International Journal of Mathematical Archive-5(8), 2014, 204-210
Available online through www.ijma.info ISSN 2229 – 5046

COUPLED FIXED POINT THEOREM IN FUZZY METRIC SPACES

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(Received On: 19-04-14; Revised & Accepted On: 08-09-14)

ABSTRACT

In this paper, we extend the notion of weakly compatible maps in the context of coupled fixed points and use this notion to generalize the result of Xin-Qi Hu [6]. Suitable example has also been given in support of the main result.

Keywords: *Fuzzy metric space; Coupled coincidence point; Weakly compatible mappings.*

AMS 2000*: MSC: 47H10, 54H25.*

1. INTRODUCTION

It proved a turning point in the development of mathematics when the notion of fuzzy set was introduced by Zadeh [12]. This notion has laid the foundation of fuzzy mathematics. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modelling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication etc.

Kramosil and Michalek [8] introduced the notion of fuzzy metric space by generalizing the concept of probabilistic metric space to the fuzzy situation. George and Veeramani[3] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek[8]. There are many view points of the notion of metric space in fuzzy topology. Works noted in [3, 7, 8] are some examples.

Bhaskar and Lakshmikantham [1], Lakshmikantham and Ćirić [9] gave some coupled fixed point theorems. Coupled fixed point theorem under contraction conditions given by Sedghi *et al.* [10] are of great importance in the theory of fixed points in fuzzy metric spaces. The results proved by Fang [2] for compatible and weakly compatible mappings under ϕ-contractive conditions provided a tool to Xin-Qi Hu [6] for proving a result, which is genuine generalization of the result of Sedghi [10].

2. PRELIMINARIES

Before we give our main result we need the following definitions:

Definition 2.1[12]: A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition 2.2[11]: A binary operation * : [0,1] × [0,1]→ [0,1] is a continuous t-norm if ([0,1], *) is a topological abelian monoid with unit 1 s.t. a * b ≤ c * d whenever a ≤ c and b ≤ d , ∀ a, b, c, d ∈ [0,1]. Some examples are below:
(1) *(a, b) = ab,
(2) *(a, b) = min.(a, b).

Definition 2.3 [5]: Let $\sup_{0<t<1} \Delta(t, t) = 1$. A t-norm Δ is said to be of H-type if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at t = 1,

where
$$\Delta^1(t) = t,\ \Delta^{m+1}(t) = t\, \Delta\, (\Delta^m(t)),\ m = 1, 2, \ldots,\ t \in [0, 1].$$

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A t-norm Δ is a H-type t-norm iff for any $\lambda \in (0, 1)$, there exists $\delta(\lambda) \in (0, 1)$ such that $\Delta^m(t) > (1-\lambda)$ for all $m \in N$, when $t > (1-\delta)$.

The t-norm Δ_M = min. is an example of t-norm of H-type.

Definition 2.4 [3]: The 3-tuple (X, M, *) is called a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions:
(FM-1) $M(x, y, 0) > 0$,
(FM-2) $M(x, y, t) = 1$ iff $x=y$,
(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(FM-5) $M(x, y, .) : (0,\infty) \to [0,1]$ is continuous, for all $x, y, z \in X$ and $s, t > 0$.

In present paper , we consider M to be fuzzy metric space with condition:
(FM-6) $\lim_{t \to \infty} M(x, y, t) = 1$, $\forall$ $x, y \in X$ and $t > 0$.

Definition 2.5[3]: Let (X, M, *) be fuzzy metric space . A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$, if $\lim_{n \to \infty} M(x_n, x, t) = 1$, for all $t > 0$;
(i) Cauchy sequence if $\lim_{n \to \infty} M(x_{n+p}, x_n, t) = 1$, for all $t > 0$ and $p > 0$.

Definition 2.6 [3]: A fuzzy metric space (X, M, *) is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma 2.1[4]: M(x, y, .) is non-decreasing for all $x, y \in X$.

Lemma 2.2[4]): Let $x_n \to x$ and $y_n \to y$, then
(i) $\lim_{n \to \infty} M(x_n, y_n, t) \geq M(x, y, t)$, for all $t > 0$,
(ii) $\lim_{n \to \infty} M(x_n, y_n, t) = M(x, y, t)$, for all $t > 0$, if M(x, y, t) is continuous.

