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FUZZY LINEAR PROGRAMMING PROBLEM WITH FUZZY HOMOGENEOUS CONSTRAINTS

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ABSTRACT

This paper proposes an algorithm for solving a fuzzy linear programming problem (FLPP) when some of its constraints are fuzzy homogeneous. Using these fuzzy homogeneous constraints a fuzzy transformation matrix $\tilde{T}$ is constructed. The $\tilde{T}$ transforms the given problem into another FLPP but with fewer fuzzy constrains. A relationship between these two problems, which ensures that the solution of the original problem can be recovered from the solution of the transformed problem, is explained. A simple numerical example illustrates the steps of the proposed algorithm.

Keywords: *Fuzzy numbers, Trapezoidal fuzzy numbers, Fuzzy linear programming problem, Fuzzy Homogeneous constraints, Fuzzy Transformation matrix.*

2000 AMS Subject Classification: *3E72, 65G30, 94D05.*

1. INTRODUCTION

Fuzzy linear programming is a special type of problem in which all relations among the variables are linear both in fuzzy constraints and the fuzzy functions to be optimized. Chadha. S. S [2] was introduced “A linear fractional program with homogeneous constraints”.

The intention here is to reduce the computing time of the optimization process when a block of constraints are fuzzy homogeneous in nature. The methods seems to be beneficial to large class of programming models containing a great number of fuzzy homogeneous constraints such homogeneous constraints are encountered in fuzzy transportation problem and fuzzy network models.

The concept of optimization decision was proposed by Bellmann and Zadeh [1]. This concept was adopted to problems of mathematical programming by Tanaka *et al.* Zimmer mann [7] presented a fuzzy approach to multi objective linear programming problems. He also studied the duality relations in fuzzy linear programming. Fuzzy linear programming problem with fuzzy coefficients was formulated by Negoita and it is called robust programming. Dubois. D and H. Parde [3] investigated linear fuzzy constraints. Tanaka and Asai [6] also proposed a formulation of fuzzy linear programming with fuzzy constraints and gave a method for its solution with bases on inequality relations between fuzzy numbers.

In this paper, Section 2 gives relevant preliminaries which are very much needed for our article. Section 3 presents an algorithm for the fuzzy transformation matrix $\tilde{T}$ is constructed. Some results and theorems are discussed in section 4. In section 5 a relevant numerical example explains the proposed algorithm. Finally, the conclusions are given in the section 6.

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2. BASICS AND DEFINITIONS

2.1. Definition

Let X denote an universal set that is X={x} then the characteristic function which assigns certain values or a membership grade to the elements of this universal set within a specified range[0,1] is known as the membership function and the set thus defined is called **a fuzzy set**. The membership grade corresponds to the degree to which an element is compatible with the concept represented by the fuzzy set. If $\mu_{\tilde{A}}$ is the membership function defining a fuzzy set $\tilde{A}$ then $\mu_{\tilde{A}}: x \rightarrow [0,1]$ where [0,1] denotes the interval of real numbers from 0 to 1.

2.2. Definition

A convex and normalized fuzzy set defined on R whose membership function is piecewise continuous is called **fuzzy numbers**

2.3. Definition

A fuzzy number $\tilde{A}$ is called **positive fuzzy number** if it is membership function is such That $\boldsymbol{\mu_{\tilde{A}} = 0}$; $\forall$ x< 0. This is denoted by $\tilde{A} > 0$.

2.4. Definition

A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is defined by the membership function

$$\boldsymbol{\mu_{\tilde{A}}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} \;; & if a_1 \le x \le a_2 \\ 1 \;; & if \; a_2 \le x \le a_3 \\ \frac{x-a_3}{a_3-a_4} \;; & if \; a_3 \le x \le a_4 \\ 0 \;; & otherwise \end{cases}$$

2.5. Definition

A ranking function R: F(R)→R which maps each fuzzy number into the real line. F(R) denotes the set of all trapezoidal fuzzy number. If R be any linear ranking function, then $R(\tilde{A}) = a_2 + a_3 + \frac{1}{2}[(a_4 + a_1) - (a_3 + a_2)]$.

