

OPERATIONAL CALCULUS ON TWO DIMENSIONAL FRACTIONAL MELLIN TRANSFORM

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ABSTRACT

Fractional integral transform is a very powerful mathematical tool which in engineering area of research. Mellin transform offers us many applications navigation, correlators, in area of statistics, probability and also in solving differential equations.

The aim of this paper is to a generalization of two dimensional fractional Mellin transform (2DFRMT) and also discuss some important results for two dimensional fractional Mellin transform.

I. INTRODUCTION

A classical theory of the Mellin transform is extended to a generalized function space which is a dual of a testing function space developed by A.H. Zamanian [1]. The Mellin transform is an integral transform named after the finnish mathematician Hjalmar Mellin (1854-1933). Fractional Mellin transform is one of the flourishing field of active research due to its wide range applications [2].

Fourier transform and Mellin transform provide us alternative ways to analyze the spectra of different signals. Mellin transform, a kind of nonlinear transformation, is widely used for its scale invariance property. Perhaps the most famous application is the computation of the solution to a potential problem in a wedge-shaped region, where the unknown function is supposed to satisfy Laplace's equation with given boundary conditions on the edges. In the visual navigation, algorithms are usually known to be computationally heavy and time-consuming [3] [4][5], while the time-to-impact computation of the imaged object is straightforward by using the Mellin transform based correlators. Fractional Mellin transform becomes used in visual navigation since it can control the range of rotation and scaling [6]. Mellin integral transforms helps us to derive different properties in statistics and probability densities of single continuous random variable [7]. Mellin transform is a natural analytical tool to study the distribution of products and quotients of independent random variables. So by using Mellin transform agricultural land classification is possible [8].

The two dimensional fractional Mellin transform with parameter θ of $f(x, y)$ denoted by $FRMT\{f(x, y)\}$ performs a linear operation, given by the integral transform $FRMT\{f(x, y)\} = F_\theta(u, v) = \int_0^\infty \int_0^\infty f(x, y) K_\theta(x, y, u, v) dx dy$ (1)

where the kernel

$$K_\theta(x, y, u, v) = x^{\frac{2\pi i u}{\sin \theta}-1} y^{\frac{2\pi i v}{\sin \theta}-1} e^{\frac{\pi i}{\tan \theta}[u^2+v^2+\log x^2+\log y^2]} \quad 0 < \theta \leq \frac{\pi}{2}. \quad (2)$$

In this paper we have defined Distributional two dimensional fractional Mellin transform having compact support in section II. Also discussed some important results for two dimensional fractional Mellin transform in section III.

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II. DEFINITIONS

2.1. Testing function space E-

An infinitely differentiable complex valued function $\phi(x, y)$ on R^n belongs to $E(R^n)$, if for each compact set $I \subset S_a, K \subset S_b$ where

$$S_a = \{x : x \in R, |x| \leq a, a > 0\} \quad S_b = \{y : y \in R, |y| \leq b, b > 0\}, \quad I, K \in R^n$$

$$\gamma_{E,q,\lambda}[\phi(x, y)] = \sup_{\substack{x \in I \\ y \in K}} |D_{x,y}^{q,\lambda} \phi(x, y)| < \infty$$

Thus $E(R^n)$ will denote the space of all $\phi(x, y) \in E(R^n)$ with compact support contained in S_a & S_b .

Note that the space E is complete and therefore a Frechet space. Moreover, we say that $f(x, y)$ is a fractional Mellin transformable if it is a member of E^* , the dual of E .

2.2. Two dimensional fractional Mellin transform (FRMT)

The two dimensional fractional Mellin transform of $f(x, y) \in E^*(R^n)$ can be defined by

$$FRMT\{f(x, y)\} = F_\theta(u, v) = \langle f(x, y), K_\theta(x, y, u, v) \rangle \quad K_\theta(x, y, u, v) = x^{\frac{2\pi i u}{\sin \theta} - 1} y^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} [u^2 + v^2 + \log x^2 + \log y^2]}$$

Right hand side of equation has a meaning as the application of $f(x, y) \in E^*$ to $K_\theta(x, y, u, v) \in E$. It can be extended to the complex space as an entire function given by $FRMT\{f(x, y)\} = F_\theta(p, k) = \langle f(x, y), K_\theta(x, y, p, k) \rangle$

The right hand side is meaningful because for each $p, k \in C^n$, $K_\theta(x, y, p, k) \in E$, as a function of x, y .

