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LINDELOFNESS AND COMPACTNESS IN FUZZY RECOMBINATION SPACE

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ABSTRACT

The recombination space can be studied in fuzzy setting by assigning different possibilities to each offspring under the recombination operator. The recombination can be treated as a binary operator on the set of all chromosomes. A fuzzy pretopology is naturally generated in the recombination set. We have studied the covering properties of recombination space from fuzzy point of view.

Keywords: *Fuzzy pretopology, Fuzzy Closure, Fuzzy recombination set, 1-compactness, 1-Lindelofness.*

2010, Mathematical Subject Classification: *92D15, 92D20, 54A05, 54A40, 54D05, 54D30.*

1. INTRODUCTION

During nuclear division the DNA (as chromatin) in a Eukaryotic cell's nucleus is coiled in to very tight compact structures called chromosomes. When preparing for nuclear division, during the S phase of interphase the chromosomes copy themselves (i.e. DNA replication occurs) and the chromosomes consists of two identical sister chromatids joined together by a centromere. A chromosome consists of gene blocks of DNA. Each gene encodes a particular protein. Alleles are different versions of the same gene. Complete set of genetic material (all chromosomes) is called a genome. In diploid (2n) organisms, the genome is composed of homologous chromosomes. One chromosome of each homologous pair comes from the mother and the other comes from the father.

Natural selection acts on genetic variation that comes from two principal sources- mutation and recombination. In recombination DNA molecules interact with one another to bring about a rearrangement of the genetic information or content in an organism. In Eukaryotic system, this process is carried out by crossing over of homologous chromosomes during meiosis and this leads to offsprings having different combination of genes from those their parents. During meiosis the paired chromosomes can both break and the broken ends can be rejoined to the original parental chromosomes or can cross over to join the homologous parental chromosomes. It is sometimes possible that breakage occurs unequally in the two chromosomes. In such case parental chromosome changes in length, one becomes longer, while the other becomes shorter .This is known as unequal crossover and it is a source of mutation in the genome.

Mathematical recombination is based on the notion of the recombination function $R: X \times X \rightarrow P(X)$. The recombination set consist of all possible offspring that are obtained by recombining two parents chromosomes x and y using a given family of crossover operators.

This recombination set along with the crossover operators forms a weaker topological structure, known as neighbourhood space which satisfies basic axioms of it under the closure operator as discussed by Stadler and Stadler [6]. A new way of constructing recombination spaces is introduced and the topological features of the resulting hypergraphs are analyzed by Stadler and Wagner [7]. It is further shown that mutation and recombination spaces are homomorphic. In the classical model [8], the recombination spaces arising from four different unequal crossover models are studied in context of pretopological spaces and it is shown that all the four models are incompatible with the metric distance measures. Ali and Phukan [1] have discussed the fuzzy version of recombination space. Further they have shown that a fuzzy pretopology naturally arises in the recombination set of two crossover models viz. unrestricted and restricted unequal crossover models and both the models are incompatible with fuzzy metric measure. The concept of countable compactness, Lindelofness, almost compactness, near compactness, 2-connectedness in a L-fuzzy

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pretopological space are introduced and their properties are discussed by Badard [2]. Further he equipped fuzzy preneighborhoods to define the fuzzy pretopology. Mashour, Ramadan and Monsef discussed some new characteristics of fuzzy almost compact spaces and fuzzy almost compactness for arbitrary fuzzy sets in a fuzzy topological spaces [4].

In this paper we have discussed some covering properties of recombination space in fuzzy sense. In the second section we have recalled the basic idea of recombination. In the fourth section we have recalled the definition of various types of compactness on a fuzzy pretopological spaces. In the sixth section we have discussed the properties lindelofness and compactness of fuzzy recombination space in unrestricted unequal crossover model.

2. RECOMBINATION SETS

The abstract definition of recombination spaces is based on the notion of the recombination functions $R: X \times X \to P(X)$. Given a pair of parental chromosomes x and y the recombination set $R(x, y)$ consists of all chromosomes that can be obtained by recombining x and y using a given family of crossover operators. Consider the following properties:

(X1) $\{x, y\} \subseteq R(x, y)$,
(X2) $R(x, y) = R(y, x)$,
(X3) $R(x, x) = \{x\}$,
(X4) For all $z \in R(x, y)$ holds $|R(x, z)| \leq |R(x, y)|$

The first condition is only for notational convenience. The second condition is a simple symmetry requirement. The third says that no new types are created from a single type by recombination and the fourth condition defines the topological implications of recombination. It essentially says that recombinants are more "similar" to the parental types than the two parental types. This is the idea that recombinants are mixtures of the two parental types. Furthermore this condition says that the more "similar" two types are, the fewer types can be created by recombination. A generalized recombination structure satisfies (X1) and (X2). The proper recombination structures of homologous crossover satisfy also (X3) and (X4) [7]. It seems natural to interpret $R(x, y)$ as neighbourhoods of x for each $y \in X$ [8].