2.6. Arithmetic operations on fuzzy numbers

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are trapezoidal fuzzy numbers then the

(i) Image of $\tilde{A} = (-a_4, -a_3, -a_2, -a_1)$

(ii) $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(iii) $\tilde{A} - \tilde{B} = (a1 - b4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

(iv) If λ is any scalar then $\lambda \tilde{A} = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4)$, if $\lambda > 0$ and $\lambda \tilde{A} = (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1)$, if $\lambda < 0$

(v) The multiplication of $\tilde{A}$ and $\tilde{B}$ is defined as

$$\tilde{A} \circ \tilde{B} = [\tfrac{a_1}{2}(b_1+b_2+b_3+b_4), \tfrac{a_2}{2}(b_1+b_2+b_3+b_4), \tfrac{a_3}{2}(b_1+b_2+b_3+b_4), \tfrac{a_4}{2}(b_1+b_2+b_3+b_4)], \text{ if } R(\tilde{B}) > \tilde{0} \text{ and}$$

$$\tilde{A} \circ \tilde{B} = [\tfrac{a_4}{2}(b_1+b_2+b_3+b_4), \tfrac{a_3}{2}(b_1+b_2+b_3+b_4), \tfrac{a_2}{2}(b_1+b_2+b_3+b_4), \tfrac{a_1}{2}(b_1+b_2+b_3+b_4)], \text{ if } R(\tilde{B}) < \tilde{0}$$

(vi) The division is defined as

$$\tilde{A} / \tilde{B} = [\frac{2a_1}{b_1+b_2+b_3+b_4}, \frac{2a_2}{b_1+b_2+b_3+b_4}, \frac{2a_3}{b_1+b_2+b_3+b_4}, \frac{2a_4}{b_1+b_2+b_3+b_4}], if \; R(\tilde{B}) > \tilde{0}, R(\tilde{B}) \neq \tilde{0} \; and$$

$$\tilde{A} / \tilde{B} = [\frac{2a_4}{b_1+b_2+b_3+b_4}, \frac{2a_3}{b_1+b_2+b_3+b_4}, \frac{2a_2}{b_1+b_2+b_3+b_4}, \frac{2a_1}{b_1+b_2+b_3+b_4}], \; if \; R(\tilde{B}) < \tilde{0}, R(\tilde{B}) \neq \tilde{0}$$

1.7. Notations

Let us denote the zero fuzzy number $\tilde{0}$ and unit fuzzy number $\tilde{1}$ as follows $\tilde{0} = (-2,-1, 1, 2)$, $\tilde{1} = (-1, 0, 1, 2)$ and $\tilde{I}_n$ denotes fuzzy identity matrix.

3. FUZZY LINEAR PROGRAM WITH FUZZY HOMOGENEOUS CONSTRAINTS

3.1. Definition

A System of fuzzy linear equations $\tilde{A}x = \tilde{b}$ is said to be a **fuzzy homogeneous constraint**, if $\tilde{b} = \tilde{0}$, such a system always has the trivial solution $x = \tilde{0}$

3.2. The development of the fuzzy transformation matrix

Let the given fuzzy problem be Maximize $\tilde{Z} = \tilde{C}x$ (3.1)

Subject to $\tilde{A}$ x=$\tilde{b}$ (3.2)

x≥ $\tilde{0}$

With $\tilde{a}_{i1}x_1 + \tilde{a}_{i2}x_2 + \tilde{a}_{i3}x_3 + \ldots + \tilde{a}_{ik}x_k + \ldots + \tilde{a}_{il}x_l + \ldots + \tilde{a}_{in}x_n = \tilde{0}$, for some i.

Let $\tilde{L}$ = {x: $\tilde{A}$X=$\tilde{b}$, x≥ $\tilde{0}$} be fuzzy constraint space. Here $\tilde{C}$ = ($\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \ldots \tilde{c}_n$) is a row vector with n fuzzy numbers, $\tilde{A}$ = ($\tilde{a}_{ij}$) is a fuzzy matrix,

$\tilde{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix}$ and $\tilde{b} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \\ \cdot \\ \cdot \\ \cdot \\ \tilde{b}_n \end{pmatrix}$ are column vectors with n fuzzy numbers

3.3. Algorithm for constructing the transformation matrix $\tilde{T}$ is constructed

Step-1: Select a homogeneous constrain from the given constraints of the problem

Step-2: Find the number of positive terms, the number of negative terms and the number of Zero terms in the homogeneous constraints. They are denoted by p, q and r respectively.