III. Some Important Results

2.1. Prove that:

$$F_\theta\{xy f(x, y)\}(u, v) = e^{-\cos \theta [u+v-\frac{i}{2\pi} \sin \theta]} F_\theta\{f(x, y)\}\left(u + \frac{\sin \theta}{2\pi i}, v + \frac{\sin \theta}{2\pi i}\right)$$

Proof: We know that

$$\begin{aligned} F_\theta\{xy f(x, y)\}(u, v) &= \int_0^\infty \int_0^\infty f(x, y) xy x^{\frac{2\pi i u}{\sin \theta} - 1} y^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} (u^2 + v^2 + \log^2 x + \log^2 y)} dx dy \\ &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i u}{\sin \theta} + 1 - 1} y^{\frac{2\pi i v}{\sin \theta} + 1 - 1} e^{\frac{\pi i}{\tan \theta} (u^2 + v^2 + \log^2 x + \log^2 y)} dx dy \\ &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} \left[u + \frac{\sin \theta}{2\pi i}\right] - 1} y^{\frac{2\pi i}{\sin \theta} \left[v + \frac{\sin \theta}{2\pi i}\right] - 1} e^{\frac{\pi i}{\tan \theta} (u^2 + v^2 + \log^2 x + \log^2 y)} dx dy \\ &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} \left[u + \frac{\sin \theta}{2\pi i}\right] - 1} y^{\frac{2\pi i}{\sin \theta} \left[v + \frac{\sin \theta}{2\pi i}\right] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left(u + \frac{\sin \theta}{2\pi i}\right)^2 + \left(v + \frac{\sin \theta}{2\pi i}\right)^2 + \log^2 x + \log^2 y \right\}} \\ &\quad e^{\frac{\pi i}{\tan \theta} \left\{ \left(\frac{-u \sin \theta}{\pi i} + \frac{\sin^2 \theta}{4\pi^2}\right) + \left(\frac{-v \sin \theta}{\pi i} + \frac{\sin^2 \theta}{4\pi^2}\right) \right\}} dx dy \\ &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} \left[u + \frac{\sin \theta}{2\pi i}\right] - 1} y^{\frac{2\pi i}{\sin \theta} \left[v + \frac{\sin \theta}{2\pi i}\right] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left(u + \frac{\sin \theta}{2\pi i}\right)^2 + \left(v + \frac{\sin \theta}{2\pi i}\right)^2 + \log^2 x + \log^2 y \right\}} \\ &\quad e^{\left\{ \left(-u \cos \theta + \frac{i}{4\pi} \cos \theta \sin \theta\right) + \left(-v \cos \theta + \frac{i}{4\pi} \cos \theta \sin \theta\right) \right\}} dx dy \\ &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} \left[u + \frac{\sin \theta}{2\pi i}\right] - 1} y^{\frac{2\pi i}{\sin \theta} \left[v + \frac{\sin \theta}{2\pi i}\right] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left(u + \frac{\sin \theta}{2\pi i}\right)^2 + \left(v + \frac{\sin \theta}{2\pi i}\right)^2 + \log^2 x + \log^2 y \right\}} \\ &\quad e^{\cos \theta \left\{ -u + \frac{i}{4\pi} \sin \theta - v + \frac{i}{4\pi} \sin \theta \right\}} dx dy \end{aligned}$$

$$\begin{aligned}
 &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{\sin \theta}{2\pi i}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{\sin \theta}{2\pi i}] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left(u + \frac{\sin \theta}{2\pi i}\right)^2 + \left(v + \frac{\sin \theta}{2\pi i}\right)^2 + \log^2 x + \log^2 y \right\}} \\
 &\quad e^{\cos \theta \left\{ -(u+v) + \frac{i}{2\pi} \sin \theta \right\}} dx dy \\
 &= e^{\cos \theta \left\{ -(u+v) + \frac{i}{2\pi} \sin \theta \right\}} \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{\sin \theta}{2\pi i}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{\sin \theta}{2\pi i}] - 1} \\
 &\quad e^{\frac{\pi i}{\tan \theta} \left\{ \left(u + \frac{\sin \theta}{2\pi i}\right)^2 + \left(v + \frac{\sin \theta}{2\pi i}\right)^2 + \log^2 x + \log^2 y \right\}} dx dy \\
 &= e^{-\cos \theta \left\{ (u+v) - \frac{i}{2\pi} \sin \theta \right\}} F_\theta \{f(x, y)\} \left(u + \frac{\sin \theta}{2\pi i}, v + \frac{\sin \theta}{2\pi i}\right)
 \end{aligned}$$