Definition 2.7[6]: Define $\boldsymbol{\Phi} = \{ \phi : R^+ \to R^+\}$, where $R^+ = [0, +\infty)$ and each $\phi \in \boldsymbol{\Phi}$ satisfies the following conditions:
(ϕ-1) ϕ is non-decreasing;
(ϕ-2) ϕ is upper semicontinuous from the right;
(ϕ-3) $\sum_{n=0}^{\infty} \phi^n(t) < +\infty$ for all $t > 0$, where $\phi^{n+1}(t) = \phi(\phi^n(t))$, $n \in N$.

Clearly, if $\phi \in \boldsymbol{\Phi}$, then $\phi(t) < t$ for all $t > 0$.

Definition 2.8 [9]: An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F: X \times X \to X$ if

$$F(x, y) = x, \quad F(y, x) = y.$$

Definition 2.9 [9]: An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings $F: X \times X \to X$ and $g: X \to X$ if

$$F(x, y) = g(x), \quad F(y, x) = g(y).$$

Definition 2.10 [2]: An element $(x, y) \in X \times X$ is called a common coupled fixed point of the mappings $F: X \times X \to X$ and $g: X \to X$ if

$$x = F(x, y) = g(x), \quad y = F(y, x) = g(y).$$

Definition 2.11 [2]: An element $x \in X$ is called a common fixed point of the mappings $F: X \times X \to X$ and $g: X \to X$ if

$$x = F(x, x) = g(x).$$

Definition 2.12 [2]: The mappings $F: X \times X \to X$ and $g: X \to X$ are called commutative if

$$gF(x, y) = F(gx, gy),$$

for all $x, y \in X$.

Definition 2.13 [2]: The mappings $F: X \times X \to X$ and $g: X \to X$ are said to be compatible if
$\lim_{n\to\infty} M(gF(x_n, y_n), F\big(g(x_n), g(y_n)\big), t) = 1$,
$\lim_{n\to\infty} M(gF(y_n, x_n), F\big(g(y_n), g(x_n)\big), t) = 1$, for all $t > 0$ whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X, such that $\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x$, $\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y$, for all $x, y \in X$.

Now we introduce the notion.

Definition 2.14: The maps $F: X \times X \to X$ and $g: X \to X$ are called weakly compatible maps if
$F(x, y) = g(x)$, $F(y, x) = g(y)$ implies $gF(x, y) = F(gx, gy)$, $gF(y, x) = F(gy, gx)$, for all $x, y \in X$.

We note that compatible maps are weakly compatible but converse need not be true.

3. MAIN RESULTS

For convenience, we denote

$$[M(x, y, t)]^n = \underbrace{M(x,y,t) * M(x,y,t) * \ldots * M(x,y,t)}_{n}, \tag{3.1}$$

for all n ∈ N. Xin-Qi Hu [6] proved the following result:

Theorem 3.1: Let (X, M, *) be a complete FM-space, where * is a continuous t-norm of H-type. Let $F: X \times X \to X$ and $g: X \to X$ be two mappings and there exists $\phi \in \Phi$ such that

$$M(F(x, y), F(u, v), \phi(t)) \geq M(gx, gu, t) * M(gy, gv, t), \text{for all } x, y, u, v \text{ in } X \text{ and } t > 0. \tag{3.2}$$

Suppose that $F(X \times X) \subseteq g(X)$ and g is continuous, F and g are compatible. Then, there exists $x \in X$ such that $x = g(x) = F(x, x)$; that is, F and g have a common fixed point in X.

We now give our main result which provides a generalization of Theorem 3.1 in the sense that we have relaxed the completeness of the space. The notion of continuity has also been relaxed and the concept of compatibility has been replaced by weak compatibility.

Theorem 3.2: Let (X, M, ∗) be a FM - Space, * being continuous t – norm of H-type. Let $F: X \times X \to X$ and $g: X \to X$ be two mappings and there exists $\phi \in \Phi$ satisfying condition (3.2).

Suppose that $F(X \times X) \subseteq g(X)$, F and g are weakly compatible, range space of one of the mappings f or g is complete. Then F and g have a coupled coincidence point.

Moreover, there exists a unique point x in X such that $x = F(x, x) = g(x)$.

Proof: Let x_0, y_0 be two arbitrary points in X. Since $F(X \times X) \subseteq g(X)$, we can choose x_1, y_1 in X such that $g(x_1) = F(x_0, y_0)$, $g(y_1) = F(y_0, x_0)$.

