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Applications of $^{**}g\alpha$-closed sets in Topological Spaces

[1]M. Vigneshwaran* and [2]A. Singaravelan

[1]Assistant Professor & [2]Research Scholar, Department of Mathematics,
Kongunadu Arts and Science College (Autonomous), Coimbatore-641029, Tamilnadu, India.

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ABSTRACT

We introduce a new class of sets namely $^{**}g\alpha$-closed sets in topological spaces and derive the properties of $^{**}g\alpha$ -closed sets. Also we find the relationship between $^{**}g\alpha$-closed sets and other existing sets. Moreover with the help of these sets, we introduce four new spaces namely, ${}_{\alpha}T_{1/2}^{***}$ spaces, T_c^{***} spaces, ${}_{\alpha}T_c^{***}$ spaces, ${}^{*}{}_{\alpha}T_{1/2}^{**}$ spaces and derived its properties.

***Key words**: $^{**}g\alpha$-closed sets, ${}_{\alpha}T_{1/2}^{***}$ spaces, T_c^{***} spaces, ${}_{\alpha}T_c^{***}$ spaces, ${}^{*}{}_{\alpha}T_{1/2}^{**}$ spaces.*

1. INTRODUCTION

Levine [15] introduced g-closed sets and studied their most fundamental properties. P.Bhattacharya and B.K.Lahiri [6], S.P.Arya and T.Nour [4], H.Maki *et al* [17, 18] introduced semi generalized-closed sets, generalized semi-closed, α-generalized closed sets and generalized α-closed sets respectively. R. Devi, *et al.* [10] introduced semi generalized-homeomorphism and generalized semi-homeomorphism in topological spaces. R. Devi, *et al* [9] introduced semi generalized-closed maps and generalized semi-closed maps. M.K.R.S Veera Kumar [23] introduced g^*-closed sets and M.Vigneshwaran, *et al* [24] introduced $^{*}g\alpha$-closed sets in topological spaces and Gnanambal [14] introduced gsp- closed sets and gpr closed sets respectively.

In this paper, we introduce a new class of sets namely $^{**}g\alpha$-closed sets in topological spaces and derive the properties. Also we find the relationship between $^{**}g\alpha$-closed sets and the other existing sets. Moreover with the help of these sets, we introduce four new spaces, ${}_{\alpha}T_{1/2}^{***}$ spaces, T_c^{***} spaces, ${}_{\alpha}T_c^{***}$ spaces, ${}^{***}{}_{\alpha}T_{1/2}$ spaces and derive its properties.

2. PRELIMINARIES

Throughout this paper (X, τ), represent topological space on we no sepration axioms are assumed unless otherwise to be mentioned. For a subsets A of(X,τ),cl(A) and int(A) denote the closure and the interior of A in (X, τ) respectively. The power set of X is denoted by P(X).

Let us recall the following definitions, which are useful in the sequel.

Definition 2.1: A subset A of a topological space (X,τ) is called
(1) a pre-open set [20] if A ⊆int(cl(A)) and a pre-closed set if cl(int(A)) ⊆ A.
(2) a semi-openset [16] if A ⊆ cl(int(A)) and a semi-closedset if int(cl(A))⊆A.
(3) an α-openset [22] if A ⊆int(cl(int(A))) and an α-closedset [22] if cl(int(cl(A)))⊆A.
(4) a semi pre-openset [2] *(= β-open*[1]) if A ⊆ cl(int(cl(A))) and a semipre-closedset [2] (=β-closed[1]) if int(cl(int(A))) ⊆ A.

*Corresponding Author: [1]M. Vigneshwaran**
[1]Assistant Professor and [2]Research Scholar, Department of Mathematics,
Kongunadu Arts and Science College (Autonomous), Coimbatore-641029, Tamilnadu, India.

Definition 2.2: A subset A of a topological space (X, τ) is called
(1) a generalized closed set (briefly g-closed) [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
(2) a semi-generalized closed set (briefly sg-closed) [6] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).
(3) a generalized semi-closed set (briefly gs-closed) [4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
(4) a generalized α-closed set (briefly $g\alpha$-closed) [18] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).
(5) an α-generalized closed set (briefly αg-closed) [17] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
(6) a generalized semi pre-closed set (briefly gsp-closed[12] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
(7) a generalized pre-closed set (briefly gp-closed[19] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
(8) a g^*-closed set [23] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
(9) a $^*g\alpha$-closed set [24] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g\alpha$-open in (X, τ).

The class of all g-closed sets (gsp-closed sets) of a space (X, τ) is denoted by $GC(X, \tau)(GSPC(X,\tau))$.

Definition 2.3: A topological space (X, τ) is said to be
(1) a T_b space [9] if every gs-closed set is closed.
(2) an ${}_\alpha T_b$ space [8] if every αg-closed set is closed.
(3) a $T_{1/2}^*$ space [23] if every g^*-closed set is closed.
(4) a $T_c$space [23] if every gs-closed set is g^*-closed.
(5) an ${}_\alpha T_c$space [23] if every αg-closed set is g^*-closed.
(6) an ${}_\alpha T_{1/2}^{**}$ space [24] if every $^*g\alpha$-closed set is closed.
(7) a T_c^{**} space [24] if every gs-closed set is $^*g\alpha$-closed.
(8) an ${}_\alpha T_c^{**}$ space [24] if every αg-closed set is $^*g\alpha$-closed.