3.2. Prove that:

$$F_\theta \{x^{ia} y^{ib} f(x, y)\}(u, v) = e^{-icos \theta [ua + vb + \frac{\sin \theta}{4\pi} (a^2 + b^2)]} F_\theta \{f(x, y)\} \left(u + \frac{a \sin \theta}{2\pi}, v + \frac{b \sin \theta}{2\pi}\right)$$

Proof: We know that

$$\begin{aligned}
 F_\theta \{x^{ia} y^{ib} f(x, y)\}(u, v) &= \int_0^\infty \int_0^\infty f(x, y) x^{ia} y^{ib} x^{\frac{2\pi i u}{\sin \theta} - 1} y^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} (u^2 + v^2 + \log^2 x + \log^2 y)} dx dy \\
 &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{a \sin \theta}{2\pi}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{b \sin \theta}{2\pi}] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left[u + \frac{a \sin \theta}{2\pi}\right]^2 + \left[v + \frac{a \sin \theta}{2\pi}\right]^2 + \log^2 x + \log^2 y \right\}} \\
 &\quad e^{\frac{\pi i}{\tan \theta} \left(\frac{-ausin \theta}{\pi} - \frac{a^2 \sin^2 \theta}{4\pi^2} - \frac{-bvsin \theta}{\pi} - \frac{b^2 \sin^2 \theta}{4\pi^2} \right)} dx dy \\
 &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{a \sin \theta}{2\pi}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{b \sin \theta}{2\pi}] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left[u + \frac{a \sin \theta}{2\pi}\right]^2 + \left[v + \frac{a \sin \theta}{2\pi}\right]^2 + \log^2 x + \log^2 y \right\}} \\
 &\quad e^{\frac{-\pi i}{\tan \theta} \left[\frac{\sin \theta}{\pi} (au + bv) + \frac{\sin^2 \theta}{4\pi^2} (a^2 + b^2) \right]} dx dy \\
 &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{a \sin \theta}{2\pi}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{b \sin \theta}{2\pi}] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left[u + \frac{a \sin \theta}{2\pi}\right]^2 + \left[v + \frac{a \sin \theta}{2\pi}\right]^2 + \log^2 x + \log^2 y \right\}} \\
 &\quad e^{-i \left[\cos \theta (au + bv) + \frac{\sin \theta \cos \theta}{4\pi} (a^2 + b^2) \right]} dx dy \\
 &= e^{-icos \theta [(au + bv) + \frac{\sin \theta}{4\pi} (a^2 + b^2)]} \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{a \sin \theta}{2\pi}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{b \sin \theta}{2\pi}] - 1} \\
 &\quad e^{\frac{\pi i}{\tan \theta} \left\{ \left[u + \frac{a \sin \theta}{2\pi}\right]^2 + \left[v + \frac{a \sin \theta}{2\pi}\right]^2 + \log^2 x + \log^2 y \right\}} \\
 &= e^{-icos \theta [(au + bv) + \frac{\sin \theta}{4\pi} (a^2 + b^2)]} F_\theta \{f(x, y)\} \left(u + \frac{a \sin \theta}{2\pi}, v + \frac{b \sin \theta}{2\pi}\right)
 \end{aligned}$$

3.3. Prove that:

$$F_\theta \{x^a y^b f(x, y)\}(u, v) = e^{-\cos \theta \left[a \left(u + \frac{a \sin \theta}{4\pi i} \right) + b \left(v + \frac{b \sin \theta}{4\pi i} \right) \right]} F_\theta \{f(x, y)\} \left(u + \frac{a \sin \theta}{2\pi i}, v + \frac{b \sin \theta}{2\pi i}\right)$$

Proof: We know that

$$\begin{aligned}
 F_\theta \{x^a y^b f(x, y)\}(u, v) &= \int_0^\infty \int_0^\infty f(x, y) x^a y^b x^{\frac{2\pi i u}{\sin \theta} + a - 1} y^{\frac{2\pi i v}{\sin \theta} + b - 1} e^{\frac{\pi i}{\tan \theta} (u^2 + v^2 + \log^2 x + \log^2 y)} dx dy \\
 &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{a \sin \theta}{2\pi}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{b \sin \theta}{2\pi}] - 1} e^{\frac{\pi i}{\tan \theta} (u^2 + v^2 + \log^2 x + \log^2 y)} dx dy \\
 &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{a \sin \theta}{2\pi}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{b \sin \theta}{2\pi}] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left[u + \frac{a \sin \theta}{2\pi}\right]^2 + \left[v + \frac{b \sin \theta}{2\pi}\right]^2 + \log^2 x + \log^2 y \right\}} \\
 &\quad e^{\frac{\pi i}{\tan \theta} \left(\frac{-au \sin \theta}{\pi} + \frac{a^2 \sin^2 \theta}{4\pi^2} - \frac{-bv \sin \theta}{\pi} + \frac{b^2 \sin^2 \theta}{4\pi^2} \right)} dx dy \\
 &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{a \sin \theta}{2\pi}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{b \sin \theta}{2\pi}] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left[u + \frac{a \sin \theta}{2\pi}\right]^2 + \left[v + \frac{b \sin \theta}{2\pi}\right]^2 + \log^2 x + \log^2 y \right\}} \\
 &\quad e^{\left\{ -au \sin \theta + \frac{ia^2 \cos \theta \sin \theta}{4\pi} - bv \sin \theta + \frac{ib^2 \cos \theta \sin \theta}{4\pi} \right\}} dx dy
 \end{aligned}$$