Step-3: From these (step 2), find the order of fuzzy identity matrix using the relation p + q + r = n, where n denotes order of fuzzy identity matrix. Order of transformation matrix is (p + q + r) X (pq)

Step-4: Construction of a fuzzy transformation matrix as follows

$\tilde{T} = (\tilde{T}_{1:}\ \tilde{T}_2) = (\tilde{e}_j : \tilde{t}_{kl})$ where $\tilde{e}_j$ is the j[th] column of fuzzy identity matrix and

$\tilde{t}_{kl} = -\tilde{a}_{il}\tilde{e}_k + \tilde{a}_{ik}\tilde{e}_l$

Step-5: Using the transformation matrix $\tilde{T}$, the fuzzy problem can be transformed as follows

Maximize $\tilde{Z} = \tilde{C}\tilde{T}w$, Subject to $\tilde{A}\,\tilde{T}$ w = $\tilde{b}$, W ≥ $\tilde{0}$

4. RESULTS ON FUZZY TRANSFORMED PROBLEM

Using the fuzzy transformation $\tilde{T}$x = w, we define the following fuzzy problem.

Maximize $\tilde{Z} = \overline{C}\,w$ (4.1)

Subject to $\overline{A}\,w = \tilde{b}$ (4.2)

w≥ $\tilde{0}$.

Here $\overline{C} = \tilde{C}\,\tilde{T}$; $\overline{A} = \tilde{A}\,\tilde{T}$, $\overline{b} = \tilde{b}\tilde{T}$. Let G = [w: $\overline{A}\,w = \overline{b}$, $w \geq \tilde{0}$] be fuzzy constraint space.

Result 4.1: If x solves $\tilde{A}$ X = $\tilde{b}$ then there exist a w(x =$\tilde{T}$ w) which solves $\overline{A}\,w = \tilde{b}$.

Result 4.2: If $\sum_{i=1}^{p} \tilde{a}_i = \sum_{j=1}^{q} \tilde{\boldsymbol{\beta}}_j = \tilde{v}$, $\tilde{a}_i \geq \tilde{0}$. ; $\tilde{\boldsymbol{\beta}}_j \geq \tilde{0}$ then there exist a matrix $\tilde{y} = (\widetilde{y_{ij}} \geq \tilde{0})$ such that $\sum_{j=1}^{q} \widetilde{y_{ij}} = \tilde{a}_i$ and $\sum_{i=1}^{p} \widetilde{y_{ij}} = \tilde{\beta}_j$.

Theorem 4.3: X* solves the program (3.1)—(3.2) if and only if w* solves the program (4.1)—(4.2).

Proof: Result (4.1) guarantees the existence of a feasible w*. Assume that x* solves the program (3.1)—(3.2) implies that $\tilde{C}x^* \geq \tilde{C}$x; ∀ x∈ L, which is same as with respect to the relation

x = $\tilde{T}$w. $\overline{C}$x* ≥ $\overline{C}$x or $\tilde{C}\,\tilde{T}$ w* ≥ $\tilde{C}\,\tilde{T}$w; ∀ w ∈ G.

This implies w* solves the program (4.1) --- (4.2).
Conversely, w* solves the program (4.1)-(4.2). Implies that
$\tilde{C}\,\tilde{T}$w* ≥ $\tilde{C}\tilde{T}$w; ∀ w ∈ G, $\overline{C}$w* ≥ $\overline{C}\ w$ and $\tilde{C}x^* \geq \tilde{C}$x; ∀ x∈ L

This implies x* solves the program (3.1)—(3.2).

Theorem 4.4: The extreme values of the two objective functions Maximize $\tilde{Z} = \tilde{C}x$ and Maximize $\tilde{Z} = \overline{C}\,w$ are equal.

Proof: Let Z* and z* be the values of Maximize $\tilde{Z} = \tilde{C}x$ and Maximize $\tilde{Z} = \overline{C}\,w$ at x* and w* respectively.This means Z* = $\tilde{C}x*$ = $\tilde{C}\,\tilde{T}$ w* = $\overline{C}$w* = z*.

5. NUMERICAL EXAMPLE

Maximize $\tilde{Z}$ = (-2,1,2,3)x_1 +(-9,4,7,10)x_2

Subject to (-1,0,1,2)x_1 + (-1,0,1,2)x_2 + (-1,0,1,2)x_3= (-7,3,5,7)
(-2,1,3,4)x_1 + (-1,0,1,2)x_2 + (-1,0,1,2)x_4 = (-9,4,7,10)
(-1,0,1,2)x_1 + (-2,-1,0,1)x_2+ (-2,-1,1,2)x_3+(-2,-1,1,2)x_4= (-2,-1,1,2)
$x_1, x_2, x_3, x_4 \geq \tilde{0}$.