Notation 2.5: For a space (X, τ),$C(X, \tau)$(resp.$SC(X, \tau)$, $\alpha C(X, \tau)$,$G\alpha C(X, \tau)$,$GC(X, \tau)$,$GSC(X, \tau)$,$\alpha GC(X, \tau)$) denote the class of all closed (resp.semi-closed, α-closed, $g\alpha$-closed, g-closed, gs-closed, αg-closed)subsets of (X, τ)**.**

3. BASIC PROPERTIES OF $^{**}g\alpha$-CLOSED SETS

We introduce the following definition.

Definition 3.1: A subset A of (X, τ) is called a $^{**}g\alpha$-closed set if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is $^*g\alpha$-open in (X, τ). The class of $^{**}g\alpha$-closed subset of (X, τ) is denoted by $^{**}G\alpha C(X, \tau)$.

Theorem 3.2: Every closed set is a $^{**}g\alpha$-closed set.

Proof: Let $A\subseteq U$ and U is $^*g\alpha$-open set in X. Since A is closed, $cl(A) = A$. Then $cl(A) = A\subseteq U$ implies $cl(A)\subseteq U$. Hence A is $^{**}g\alpha$-closed set.

The converse of the above theorem need not be true. It can be seen from the following example.

Example 3.3: Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a, b\}\}$; $\tau^c = \{X, \phi, \{c\}\}$, $^{**}g\alpha$-closed sets $=\{X, \phi, \{c\}, \{b, c\}, \{a, c\}\}$.Here $\{a, c\}$ is a $^{**}g\alpha$-closed set but not a closed set of (X, τ).

Theorem 3.4 Every g^*-closed set is a $^{**}g\alpha$-closed set.

Proof: Let $A\subseteq U$ and U is $^*g\alpha$-open. Since every $^*g\alpha$-open set is g-open. Hence U is g-open. Since A is g^*-closed, $cl(A)\subseteq U$. Hence A is $^{**}g\alpha$-closed set.

The converse of the above theorem need not be true. It can be seen from the following example.

Example 3.5: Let $X = \{a, b, c\}$ with $\tau = \{X, \phi,\{c\}\}$; $\tau^c = \{X, \phi ,\{a, b\}\}$,$g^*$-closed sets $=\{ X, \phi, \{a, b\}\}$,$^{**}g\alpha$-closed sets $=\{ X, \phi, \{a\}, \{b\},\{a, b\}, \{b, c\}, \{a, c\}\}$.
Here $\{b, c\}$ is a $^{**}g\alpha$-closed set but not a g^*-closed set of (X, τ).

Theorem 3.6: Every $^*g\alpha$-closed set is a $^{**}g\alpha$-closed set.

Proof: Let $A\subseteq U$ and U is a $^*g\alpha$ -open set in X. Since every $^*g\alpha$ -open set is $g\alpha$-open then U is $g\alpha$-open set. Since A is $^*g\alpha$-closed, $cl(A) \subseteq U$. Hence A is a $^{**}g\alpha$-closed set.

The converse of the above theorem need not be true. It can be seen from the following example.

Example 3.7: Let X = {a, b, c} with τ = {X, ϕ , {c}}; τ^c = {X, ϕ,{a, b}},*gα-closed set={ X, ϕ, {a, b}}, **gα-closed set={ X, ϕ , {a, b}, {b, c},{a, c}}.
Here {b, c} is a **gα-closed set but not a *gα-closed set of (X, τ).

Theorem 3.8: Every **gα-closed set is g-closed set.

Proof: Let A⊆U and U is open set in X. Since every open set is *gα-open, U is *gα-open. Since A is a **gα-closed, cl(A) ⊆ U. Hence A is g-closed.

The converse of the above theorem need not be true. It can be seen from the following example.

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Example 3.9: Let X = {a, b, c}, with τ={X, ϕ, {a}, {b, c}}; τ^c = {X, ϕ, {a}, {b, c}},g-closed set={X, ϕ, {a},{b},{c}, {a, b},{b, c}, {a, c}},**gα-closed set={ X, ϕ , {a}, {b, c}}.

Here {a, b} is a g-closed set but not a **gα-closed set of (X, τ).

Theorem 3.10: Every **gα-closed set is a αg-closed set.

Proof: Let A⊆U and U is open set. Since every open set is *gα-open, U is *gα-open. Since A is a**gα-closed, cl(A) ⊆ U. we know that αcl(A) ⊆ cl(A) ⊆U implies αcl(A) ⊆ U. Hence A is a αg-closed set.

The converse of the above theorem need not be true. It can be seen from the following example.

Example 3.11: Let X = {a, b, c} with τ = {X, ϕ,{a}, {b, c}}; τ^c = {X, ϕ, {a}, {b, c}},αg-closed set={X, ϕ, {a},{b},{c},{a, b},{b, c}, {a, c}},**gα-closed set={ X, ϕ, {a},{b, c}} **.**

Here {a, b} is a αg-closed set but not a **gα-closed set of (X, τ).