Continuing in this way we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$g(x_{n+1}) = F(x_n, y_n) \text{ and } g(y_{n+1}) = F(y_n, x_n), \text{ for al } n \geq 0.$$

Step-1: We first show that $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences.

Since * is a t-norm of H-type, for any $\epsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p} \geq (1-\epsilon), \text{ for all } p \in N. \tag{3.3}$$

Since $\lim_{t\to\infty} M(x,y,t) = 1$, for all x, y in X, there exists $t_0 > 0$ such that

$$M(gx_0, gx_1, t_0) \geq (1-\delta) \text{ and } M(gy_0, gy_1, t_0) \geq (1-\delta).$$

Also, since $\phi \in \Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in N$ such that

$$t > \sum_{k=n_0}^{\infty} \phi^k(t_0). \tag{3.4}$$

Using condition (3.2), we have

$$\begin{aligned} M(gx_1, gx_2, \phi(t_0)) &= M(F(x_0, y_0), F(x_1, y_1), \phi(t_0)) \\ &\geq M(gx_0, gx_1, t_0) * M(gy_0, gy_1, t_0), \end{aligned}$$

$$\begin{aligned} M(gy_1, gy_2, \phi(t_0)) &= MF(F(y_0, x_0), F(y_1, x_1), \phi(t_0)) \\ &\geq M(gy_0, gy_1, t_0) * M(gx_0, gx_1, t_0). \end{aligned}$$

Similarly, we can also get

$$\begin{aligned} M(gx_2, gx_3, \phi^2(t_0)) &= M(F(x_1, y_1), F(x_2, y_2), \phi^2(t_0)) \\ &\geq M(gx_1, gx_2, \phi(t_0)) * M(gy_1, gy_2, \phi(t_0)) \\ &\geq [M(gx_0, gx_1, t_0)]^2 * [M(gy_0, gy_1, t_0)]^2, \end{aligned}$$

$$\begin{aligned} M(gy_2, gy_3, \phi^2(t_0)) &= M(F(y_1, x_1), F(y_2, x_2), \phi^2(t_0)) \\ &\geq [M(gy_0, gy_1, t_0)]^2 * [M(gx_0, gx_1, t_0)]^2. \end{aligned}$$

Continuing in this way, we can get

$$M(gx_n, gx_{n+1}, \phi^n(t_0)) \geq [M(gx_0, gx_1, t_0)]^{2^{n-1}} * [M(gy_0, gy_1, t_0)]^{2^{n-1}},$$
$$M(gy_n, gy_{n+1}, \phi^n(t_0)) \geq [M(gy_0, gy_1, t_0)]^{2^{n-1}} * [M(gx_0, gx_1, t_0)]^{2^{n-1}}.$$

So, from (3.3) and (3.4), for $m > n \geq n_0$, we have

$$\begin{aligned} M(gx_n, gx_m, t) &\geq M(gx_n, gx_m, \textstyle\sum_{k=n_0}^{\infty} \phi^k(t_0)) \\ &\geq M(gx_n, gx_m, \textstyle\sum_{k=n}^{m-1} \phi^k(t_0)) \\ &\geq M(gx_n, gx_{n+1}, \phi^n(t_0)) * M(gx_{n+1}, gx_{n+2}, \phi^{n+1}(t_0)) * \ldots * M(gx_{m-1}, gx_m, \phi^{m-1}(t_0)) \\ &\geq \{[M(gx_0, gx_1, t_0)]^{2^{n-1}} * [M(gy_0, gy_1, t_0)]^{2^{n-1}}\} * \\ &\qquad * \{[M(gx_0, gx_1, t_0)]^{2^{n}} * [M(gy_0, gy_1, t_0)]^{2^{n}}\} * \\ &\qquad \ldots\ldots\ldots \\ &\qquad * \{[M(gx_0, gx_1, t_0)]^{2^{m-2}} * [M(gy_0, gy_1, t_0)]^{2^{m-2}}\} \\ &= [M(gx_0, gx_1, t_0)]^{2^{n-1}(2^{m-n}-1)} * [M(gy_0, gy_1, t_0)]^{2^{n-1}(2^{m-n}-1)} \\ &\geq \underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{2^n(2^{m-n}-1)} \geq (1-\epsilon), \end{aligned}$$

which implies that

$$M(gx_n, gx_m, t) \geq (1-\epsilon), \text{ for all } m, n \in N \text{ with } m > n \geq n_0 \text{ and } t > 0.$$

So $\{gx_n\}$ is a Cauchy sequence. Similarly, we can get that $\{gy_n\}$ is a Cauchy sequence.