$$\begin{aligned}
 &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{a \sin \theta}{2\pi i}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{b \sin \theta}{2\pi i}] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left[u + \frac{a \sin \theta}{2\pi i} \right]^2 + \left[v + \frac{b \sin \theta}{2\pi i} \right]^2 + \log^2 x + \log^2 y \right\}} \\
 &\quad e^{\left\{ -(au + bv) \cos \theta - \frac{\cos \theta \sin \theta}{4\pi i} (a^2 + b^2) \right\}} dx dy \\
 &= \int_0^\infty \int_0^\infty f(x, y) x^{\frac{2\pi i}{\sin \theta} [u + \frac{a \sin \theta}{2\pi i}] - 1} y^{\frac{2\pi i}{\sin \theta} [v + \frac{b \sin \theta}{2\pi i}] - 1} e^{\frac{\pi i}{\tan \theta} \left\{ \left[u + \frac{a \sin \theta}{2\pi i} \right]^2 + \left[v + \frac{b \sin \theta}{2\pi i} \right]^2 + \log^2 x + \log^2 y \right\}} \\
 &\quad e^{-\cos \theta \left\{ (au + bv) + \frac{\sin \theta}{4\pi i} (a^2 + b^2) \right\}} dx dy \\
 &= e^{-\cos \theta \left\{ (au + bv) + \frac{\sin \theta}{4\pi i} (a^2 + b^2) \right\}} F_\theta \{f(x, y)\} \left(u + \frac{a \sin \theta}{2\pi i}, v + \frac{b \sin \theta}{2\pi i} \right)
 \end{aligned}$$

3.4. Prove that:

$$\begin{aligned}
 F_\theta \{f(ax, by)\}(u, v) &= e^{\frac{\pi i}{\tan \theta} (\log^2 a + \log^2 b) \sin^2 \theta} e^{\frac{2\pi i}{\tan \theta} (u \log a + v \log b) \cos \theta} a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} \\
 F_\theta \{f(x, y)\}(u - \log a \cos \theta, v - \log b \cos \theta)
 \end{aligned}$$

Proof: We know that

$$F_\theta \{f(ax, by)\}(u, v) = \int_0^\infty \int_0^\infty f(ax, by) x^{\frac{2\pi i u}{\sin \theta} - 1} y^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} (u^2 + v^2 + \log^2 x + \log^2 y)} dx dy$$

Putting $ax = p, by = q$

$$\begin{aligned}
 x &= \frac{p}{a}, \quad y = \frac{q}{b} \\
 dx &= \frac{dp}{a}, \quad dy = \frac{dq}{b}
 \end{aligned}$$