Theorem 3.12: Every **gα-closed set is a gs-closed set.

Proof: Let A⊆U and U is an open set in X. Since every open set is *gα-open. Then U is *gα-open. Since A is **gα-closed set, cl(A) ⊆ U. We know that Scl(A) ⊆cl(A) ⊆U implies Scl(A) ⊆ U. Hence A is a gs-closed set.

The converse of the above theorem need not be true. It can be seen from the following example.

Example 3.13: Let X = {a, b, c}, with τ = {X, ϕ, {a}, {b, c}}; τ^c = {X, ϕ, {a}, {b, c}},gs-closed set={X, ϕ, {a},{b},{c},{a, b},{b, c}, {a, c}},**gα-closed set={ X, ϕ, {a}, {b, c}}.

Here {a, c} is a gs-closed set but not a **gα-closed set of (X, τ).

Theorem 3.14: Every **gα-closed set is a gsp-closed set.

Proof: Let A⊆U and U is an open set in X. Since every open set is *gα-open, U is *gα-open. Since A is **gα-closed, cl(A) ⊆U. We know that spcl(A) ⊆ cl(A) ⊆ U implies spcl(A) ⊆ U. Hence A is a gsp-closed set.

The converse of the above theorem need not be true. It can be seen from the following example.

Example 3.15: Let X = {a, b, c} with τ = {X, ϕ, {a}, {c}, {a, c}}; τ^c = {X, ϕ, {b}, {a, b}, {b, c}}**,** gsp-closed set={X, ϕ, {a},{b},{c},{a, b},{b, c}, {a, c}}**,****gα-closed set={ X , ϕ, {b}, {a, b}, {b, c}}.

Here {a} is a gsp-closed set but not a **gα-closed set of (X, τ).

Theorem 3.16: Every **gα-closed set is a gpr-closed set.

Proof: Let A⊆U and U is an open set in X. Since every regular open set is *gα-open, U is *gα-open. Since A is **gα-closed, cl(A) ⊆ U. We know that Pcl(A) ⊆ cl(A) ⊆ U implies Pcl(A) ⊆ U. Hence A is a gpr-closed set.

The converse of the above theorem need not be true. It can be seen from the following example.

Example 3.17: Let X = {a, b, c} with τ = {X, ϕ , {a}}; τ^c = {X, ϕ , {b, c}}**,** gpr-closed set={X, ϕ, {a},{b},{c},{a, b},{b, c}, {a, c}},**gα-closed set={ X, ϕ, {b},{c},{a, b},{b, c}, {a, c}}
Here {a} is a gpr-closed set but not a **gα-closed set of (X, τ).

Theorem 3.18: Every $^{**}g\alpha$-closed set is a gp-closed set.

Proof: Let A⊆U and U is an open set in X. Since every open set is $^{*}g\alpha$ -open, U is $^{*}g\alpha$-open set. Since A is $^{**}g\alpha$-closed, cl(A) ⊆ U. We know that Pcl(A) ⊆ cl(A) ⊆ U implies Pcl(A) ⊆ U. Hence A is a gp-closed set.

The converse of the above theorem need not be true. It can be seen from the following example.

Example 3.19: Let X = {a, b, c}with τ = {X, ϕ, {a, b}}; τ^c = {X, ϕ, {c}},gp-closed set={X, ϕ, {a},{b},{c},{b, c}, {a, c}},$^{**}g\alpha$-closed set={ X, ϕ , {c},{b, c}, {a, c}}.

Here {a} is a gp-closed set but not a $^{**}g\alpha$-closed set of (X, τ).

Remark 3.20: $^{**}g\alpha$- closedness is independent of α-closedness and semi-closedness, it can be seen from the following examples

Examples 3.21: Let X = {a, b, c} with τ = {X, ϕ, {b}, {a, b}};τ^c = {X, ϕ, {a, c}, {c}}, $^{**}g\alpha$-closed set={X, ϕ , {c}, {b, c}, {a, c}},α-closed set={ X, ϕ , {a}, {c}, {a, c}},semi-closed set={ X, ϕ , {a}, {c}, {a, c}}.

Let A = {b, c} is a $^{**}g\alpha$-closed sets, but not a α-closed set and semi-closed set in (X, τ).

Examples 3.22: Let X = {a, b, c} with τ = {X, ϕ, {b}, {a, b}};τ^c = {X, ϕ, { c}, {a, c}}, $^{**}g\alpha$-closed set={X, ϕ, {c}, {b, c}, {a, c}},α-closed set={ X, ϕ , {a}, {c}, {a, c}}, semi-closed set={ X, ϕ, {a}, {c}, {a, c}}.

Here {a} is a α-closed set and semi-closed set, but not a $^{**}g\alpha$-closed set in (X, τ).