Step-2: To show that F and g have a coupled coincidence point.

Without loss of generality, we assume that g(X) is complete, then there exists points x, y in g(X) so that $\lim_{n\to\infty} g(x_{n+1}) = x$, $\lim_{n\to\infty} g(y_{n+1}) = y$.

Again x, y ∈ g(X) implies the existence of p, q in X so that g(p) = x, g(q) = y and hence

$$\lim_{n\to\infty} g(x_{n+1}) = \lim_{n\to\infty} F(x_n, y_n) = g(p) = x,$$

$$\lim_{n\to\infty} g(y_{n+1}) = \lim_{n\to\infty} F(y_n, x_n) = g(q) = y.$$

From (3.2),

$$M(F(x_n, y_n), F(p, q), \phi(t)) \geq M(gx_n, g(p), t) * M(gy_n, g(q), t)$$

Taking limit as $n \to \infty$, we get

$$M(g(p), F(p, q), \phi(t)) = 1 \text{ that is, } F(p, q) = g(p) = x.$$

Similarly, f(q, p) = g(q) = y.

But f and g are weakly compatible, so that F(p, q) = g(p) = x and F(q, p) = g(q) = y implies gF(p, q) = F(g(p), g(q)) and gF(q, p) = f(g(q), g(p)) that is g(x) = F(x, y) and g(y) = f(y, x).

Hence F and g have a coupled coincidence point.

Step-3: To show that g(x) = y, g(y) = x.

Since * is a t-norm of H-type, for any $\epsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p} \geq (1-\epsilon), \text{ for all } p \in N.$$

Since $\lim_{t\to\infty} M(x, y, t) = 1$, for all x, y in X, there exists $t_0 > 0$ such that
$M(gx, y, t_0) \geq (1-\delta)$ and $M(gy, x, t_0) \geq (1-\delta)$.

Also, since $\phi \in \Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in N$ such that

$$t > \textstyle\sum_{k=n_0}^{\infty} \phi^k(t_0).$$

Using condition (3.2), we have

$$\begin{aligned} M(gx, gy_{n+1}, \phi(t_0)) &= M(F(x, y), F(y_n, x_n), \phi(t_0)) \\ &\geq M(gx, gy_n, t_0) * M(gy, gx_n, t_0), \end{aligned}$$

Letting $n \to \infty$, we get

$$M(gx, y, \phi(t_0)) \geq M(gx, y, t_0) * M(gy, x, t_0)$$

By this way, we can get for all $n \in N$,

$$M(gx, y, \phi^n(t_0)) \geq M(gx, y, \phi^{n-1}(t_0)) * M(gy, x, \phi^{n-1}(t_0))$$
$$\geq [M(gx, y, t_0)]^{2^{n-1}} * [M(gy, x, t_0)]^{2^{n-1}}$$

Thus, we have

$$M(gx, y, t) \geq M(gx, y, \textstyle\sum_{k=n_0}^{\infty} \phi^k(t_0))$$
$$\geq M(gx, y, \phi^{n_0}(t_0))$$
$$\geq [M(gx, y, t_0)]^{2^{n_0-1}} * [M(gy, x, t_0)]^{2^{n_0-1}}$$
$$\geq \underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{2^{n_0}} \geq (1-\epsilon)$$

So, for any $\epsilon > 0$, we have

$$M(gx, y, t) \geq (1-\epsilon), \text{ for all } t > 0.$$

This implies $g(x) = y$. Similarly, $g(y) = x$.

Step-4: Next we shall show that $x = y$.

Since * is a t-norm of H-type, for any $\epsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p} \geq (1-\epsilon), \text{ for all } p \in N.$$

Since $\lim_{t\to\infty} M(x, y, t) = 1$, for all x, y in X, there exists $t_0 > 0$ such that

$$M(x, y, t_0) \geq (1-\delta).$$

Also, since $\phi \in \Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in N$ such that

$$t > \textstyle\sum_{k=n_0}^{\infty} \phi^k(t_0).$$

Using condition (3.2), we have

$$M(gx_{n+1}, gy_{n+1}, \phi(t_0)) = M(F(x_n, y_n), F(y_n, x_n), \phi(t_0))$$
$$\geq M(gx_n, gy_n, t_0) * M(gy_n, gx_n, t_0)$$

Letting $n \to \infty$, we get

$$M(x, y, \phi(t_0)) \geq M(x, y, t_0) * M(y, x, t_0)$$

Thus, we have

$$M(x, y, t) \geq M(x, y, \textstyle\sum_{k=n_0}^{\infty} \phi^k(t_0))$$
$$\geq M(x, y, \phi^{n_0}(t_0))$$
$$\geq [M(x, y, t_0)]^{2^{n_0-1}} * [M(y, x, t_0)]^{2^{n_0-1}}$$
$$\geq \underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{2^{n_0}} \geq (1-\epsilon),$$

which implies that $x = y$.