By (X1) we have $x \in R(x, y)$ for all x, y. Thus the recombination sets form a neighbourhood basis if and only if for all x, y, z there is a v such that

$$R(x, v) \subseteq R(x, y) \cap R(x, z) \tag{1}$$

In general this condition will not be satisfied. We may, however, consider the recombination sets as a sub-basis of the neighbourhood filters and construct the coarsest pretopology in which the recombination sets are neighbourhoods by adding the intersections of any finite number of recombination sets to the basis. In the case of finite genome sets X there is always a smallest neighbourhood $N(x)$, i.e., a minimal element of the neighbourhood basis. This is true in general if the neighbourhood filters have a finite basis, i.e., in Alexandroff spaces. Provided X is finite we can extract the vicinities directly from the *(sub) basis* of recombination sets:

$$N(x) = \cap_{x \in X} R(x, y) \tag{2}$$

If X is infinite, however, the vicinity $N(x)$ defined in the Equ. (2) need not be a neighbourhood of x in general. The intersection of a finite number of neighbourhoods of course is again a neighbourhood. Equation (2) however defines neighbourhoods if the size of the recombination sets $R(x, y)$ is bounded.

3. FUZZY SET

In this section we discuss some preliminaries on fuzzy sets. Fuzzy (sub)sets are generalization of classical (sub)sets. The boundary of a fuzzy subset is not precisely defined as an element of the universal set belongs to the fuzzy subset with some level of membership. Fuzzy set expresses the concept of graded membership. Mathematically a fuzzy set A of a universal set X is a function

$$\mu_A: X \to [0,1]$$

For each $x \in X$, $\mu_A(x)$is called the membership grade of x in A. For convenience the fuzzy set as well as the corresponding membership function is represented by A**.**

For a non empty set X, $I^X = \{A: X \to [0,1]\}$.

The elements of I^X are called fuzzy subsets of X. 0_X and 1_X are functions on X identically equal to 0 and 1 respectively. Given two fuzzy sets A and B, their standard intersection $A \cap B$, standard union $A \cup B$ and standard complement A^C are defined for all $x \in X$ by the equations

$$\mu_{A\cap B}(x) = min[\,\mu_A(x), \mu_B(x)\,]$$
$$\mu_{A\cup B}(x) = max[\,\mu_A(x), \mu_B(x)\,]$$
$$\mu_{A^c}(x) = 1 - \mu_A(x)$$

For infinite collection of fuzzy subsets, *min* and *max* are respectively replaced by *infimum* and *supremum*.

Also $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)\ \ \forall x \in X$

4. LINDELOFNESS AND COMPACTNESS IN A FUZZY PRETOPOLOGICAL SPACE

Definition: 4.1 [2] A fuzzy pretopology on a set X is described by an application Cl of I^X into I^X where I^X denotes the set of all fuzzy subsets on X which verifies:
P l : $Cl(0_X) = 0_X$,
P2 : $Cl(A) \supset A$ for every $A \in I^X$
(X, Cl) is then said to be a fuzzy pretopological space.

Sometimes we suppose that Cl verifies some additional properties, for instance:

Definition: 4.2 [2] Let (X, Cl) be a fuzzy pretopological space, and let us consider the following properties:
P 3: For every $A, B \in I^X$, such that $A \supset B$ we have $Cl(A) \supset Cl(B)$. (X, Cl) is then said to be of type I.
P 4: For every $A, B \in I^X$, we have $Cl(A \cup B) \supset Cl(B) \cup Cl(B)$. (X, Cl) is then said to be of type D.
P 5: For every $A \in I^X$, we have $Cl^2(A) = Cl(A)$. (X, Cl) is then said to be of type S.

A fuzzy pretopological space which is of type I, D, S is a fuzzy topological space and *Cl* is its Kuratowsky closure.