Remark 3.23: $^{**}g\alpha$- closedness is independent of pre-closedness and semi-pre closedness, it can be seen from the following examples

Examples 3.24: Let X = {a, b, c} with τ = {X, ϕ, {b}, {a, b}};τ^c = {X, ϕ, {c}, {a, c}} ,$^{**}g\alpha$-closed set={X, ϕ, {c}, {b, c}, {a, c}},Pre–closed set = {X, ϕ , {a}, {c}, {a, c}},Semi-pre closed set ={X, ϕ, {a}, {c}, {a, c}}

Here {b, c} is a $^{**}g\alpha$-closed sets, but not a pre-closed set and semi-pre closed set in (X, τ).

Examples 3.25: Let X = {a, b, c} with τ = {X, ϕ, {b}, {a, b}};τ^c = {X, ϕ, {c}, {a, c}},$^{**}g\alpha$-closed set={X, ϕ, {c}, {b, c}, {a, c}},pre-closed set={ X , ϕ, {a}, {c}, {a, c}},semi-pre closed set={ X , ϕ, {a}, {c}, {a, c}}

Let B = {a} is a pre-closed set and semi-pre closed set, but not a $^{**}g\alpha$-closed set in (X, τ).

Remark 3.26: $^{**}g\alpha$- closedness is independent of sg-closedness and gα- closedness, it can be seen from the following examples

Example 3.27: Let X = {a, b, c} with τ = {X, ϕ, {b}, {a, b}};τ^c = {X, ϕ, {c}, {a, c}},$^{**}g\alpha$-closed set={ X, ϕ , {c}, {b, c}, {a, c}}, sg-closed set={ X, ϕ, {a}, {c}, {a, c}},gα-closed set={ X, ϕ, {a}, {c}, {a, c}}

Let A = {b, c} is a $^{**}g\alpha$-closed set, but not a sg-closed set and gα- closed set in (X, τ).

Examples 3.28: Let X = {a, b, c} with τ = {X, ϕ, {b}, {a, b}}; τ^c = {X, ϕ, {c}, {a, c}},$^{**}g\alpha$-closed set={ X, ϕ, {c}, {b, c}, {a, c}},sg-closed set={ X, ϕ , {a}, {c}, {a, c}}, gα-closed set={ X, ϕ, {a}, {c}, {a, c}}

Here {a} is a sg-closed set and gα- closed set, but not a $^{**}g\alpha$-closed set in (X, τ).

Theorem 3.29: If A and B are $^{**}g\alpha$-closed sets. Then AU B is also a $^{**}g\alpha$-closed set.

Proof: Let A and B are $^{**}g\alpha$-closed sets. Let AU B⊆U, U is $^{*}g\alpha$-open. Since A and B are $^{**}g\alpha$-closed sets, cl(A)⊆ U and cl(B) ⊆ U. This implies that cl(AU B) = cl(A) U cl(B)⊆U implies cl(AU B) ⊆ U. Therefore AUB is $^{**}g\alpha$-closed set.

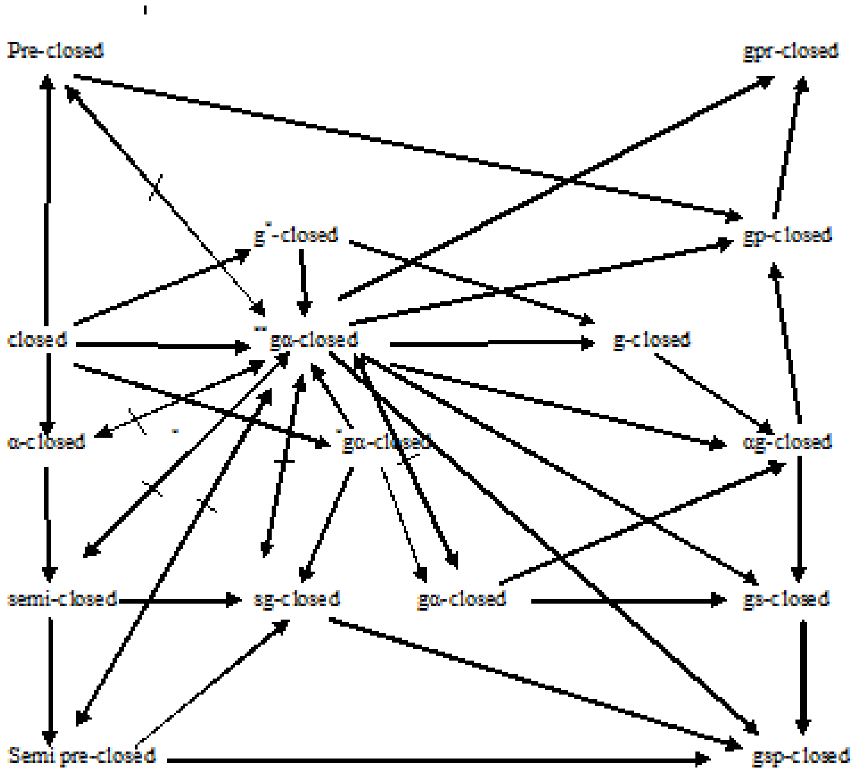
Remark 3.30: The intersection of two $^{**}g\alpha$-closed set is again $^{**}g\alpha$-closed set.