Thus, we have proved that F and g have a common fixed point x in X.

Step-5: We now prove the uniqueness of x.

Let z be any point in X such that $z \neq x$ with $g(z) = z = F(z, z)$.

Since * is a t-norm of H-type, for any $\epsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{p} \geq (1-\epsilon), \text{ for all } p \in N.$$

Since $\lim_{t\to\infty} M(x, y, t) = 1$, for all x, y in X, there exists $t_0 > 0$ such that

$$M(x, z, t_0) \geq (1-\delta).$$

Also, since $\phi \in \Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any $t > 0$, there exists $n_0 \in N$ such that

$$t > \textstyle\sum_{k=n_0}^{\infty} \phi^k(t_0).$$

Using condition (3.2), we have

$$\begin{aligned} M(x, z, \phi(t_0)) &= M(F(x, x), F(z, z), \phi(t_0)) \\ &\geq M(g(x), g(z), t_0) * M(g(x), g(z), t_0) \\ &= M(x, z, t_0) * M(x, z, t_0) \\ &= [M(x, z, t_0)]^2 \text{ thus, we have} \end{aligned}$$

$$\begin{aligned} M(x, z, t) &\geq M(x, z, \textstyle\sum_{k=n_0}^{\infty} \phi^k(t_0)) \\ &\geq M(x, z, \phi^{n_0}(t_0)) \\ &\geq \left([M(x, z, t_0)]^{2^{n_0-1}}\right)^2 \\ &= \left(M(x, z, t_0)\right)^{2^{n_0}} \\ &\geq \underbrace{(1-\delta) * (1-\delta) * \ldots * (1-\delta)}_{2^{n_0}} \geq (1-\epsilon), \end{aligned}$$

which implies that x = y. Hence F and g have a unique common fixed point in X.

Next, we give an example in support of Theorem 3.2.

Example 3.1: Let X = [-4, 4), a * b = ab for all a, b ϵ [0, 1] and $\varphi(t) = \frac{t}{t+1}$. Then (X, M, *) is a Fuzzy Metric space, where

$$M(x, y, t) = [\varphi(t)]^{|x-y|}, \text{ for all x, y in X and } t > 0.$$

Let $\emptyset(t) = \frac{t}{2}$, g(x) = x and the mapping F: X × X → X be defined by $F(x, y) = \frac{x^2}{16} + \frac{y^2}{16} - 4$.

It is easy to check that F(X × X) = [-4, -2] ⊆ [-4, 4) = g(X). Further, F(X × X) is complete and the pair (F, g) is weakly compatible. We now check the condition (3.2),

$$\begin{aligned} M(F(x, y), F(u, v), \emptyset(t)) &= M(F(x, y), F(u, v), \tfrac{t}{2}) \\ &= \left[\varphi\left(\frac{t}{2}\right)\right]^{|F(x,y) - F(u,v)|} \\ &= \left[\frac{t}{t+2}\right]^{|x^2+y^2-u^2-v^2|/16} \\ &\geq \left[\frac{t}{t+2}\right]^{|x^2+y^2-u^2-v^2|/8} \\ &\geq \left[\frac{t}{t+1}\right]^{|x-u|+|y-v|} \\ &= \left[\frac{t}{t+1}\right]^{|x-u|} \left[\frac{t}{t+1}\right]^{|y-v|} \\ &= M(gx, gu, t) * M(gy, gv, t), \text{ for every } t > 0. \end{aligned}$$

Hence, all the conditions of Theorem 3.2, are satisfied. Thus F and g have a unique common coupled fixed point in X. Indeed, x = $4(1 - \sqrt{3})$ is a unique common coupled fixed point of F and g.

ACKNOWLEDGMENTS

This Research is supported by the University Grant Commission of India (Grant No. 39-29/2010(SR)).

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Source of support: University Grant Commission of India, Conflict of interest: None Declared