Definition: 4.3 [2] Let (*X*, *Cl*) be a fuzzy pretopological space. The interior is a function $i_{Cl}: I^X \to I^X$ defined as $i_{Cl}(A) = \left(Cl(A^C)\right)^C$ where $A \in I^X$.

Then the properties P1 to P5 becomes
P l: $i_{Cl}(0_X) = 0_X$
P2: $i_{Cl}(A) \subseteq A$ for every $A \in I^X$,
P3: For every $A, B \in I^X$, such that $A \subseteq B$ we have $i_{Cl}(A) \subseteq i_{Cl}(B)$.
P4: For every $A, B \in I^X$ we have $i_{Cl}(A \cap B) = i_{Cl}(A) \cap i_{Cl}(B)$.
P5: For every $A \in I^X$ we have ${i_{Cl}}^2(A) = i_{Cl}(A)$.

Definition: 4.4 [2] A family of fuzzy preneighbourhoods at the point $x \in X$ is a family **B**(*x*) of fuzzy subsets μ_V which verify $\mu_V(x) = 1$.

Definition: 4.5 [2] Let φ be an application of I^X in to $[0,1]$, φ is said to be a degree of non-vacuity if it verifies:
(i) $\varphi(0_X) = 0$
(ii) $\varphi(A) = 1$ if there exists *x* such that $\mu_A(x) = 1$
(iii) $A \supset B$ implies $\varphi(A) \supset \varphi(B)$.

In particular $\varphi(A) = sup_{x\epsilon X}\, \mu_A(x)$ is the degree of non-vacuity.

If we define a fuzzy subset $\bar{\mu}$ of *X* such that
$$\bar{\mu}(x) = inf_{V\epsilon \mathbf{B}(x)} \varphi(V \cap \mu)$$
then we get a *fuzzy closure* operator on *X*.

This gives an connection between preneighbourhoods and closure operator.

Definition: 4.6[2] Let (E, Cl) be a type-I fuzzy pretopological space, then *E* is said to be 1-compact if and only if for every family $A_{i/i\epsilon I}$ of fuzzy subsets of *E* which verifies $\cap_{i\epsilon I_0} A_i \neq 0_X$ (for every finite subset I_0 of *I*), we can assert that $\cap_{i\in I} Cl(A_i) \neq 0_X$.

Definition: 4.7[4]Let (E, Cl) be a type-I fuzzy pretopological space, then *E* is said to be 1-Lindelof if and only if for every family $A_{i/i\epsilon I}$ of fuzzy subsets of *E* which verifies $\cap_{i\epsilon I_0} A_i \neq 0_X$ where I_0 is a countable subset of *I*, we can assert that $\cap_{i\in I} Cl(A_i) \neq 0_X$.

Definition: 4.8[4] Let (E, Cl) be a type-I fuzzy pretopological space, then *E* is said to be countable1-compact if and only if for every family $A_{i/i\epsilon I_0}$, of fuzzy subsets of *E* which verifies $\cap_{i\epsilon I_0} A_i \neq 0_X$ where I_0 is a finite subset of *I*, we can assert that $\cap_{i\in I_0} i_{Cl}\{Cl(A_i)\} \neq 0_X$

Definition: 4.9[4] Let (E, Cl) be a type-I fuzzy pretopological space, then E is said to be almost 1-compact if and only if for every family $A_{i/i\epsilon I}$ of fuzzy subsets of E which verifies $\cap_{i\in I_0} i_{Cl}(A_i) \neq 0_X$ where I_0 is a finite subset of I, we can assert that $\cap_{i\in I} Cl(A_i) \neq 0_X$.

Definition: 4.10 [4] Let (E, Cl) be a type-I fuzzy pretopological space, then E is said to be nearly 1-compact if and only if for every family $A_{i/i\epsilon I}$ of fuzzy subsets of E which verifies $\cap_{i\epsilon I_0} i_{Cl}(A_i) \neq 0_X$ where I_0 is a finite subset of I, we can assert that $\cap_{i\in I} i_{Cl}\{Cl(A_i)\} \neq 0_X$.