Theorem 3.31: Let X be a topological space. A subset A of (X, τ) is $^{**}g\alpha$-open if and only if U⊆(Int(A)), Whenever U is $^{*}g\alpha$-closed set and U ⊆ A.

Proof: Let A be a $^{**}g\alpha$-open set and U is $^{*}g\alpha$- closed set such that U⊆ A implies X-A ⊆ X-U and X-A is a $^{**}g\alpha$-closed set. So cl(X-A) ⊆ X-U implies (X-(X-U)) ⊆ (X- cl(X-A)) = U. But (X- cl(X-A)) = Int(A). Thus U ⊆Int(A). Conversely, suppose A is subset such that U ⊆Int(A). Whenever U is $^{*}g\alpha$- closed and U ⊆ A. We show that X-A is $^{**}g\alpha$- closed set. Let X-A⊆U, Where U is $^{*}g\alpha$-open. Since X-A ⊆ U implies X-U ⊆ A. By assumption that we must have X-U ⊆Int(A) or X-Int(A)⊆ U. Now X-Int(A) = cl(X-A) which implies that cl (X-A)⊆ U and X-A is $^{**}g\alpha$-closed set.

Remark 3.32: The following diagram shows that relationships between $^{**}g\alpha$-closed sets and some other sets in theorem 3.2, 3.4, 3.6, 3.8, 3.10, 3.12, 3.14, 3.16,3.18 and remark 3.20, 3.23,3.26 and reference [23], [24].

A ⟶ B (A ⟷̸ B) represents A implies B but not conversely (A and B are independent each other).



4. APPLICATIONS OF $^{**}g\alpha$-CLOSED SETS

We introduce the following definition.

Definition 4.1: A space (X, τ) is called a $_{\alpha}T_{1/2}^{***}$ space if every $^{**}g\alpha$-closed set is closed.

Theorem 4.2: Every $T_{1/2}$ space is an $_{\alpha}T_{1/2}^{***}$ space.

Proof: Let A be a $^{**}g\alpha$-closed set of (X, τ). Since every $^{**}g\alpha$-closed set is g-closed, A is a g-closed. Since (X, τ) is a $T_{1/2}$ space, A is closed. Therefore (X, τ) is an $_{\alpha}T_{1/2}^{***}$ space.

The converse of the above theorem need not be true. It can be seen from the following example.

Example4.3: Let X={a, b, c} with τ ={X , ϕ,{a},{b ,c}};τ^c={X, ϕ,{a}, {b, c}},g-closed set={X, ϕ, {a},{b},{c}, {a, b},{b, c}, {a, c}},$^{**}g\alpha$-closed set={ X, ϕ , {a}, {b ,c}}.

Here (X, τ) is an $_{\alpha}T_{1/2}^{***}$ space but not a $T_{1/2}$ space. Since {a, b} is a g-closed set, but not a closed set of (X, τ).

Theorem 4.4: Every T_b space is an $_{\alpha}T_{1/2}^{***}$ space.

Proof: Let A be a $^{**}g\alpha$-closed set of (X, τ). Since every $^{**}g\alpha$-closed set is gs-closed, A is a gs-closed. Since (X, τ) is a T_b space, A is closed. Therefore (X, τ) is an $_{\alpha}T_{1/2}^{***}$ space.

The space in the following example is an $_{\alpha}T_{1/2}^{***}$ space, but not a T_b space.

Example 4.5: Let X = {a, b, c} with τ ={X, ϕ, {a}, {b, c}};τ^c={X, ϕ ,{a}, {b ,c}},gs-closed set={ X, ϕ , {a},{b},{c}, {a, b},{b, c}, {a, c}},$^{**}g\alpha$-closed set={ X, ϕ,{a}, {b, c}}

Here (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$ space but not a T_b space. Since {a, b} is a gs-closed set but not a closed set.

Theorem 4.6: Every ${}_{\alpha}T_b$ space is ${}_{\alpha}T_{1/2}^{***}$ space.

Proof: Let A be a $^{**}g\alpha$-closed set of (X, τ). Since every $^{**}g\alpha$-closed set is αg-closed, A is a αg-closed. Since (X, τ) is a ${}_{\alpha}T_b$ space, A is closed. Therefore (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$ space.

The space in the following example is an ${}_{\alpha}T_{1/2}^{***}$ space, but not an ${}_{\alpha}T_b$ space.

Example 4.7: Let X={a, b, c} with τ ={X, ϕ ,{a},{b, c}};τ^c={X, ϕ , {a}, {b, c}},αg-closed set={X , ϕ, {a},{b},{c}, {a, b},{b, c}, {a, c}},$^{**}g\alpha$–closed set={ X, ϕ, {a}, {b, c}}

Here (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$ space but not an ${}_{\alpha}T_b$ space. Since {a, b} is an αg-closed set but not a closed set.

The following theorem gives a characterization of ${}_{\alpha}T_{1/2}^{***}$ space.

Theorem 4.8: If (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$space, then every singleton of X is either $^{*}g\alpha$-closed or open.