Its solution is found at $x_1 = (\frac{-23}{4}, \frac{-3}{4}, \frac{7}{2}, 6)$; $x_2 = (\frac{-15}{4}$,1/4, 5/2, 4); x_3 =(-17,-3,11/2,33/2) and x_4 = (-2,-1,1,2) with $\tilde{Z}*$ =(-37/2,13/2,29/2,43/2).

Now the above problem can be written as

Maximize $\tilde{Z}$ = (-2,1,2,3)x_1 +(-9,4,7,10)x_2

$$\begin{bmatrix} (-1,0,1,2) & (-1,0,1,2) & (-1,0,1,2) & (-2,-1,1,2) \\ (-2,1,3,4) & (-1,0,1,2) & (-1,0,1,2) & (-1,0,1,2) \\ (-1,0,1,2) & (-2,-1,0,1) & (-2,-1,1,2) & (-2,-1,1,2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} (-7,3,5,7) \\ (-9,4,7,11) \\ (-2,-1,1,2) \end{bmatrix}$$

Here
(-1,0,1,2)x_1 + (-2,-1,0,1)x_2 + (-2,-1,1,2)x_3 + (-2,-1,1,2)x_4 = (-2,-1,1,2) is a fuzzy homogeneous constraint.

$\tilde{A} = [(\tilde{0}, \tilde{0}): (\tilde{1}), (\tilde{1})]$ where p=1, q=1and r =2. This implies n = p + q + r = 4.

$$\therefore \tilde{I}_4 = \begin{bmatrix} (-1,0,1,2) & (-2,-1,1,2) & (-2,-1,1,2) & (-2,-1,1,2) \\ (-2,-1,1,2) & (-1,0,1,2) & (-2,-1,1,2) & (-2,-1,1,2) \\ (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) & (-2,-1,1,2) \\ (-2,-1,1,2) & (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) \end{bmatrix}$$

Now, $\tilde{t}_{kl} = -\tilde{a}_{il}\tilde{e}_k + \tilde{a}_{ik}\tilde{e}_l$

$$\tilde{t}_{kl} = -(-2,-1,0,1)\begin{bmatrix} (-1,0,1,2) \\ (-2,-1,1,2) \\ (-2,-1,1,2) \\ (-2,-1,1,2) \end{bmatrix} + (-1,0,1,2)\begin{bmatrix} (-2,-1,1,2) \\ (-1,0,1,2) \\ (-2,-1,1,2) \\ (-2,-1,1,2) \end{bmatrix} = \begin{bmatrix} (-1,0,1,2) \\ (-1,0,1,2) \\ (0,0,0,0) \\ (0,0,0,0,) \end{bmatrix}$$

$$\text{From these } \tilde{T} = \begin{bmatrix} (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) \\ (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) \\ (-1,0,1,2) & (-2,-1,1,2) & (0,0,0,0) \\ (-2,-1,1,2) & (-1,0,1,2) & (0,0,0,0) \end{bmatrix}$$

The given problem transformed in to the form

Maximize $\tilde{Z} = \tilde{C}\,\tilde{T}$W, Subject to $\tilde{A}\,\tilde{T}\,w = \tilde{b}$ and w$\geq \tilde{0}$.

It is equivalent to the given problem. Thus our problem becomes

Maximize $\tilde{Z}$ = (-2,-1,1,2)w_1 + (-2,-1,1,2)w_2 + (-11,5,9,13)w_3

Subject to (-1,0,1,2)w_1 + (-2,-1,1,2)w_2 + (-2,0,2,4)w_3 = (-7,3,5,7)
(-2,-1,1,2)w_1 + (-2,-1,1,2)w_2 + (-3,-1,1,3)w_3 = (-9,4,7,10)
$w_1, w_2, w_3 \geq \tilde{0}$.