$$\begin{aligned}
 F_\theta \{f(ax, by)\}(u, v) &= \int_0^\infty \int_0^\infty f(p, q) \left(\frac{p}{a} \right)^{\frac{2\pi i u}{\sin \theta} - 1} \left(\frac{q}{b} \right)^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} (u^2 + v^2 + \log^2 \left(\frac{p}{a} \right) + \log^2 \left(\frac{q}{b} \right))} \frac{dp}{a} \frac{dq}{b} \\
 &= \int_0^\infty \int_0^\infty f(p, q) a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} p^{\frac{2\pi i u}{\sin \theta} - 1} q^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} [u^2 + v^2 + (\log p - \log a)^2 + (\log q - \log b)^2]} dp dq \\
 &= a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} \int_0^\infty \int_0^\infty f(p, q) p^{\frac{2\pi i u}{\sin \theta} - 1} q^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} [u^2 + v^2]} e^{\frac{\pi i}{\tan \theta} [\log^2 p + \log^2 a + \log^2 q + \log^2 b]} \\
 &\quad e^{\frac{\pi i}{\tan \theta} [-2\log p \log a - 2\log q \log b]} dp dq \\
 &= a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} \int_0^\infty \int_0^\infty f(p, q) p^{\frac{2\pi i u}{\sin \theta} - 1} q^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} [u^2 + v^2]} e^{\frac{\pi i}{\tan \theta} [\log^2 p + \log^2 a + \log^2 q + \log^2 b]} \\
 &\quad e^{\log p \left(\frac{-2\pi i}{\tan \theta} \log a \right)} e^{\log q \left(\frac{-2\pi i}{\tan \theta} \log b \right)} dp dq \\
 &= a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} \int_0^\infty \int_0^\infty f(p, q) p^{\frac{2\pi i u}{\sin \theta} - 1} q^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} [u^2 + v^2]} e^{\frac{\pi i}{\tan \theta} [\log^2 p + \log^2 a + \log^2 q + \log^2 b]} \\
 &\quad p^{\left(\frac{-2\pi i}{\tan \theta} \log a \right)} q^{\left(\frac{-2\pi i}{\tan \theta} \log b \right)} dp dq \\
 &= a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} \int_0^\infty \int_0^\infty f(p, q) p^{\frac{2\pi i u}{\sin \theta} - \frac{2\pi i}{\tan \theta} \log a - 1} q^{\frac{2\pi i v}{\sin \theta} - \frac{2\pi i}{\tan \theta} \log b - 1} e^{\frac{\pi i}{\tan \theta} [u^2 + v^2]} \\
 &\quad e^{\frac{\pi i}{\tan \theta} [\log^2 p + \log^2 a + \log^2 q + \log^2 b]} dp dq \\
 &= a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} \int_0^\infty \int_0^\infty f(p, q) p^{\frac{2\pi i u}{\sin \theta} - 1} q^{\frac{2\pi i v}{\sin \theta} - 1} e^{\frac{\pi i}{\tan \theta} [u - \cos \theta \log a]} e^{\frac{\pi i}{\tan \theta} [v - \cos \theta \log b]} \\
 &\quad e^{\frac{\pi i}{\tan \theta} [(u - \cos \theta \log a)^2 + (v - \cos \theta \log b)^2]} e^{\frac{\pi i}{\tan \theta} [\log^2 p + \log^2 a + \log^2 q + \log^2 b]} \\
 &\quad e^{\frac{\pi i}{\tan \theta} [2\cos \theta (\log a + \log b)]} e^{\frac{-\pi i \cos^2 \theta}{\tan \theta} [\log^2 a + \log^2 b]} dp dq \\
 &= a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} \int_0^\infty \int_0^\infty f(p, q) p^{\frac{2\pi i}{\sin \theta} [u - \cos \theta \log a] - 1} q^{\frac{2\pi i}{\sin \theta} [v - \cos \theta \log b] - 1} \\
 &\quad e^{\frac{\pi i}{\tan \theta} [(u - \cos \theta \log a)^2 + (v - \cos \theta \log b)^2]} e^{\frac{\pi i}{\tan \theta} [\log^2 p + \log^2 a + \log^2 q + \log^2 b]} \\
 &\quad e^{\frac{\pi i}{\tan \theta} [2\cos \theta (\log a + \log b)]} e^{\frac{-\pi i \cos^2 \theta}{\tan \theta} [\log^2 a + \log^2 b]} dp dq
 \end{aligned}$$

$$\begin{aligned}
 &= a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} e^{\frac{\pi i (1-\cos^2 \theta)}{\tan \theta} [\log^2 a + \log^2 b]} e^{\frac{2\pi i}{\tan \theta} (u \log a + v \log b) \cos \theta} \\
 &\quad \int_0^\infty \int_0^\infty f(p, q) p^{\frac{2\pi i}{\sin \theta} [u - \cos \theta \log a] - 1} q^{\frac{2\pi i}{\sin \theta} [v - \cos \theta \log b] - 1} \\
 &\quad e^{\frac{\pi i}{\tan \theta} [(u - \cos \theta \log a)^2 + (v - \cos \theta \log b)^2 + \log^2 p + \log^2 q]} dp dq \\
 &= a^{\frac{-2\pi i u}{\sin \theta}} b^{\frac{-2\pi i v}{\sin \theta}} e^{\frac{\pi i (\sin^2 \theta)}{\tan \theta} [\log^2 a + \log^2 b]} e^{\frac{2\pi i}{\tan \theta} (u \log a + v \log b) \cos \theta} \\
 &\quad F_\theta \{f(x, y)\} (u - \log a \cos \theta, v - \log b \cos \theta)
 \end{aligned}$$

IV. CONCLUSION

In this paper we have defined Distributional two dimensional fractional Mellin transform with compact support. Some results for two dimensional fractional Mellin transform is proved.

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