Proof: Let x ϵX and suppose that {x} is not a $^{*}g\alpha$–closed set of (X, τ). Then X/{x} is not a $^{*}g\alpha$-open. This implies that X is the only $^{*}g\alpha$-open set containing X/{x}. So X/{x} is a $^{**}g\alpha$-closed set of (X, τ). Since (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$ space. X/{x} is closed (or) equivalently {x} is open in (X, τ).

Theorem 4.9: Every ${}_{\alpha}T_{1/2}^{***}$ space is ${}_{\alpha}T_{1/2}^{**}$ space.

Proof: Let A be $^{*}g\alpha$-closed set of (X, τ). Since every $^{*}g\alpha$-closed set is $^{**}g\alpha$-closed set. Since (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$space, A is closed. Therefore (X, τ) is an ${}_{\alpha}T_{1/2}^{**}$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Exampl 4.10: Let X= {a, b, c} with τ ={X, ϕ, {c}};τ^c={X , ϕ, {a, b}}
$^{*}g\alpha$-closed set={X, ϕ , {a, b}},$^{**}g\alpha$–closed set={ X, ϕ, {a},{b},{a, b},{b, c}, {a, c}]

Here (X, τ) is an ${}_{\alpha}T_{1/2}^{**}$ space. Let A = {a, c} be a$^{**}g\alpha$–closed set but not closed set. Therefore (X, τ) is not an ${}_{\alpha}T_{1/2}^{***}$ space.

Theorem 4.11: Every ${}_{\alpha}T_{1/2}^{***}$ space is a $T_{1/2}^{*}$ space.

Proof: Let A-be g^*-closed set of (X, τ). Since every g^*-closed set is $^{**}g\alpha$–closed set. Since (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$space, A is closed. Therefore (X, τ) is a $T_{1/2}^{*}$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.12: Let X={a, b, c} with τ ={X, ϕ,{c}};τ^c={X, ϕ, {a, b}},g^*-closed set={ X, ϕ, {a, b}}, $^{**}g\alpha$–closed set={X, ϕ, {a},{b},{a, b},{b, c}, {a, c}}.

Here (X, τ) is a $T_{1/2}^{*}$ space. Let A = {a, c} be a$^{**}g\alpha$–closed set, but not closed set. Therefore (X, τ) is not an ${}_{\alpha}T_{1/2}^{***}$ space.

Definition 4.13: A space of (X, τ) is called a T_c^{***}space if every gs-closed set is $^{**}g\alpha$-closed.

The following Theorem gives a characterization of T_c^{***}spaces.

Theorem 4.14: If (X, τ) is a T_c^{***}space, then every singleton of X is either closed or$^{**}g\alpha$-open.

Proof: Let xϵ X and suppose that {x} is not a closed set of (X, τ). Then X/{x} is not an open. This implies x is the only open set containing X/{x}. So, X/{x} is gs-closed set of (X, τ). Since (X, τ) is T_c^{***} space. X/{x} is a $^{**}g\alpha$-closed set or equivalently {x} is $^{**}g\alpha$-open in (X, τ).

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.15: Let X = {a, b, c} with τ = {ϕ, X,{a},{c}, {a, c}};τ^c={X, ϕ, {b, c}, {a, b}, {b}}$^{**}g\alpha$-open set={ X, ϕ, {a},{c}, {a, c}}, gs-closed set={X, ϕ , {a},{b},{c},{a, c}, {a, b}}, $^{**}g\alpha$-closed set={X , ϕ,{a,}, {a, b}, {a, c}}. Here {a} and {c} are $^{**}g\alpha$-open sets and {b} is a closed set but (X, τ) is not a T_c^{***} space. Since {a} is a gs-closed but not a $^{**}g\alpha$-closed set of (X, τ).

Theorem 4.16: Every T_b space is a T_c^{***}space.

Proof: Let A be a gs- closed set of (X, τ). Since (X, τ) is a T_b space, A is closed. Since every closed set is $^{**}g\alpha$-closed, A is $^{**}g\alpha$-closed set. Therefore (X, τ) is a T_c^{***} space.

The converse of the above theorem is not true. It can be seen by the following example.

Example4.17: Let X = {a, b, c} with τ ={X, ϕ , {a, b}};τ^c={X, ϕ , {c}},$^{**}g\alpha$-closed set={ X, ϕ , {c},{b, c}, {a, c}}

Here {b, c} is a $^{**}g\alpha$-closed set of (X, τ), but it is not a closed set of (X, τ).

Theorem 4.18: Every T_c^{***}space is a T_d space.

Proof: Let A be gs-closed set of (X, τ). Since (X, τ) is a T_c^{***} space. A is $^{**}g\alpha$-closed set. Since every $^{**}g\alpha$-closed set is g-closed, A is g-closed set. Therefore (X, τ) is a T_d-space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.19: Let X = {a, b, c} with τ ={X, ϕ, {a}, {b, c}};τ^c={X, ϕ ,{a}{b,c}},gs-closed set={X, ϕ, {a},{b},{c}, {a, b},{b, c}, {a, c}},$^{**}g\alpha$-closed set={ X, ϕ , {a}, {b ,c}}

Here {b} is a gs-closed set, but not $^{**}g\alpha$-closed set.