INITIAL TABLE

		$\tilde{C}_j$	(-2,-1, 1,2) (-2,-1, 1,2) (-11,5, 9,11)	Ratio Minimum θ
$\tilde{C}_{\tilde{B}}$	$Y_{\tilde{B}}$	$\tilde{X}_{\tilde{B}}$	w_1 w_2 w_3	
(-2,-1, 1,2) (-2,-1, 1,2)	W_1 W_2	(-7,3, 5,7) (-9,4,7,10)	(-1,0 ,1,2) (-2,-1, 1,2) (-2,0, 2,4) (-2,-1, 1,2) (-1,0,1,2) (-3,1,4,6)	-7/2,3/2,5/2,7/2 -9/4,1,7/4,5/2
$\vec{Z}_j - \vec{C}_j$	$\vec{Z} = \tilde{0}$		(-4,-2,2,4) (-4,-2,2,4) (-25,15,1,23)	

Since there is one $\tilde{Z}_j - \tilde{C}_j$ = (-25, 15, 1, 23) $<\ \tilde{0}$. Therefore go to next iteration.

Here w_2 Leaves from basis and w_3 enters in to the basis.

First iteration

		$\tilde{\boldsymbol{C}}_j$	(-2,-1, 1,2)	(-2,-1, 1,2)	(-11,5, 9,13)
$\tilde{\boldsymbol{C}}_{\tilde{B}}$	$\boldsymbol{Y}_{\tilde{B}}$	$\boldsymbol{X}_{\tilde{B}}$	w_1	w_2	w_3
(-2,-1, 1,2) (-11,5,9,13)	W_1 W_3	(-12,-1/2,3,23/2) (-9/4,1,7/4,5/2)	(-2,-1/2 ,3/2,3) (-1/2,-1/4, 1/4,1/2)	(-3,-3/2, 1,5/2) (-1/4,0,1/4,1/2)	(-5,-2,3/2, 11/2) (-3/4,1/4,1,3/2)
$\tilde{Z}_j - \tilde{C}_j$		$\tilde{Z}$=(-33/2,15/2,27/2,39/2)	(-4,-2,2,4)	(-23/4,-1/4,15/4,25/4)	(-24,-4,4,24)

All $\tilde{Z}_j - \tilde{C}_j \geq \tilde{0}$, we reached optimum solution.

Maximize $\tilde{Z}$ = (-11,5,9,13) ⊙ (-9/4,1,7/4,5/2) = (-33/2,15/2,27/2,39/2)

when w_1 = (-12,-1/2,3,23/2), w_2= (-2,-1, 1,2) and w_3 = (-9/4,1,7/4,5/2).

The solution of the original problem (x =$\tilde{T}$w) is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{bmatrix} (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) \\ (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) \\ (-1,0,1,2) & (-2,-1,1,2) & (0,0,0,0) \\ (-2,-1,1,2) & (-1,0,1,2) & (0,0,0,0) \end{bmatrix} \begin{bmatrix} \left(\mathbf{-12}, \frac{-1}{2}, \mathbf{3}, \frac{23}{2}\right) \\ (\mathbf{-2}, \mathbf{-1}, \mathbf{1}, \mathbf{2}) \\ \left(\frac{-9}{4}, \mathbf{1}, \frac{7}{4}, \frac{5}{2}\right) \end{bmatrix} = \begin{bmatrix} \left(\frac{-7}{2}, \mathbf{1}, \frac{5}{2}, \mathbf{5}\right) \\ \left(\frac{-7}{2}, \mathbf{-1}, \frac{5}{2}, \mathbf{5}\right) \\ (\mathbf{-1}, \mathbf{0}, \mathbf{1}, \mathbf{2}) \\ (\mathbf{-2}, \mathbf{-1}, \mathbf{1}, \mathbf{2}) \end{bmatrix}$$

6. CONCLUSION

The algorithm explained in section 3, can be extended to define $\tilde{T}$ if $\tilde{A}$ X=$\tilde{b}$ has more than one fuzzy homogeneous constraints. In case, there are k fuzzy homogeneous constraints, we define k fuzzy transformation matrices $\tilde{T}(1), \tilde{T}(2), \tilde{T}(3)\ldots\ldots\ \tilde{T}(k)$. $\tilde{T}(2)$is determined once $\tilde{A}\tilde{T}(1)$ has been computed. In general, $\tilde{T}(k)$ is determined only when $\tilde{A}\tilde{T}(1).\tilde{T}(2).\tilde{T}(3).\ldots\ldots\ldots\tilde{T}(k-1)$. has been computed.

This algorithm reduces the number of constraints, the main factor of the fuzzy optimization problem, can be used efficiently for solving large-scale fuzzy linear programming problems. We can use this algorithm to fuzzy linear fractional programming problem with fuzzy homogeneous constraints also.

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