In a Fuzzy Pretopological space of type I, 1- compact (respectively 2-compact)$\rightarrow$ nearly 1-compact (respectively nearly 2- compact)$\rightarrow$ almost 1- compact (respectively almost 2- compact)

5. FUZZY PRETOPOLOGY IN A RECOMBINATION SPACE

The recombinants obtained from recombination event has different possibilities. If assigned some values of possibilities to each recombinant then the recombination set can be looked upon as a fuzzy set [1]. The fuzzy recombination set obtained from two chromosomes x and y can be denoted by μ_{xy}. The set μ_{xy} consists of all possible recombinants between chromosomes with x and y numbers of gene copies with different grade of membership. If X denotes the set of chromosomes then the recombinant set corresponding to a pair (x, y) is a membership function $\mu_{xy}: X \rightarrow [0,1]$. Here, the fuzzy subset μ_{xy} is considered as a fuzzy preneighbourhood of x as well as y for each x , $y \in X$ by considering
$\mu_{xy}(x) = 1$ and $\mu_{xy}(y) = 1$ respectively.

The concept of fuzzy pretopology on the set of chromosomes is obtained from the closure operator defined by

$$Cl(\mu)(x) = inf_{V\epsilon \mathbf{B}(x)}\varphi(V \cap \mu) \quad (3)$$

where μ is an arbitrary fuzzy subset on X and **B**(x) is the set of all preneighbourhood of x.

The fuzzy recombination set along with the fuzzy closure defined in (3) satisfies the basic axioms of the type-I fuzzy pretopology, so we have a fuzzy pretopology on the set of all chromosomes. In our text we will refer this fuzzy pretopological space as fuzzy recombination space.

1.1. Unrestricted Unequal Crossover

In the unrestricted unequal crossover model [8] an extreme form of unequal cross over is assumed that is a crossover may happen with equal probability at all possible intergenic regions as well as at both ends of the gene cluster. Each possible crossover event produces two recombinant chromosomes. In most of the cases the recombination events will yield chromosomes with different number of gene copies than the original ones. By abuse of notation x is a symbol for a chromosome with certain number of gene copies as well as for the number of gene copies on the chromosome. The recombination set $R(x, y)$ consists of all possible recombinants between chromosomes with x and y copies of gene i.e.,

$$R(x, y) = \{0,1, \dots, x + y\},$$

where $N(x) = R(x, 0) = \{0,1, \dots, x\}$ represents the smallest neighbourhood.

It is observed that $N(y) \subseteq N(x)$ if and only if $y \leq x$. Hence in this form of unequal crossover the neighbourhood share at least $\{0\}$ if not a much larger set.

In the fuzzy pretopological model [1] it is considered that each element of $R(x, y)$ has different possibilities of occurrence. If x is a chromosome then the recombination set $R(x, 0)$being the smallest neighbourhood is contained in the support of all fuzzy recombination subsets of the form μ_{xy} on X.

6. LINDELOFNESS AND COMPACTNESS IN UNEQUAL CROSSOVER

In this section we have discussed only the 1-Lindelofness and 1-compactness of fuzzy recombination space for the case of unequal crossover discussed above.

Let $A_{i/i\epsilon I}$ be a family of fuzzy subsets on the set of chromosomes X such that $\cap_{i\epsilon I_0} A_i \neq 0_X$ for a countable subset I_0 of I

Now

$$\cap_{i\epsilon I_0} A_i \neq 0_X$$

$\Rightarrow inf_{i\epsilon I_0} A_i(x_0) \neq 0$ for some $x_0 \epsilon X$.

Now for such $x_0, A_i(x_0) \neq 0\ \forall\ i \epsilon I_0$. This implies x_0 belongs to support of all A_i. Since the A_i's are fuzzy recombination sets, their support is nonempty as it contains atleast the smallest neighbourhood.

As discussed above the neighbourhoods share at least $\{0\}$ if not a much larger set, we can assure that there exist at least one such x_0.

If we choose $x_0 = 0$ then x_0 belongs to support of all A_i and
$Cl(A_i)(x_0) > A_i(x_0) > 0\ \ \forall\ i \in I$
$Cl(A_i)(x_0) \neq 0\ \forall\ i \in I$
And so $inf_{i \in I} Cl(A_i) \neq 0_X$

Therefore $\cap_{i \epsilon I}\ Cl(A_i) \neq 0_X$.

This shows that the space is 1-Lindelof. The same is true is if I_0 is a finite subset of I.

Hence the space is also 1-compact. Consequently it is almost 1- compact as well as nearly 1- compact.

7. CONCLUSION

There are various models in unequal crossover. In this paper we have discussed the properties 1- Lindelofness and 1-compactness of the fuzzy recombination space in unrestricted unequal crossover model. It is observed that the space is 1-Lindelof and 1- Compact in this model. Some more properties like nearly 1- compactness, almost 1-compactness also follows in the same model from the observation.

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