Theorem 4.20: Every T_c^{***} space is a $_{\alpha}T_d$ space.

Proof: Let A be an αg-closed set of (X, τ). Since every αg-closed set is gs-closed, A is gs-closed. Since (X, τ) is a T_c^{***}space, A is $^{**}g\alpha$-closed. Since every $^{**}g\alpha$-closed set g-closed, A is g-closed set. Therefore (X, τ) is an $_{\alpha}T_d$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.21: Let X = {a, b, c} with τ ={X, ϕ, {a}, {b, c}}; τ^c={X, ϕ ,{a},{b, c}}‚αg-closed set={X, ϕ, {a},{b},{c},{a, b},{b, c}, {a, c}}‚gs-closed set={X, ϕ , {a},{b},{c},{a, b},{b, c}, {a, c}}‚$^{**}g\alpha$-closed set={X, ϕ, {a}, {b, c}}.Here {a, b} is an αg-closed set, but not a $^{**}g\alpha$-closed set.

Theorem 4.22: The space (X, τ) is a T_b space if and only if it is a T_c^{***} space and $_{\alpha}T_{1/2}^{***}$.

Proof: Necessity part: Let A be a gs- closed set of (X, τ). Since (X, τ) is a T_b space, A is closed. Since every closed set is $^{**}g\alpha$-closed. A is $^{**}g\alpha$-closed set. Therefore (X, τ) is a T_c^{***} space Let A be a $^{**}g\alpha$-closed set of (X, τ). Since every $^{**}g\alpha$-closed set is gs-closed, A is a gs-closed. Since (X, τ) is a T_b space, A is closed. Therefore (X, τ) is an $_{\alpha}T_{1/2}^{***}$ space.

Sufficient part: Let A-be a gs-closed set of (X, τ). Since (X, τ) is a T_c^{***} space, A is $^{**}g\alpha$-closed set. Since (X, τ) is an $_{\alpha}T_{1/2}^{***}$ space, A is closed. Therefore (X, τ) is a T_b space.

Theorem 4.23: Every T_c-space is a T_c^{***}space.

Proof: Let A be a gs-closed set. Since (X, τ) is a T_c space, A is a g^*-closed set. Since every g^*-closed set is $^{**}g\alpha$-closed set, A is $^{**}g\alpha$-closed. Therefore (X, τ) is a T_c^{***}space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.24: Let X = {a, b, c} with τ ={X, ϕ, {c}}; τ^c={X, ϕ, {a, b}},g^*-closed set={X, ϕ, {a, b}}, gs-closed set={ X, ϕ , {a},{b},{a, b},{b, c}, {a, c}},$^{**}g\alpha$-closed set={ X, ϕ, {a}, {b}, {a, b},{b, c}, {a, c}}.

Here (X, τ) is a T_c^{***} space. Let A = {a, c} be a gs-closed set but not a g*-closed set. Therefore (X, τ) is not a T_c space.

Theorem 4.25: Every T_c^{**} space is a T_c^{***}space.

Proof: Let A be a gs-closed set of (X, τ). Since (X, τ) is a T_c^{***}space, A is $^{*}g\alpha$-closed. Since every $^{*}g\alpha$-closed set is $^{**}g\alpha$-closed. Therefore (X, τ) is a T_c^{***} space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.26: Let X = {a, b, c} with τ ={X, ϕ , {c}} ;τ^c={X, ϕ, {a, b}},$^{*}g\alpha$-closed set={X, ϕ , {a, b}}, gs-closed set={ X, ϕ , {a},{b},{a, b},{b, c}, {a, c}},$^{**}g\alpha$-closed set={ X, ϕ , {a}, {b}, {a, b},{b, c}, {a, c}}

Here (X, τ) is a T_c^{***} space. Let A = {a, c} be a gs-closed set but not a $^{*}g\alpha$-closed set. Therefore (X, τ) is not a T_c^{**} space.

Remark 4.27: T_c^{***} space and ${}_{\alpha}T_{1/2}^{***}$ space are independent of each other. It can be seen by the following examples.

Example 4.28: Let X = {a, b, c} with τ ={X, ϕ, {a}, {b, c}}; τ^c={X, ϕ,{b, c}, {a}},αg-closed set={X, ϕ, {a},{b},{c},{a, b},{b, c}, {a, c},$^{**}g\alpha$-closed set={X, ϕ, {a},{b},{c},{a, b},{b, c}, {a, c}} ,gs-closed set={X, ϕ, {a},{b},{c},{a, b}, {a, c} }.

Here (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$ space but not T_c^{***} space. Since {b} is a gs-closed set but not closed set.

Example 4.29: Let X= {a, b, c} with τ ={X, ϕ, {a, b}}; τ^c= {X, ϕ, {c}}, $^{**}g\alpha$-closed set={ X, ϕ, {c},{b, c}, {a, c}}

Here (X, τ) is a T_c^{***} space but not an ${}_{\alpha}T_{1/2}^{***}$ space. Since {b, c} is a $^{**}g\alpha$-closed set but not closed set.

