.3 = {3} Therefore $S^1.1 \neq S^1.3$ so that 1 is not $\mathscr{L}$- related to 3.

4. CONCLUSION

In a near idempotent semigroup S, $\delta = \xi$ if and only if S is near – commutative. A near commutative near – idempotent semigroup is a semilattice of near – null semigroups. A near – idempotent semigroup S is near – commutative near idempotent if and only if S is both left – normal near – idempotent and right normal – near idempotent semigroup.

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