Definition 4.30: A space (X, τ) is called an ${}_{\alpha}T_c^{***}$ space if every αg-closed set is $^{**}g\alpha$-closed.

Theorem 4.31: Every ${}_{\alpha}T_c^{***}$ space is a ${}_{\alpha}T_d$ space.

Proof: Let A be a αg-closed set of (X, τ).since (X, τ) is an ${}_{\alpha}T_c^{***}$ space, A is $^{**}g\alpha$-closed. Since every $^{**}g\alpha$-closed set is g-closed, A is g-closed set. Therefore (X, τ) is an ${}_{\alpha}T_d$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.32: Let X ={a, b, c} with τ ={X, ϕ, {a}, {b, c}}; τ^c={X, ϕ, {a}{b, c}},αg-closed set={X, ϕ, {a},{b},{c}, {a, b},{b, c}, {a, c}},$^{**}g\alpha$-closed set={X , ϕ,{a},{b, c}},g- closed set={ X , ϕ, {a},{b},{c},{a, b},{b, c}, {a, c}}.

Here (X, τ) is a ${}_{\alpha}T_d$ space but not an ${}_{\alpha}T_c^{***}$ space. Since {c} is an αg -closed set but not $^{**}g\alpha$ -closed set.

Theorem 4.33: Every T_c^{***} space is an ${}_{\alpha}T_c^{***}$space.

Proof: Let A be a αg-closed set of (X, τ). Since every αg-closed set is gs-closed, A is gs-closed. Since (X, τ) is a T_c^{***} space, A is $^{**}g\alpha$-closed set. Therefore (X, τ) is an ${}_{\alpha}T_c^{***}$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.34: Let X = {a, b, c} with τ ={X, ϕ, {a}, {b},{a, b}}; τ^c={X, ϕ , { c}, {a, c}, {b, c}},αg-closed set={ X, ϕ, {c},{b, c}, {a, c}},gs-closed set={ X, ϕ, {a},{b},{c},{a, b},{b, c}, {a, c}},$^{**}g\alpha$- closed set={ X, ϕ, { c}, {a, c}, {b, c}}.Here (X, τ) is a ${}_{\alpha}T_c^{***}$ space but not a T_c^{***} space. Since {a, b} is a gs-closed set but not $^{**}g\alpha$-closed set.

Theorem 4.35: Every T_b space is an ${}_{\alpha}T_c^{***}$space.

Proof: Let A be a αg-closed set of (X, τ). Since every αg-closed set is gs-closed, A is gs-closed. Since (X, τ) is a T_b space, A is closed set. Since every closed set is $^{**}g\alpha$-closed, A is a $^{**}g\alpha$-closed set. Therefore (X, τ) is an ${}_{\alpha}T_c^{***}$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.36: Let X ={a, b, c}with τ ={X, ϕ, {a, b}}; τ^c={X, ϕ, {c}},αg-closed set={ X, ϕ, {c}, {a, c}, {b, c}}, gs-closed set={ X, ϕ, {c}, {a, c}, {b ,c}},$^{**}g\alpha$- closed set={X, ϕ , {c},{b, c}, {a, c}}

Here (X, τ) is an ${}_{\alpha}T_c^{***}$ space but not a T_b space. Since {a, c} is a gs-closed set but not closed set.

Theorem 4.37: Every ${}_{\alpha}T_b$ space is an ${}_{\alpha}T_c^{***}$space.

Proof: Let A be a αg-closed set of (X, τ) since (X, τ) is an $_{\alpha}T_b$ space, A is closed. Since every closed set is $^{**}g\alpha$-closed, A is $^{**}g\alpha$-closed set. Therefore (X, τ) is an $_{\alpha}T_c^{***}$space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.38: Let X = {a, b, c} with τ = {X, ϕ, {a, b}}; τ^c= {X, ϕ, {c}},αg-closed set={ X, ϕ, {b, c}, {a, c}, {c}},$^{**}g\alpha$-closed set={ X, ϕ, { c},{a, c}, {b, c}}

Here (X, τ) is an $_{\alpha}T_c^{***}$ space but not an $_{\alpha}T_b$ space. Since {b, c} is an αg-closed set but not closed set.

Theorem 4.39: Every $_{\alpha}T_c$ space is an $_{\alpha}T_c^{***}$space.

Proof: Let A be a αg-closed set. Since (X, τ) is a $_{\alpha}T_c$ space, A is a g^*-closed set. Since every g^*-closed set is $^{**}g\alpha$-closed set, A is $^{**}g\alpha$-closed. Therefore (X, τ) is a T_c^{***}space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.40: Let X = {a, b, c} with τ ={X, ϕ, {c}}; τ^c={X, ϕ , {a, b}}, g^*-closed set={ X, ϕ, {a, b}}
αg-closed set={ X, ϕ , {a},{b},{a, b},{b, c}, {a, c}},$^{**}g\alpha$- closed set={ X, ϕ, {a},{b}, {a, b},{b, c}, {a, c}}

Here (X, τ) is a $_{\alpha}T_c^{***}$ space but not a $_{\alpha}T_c$ space. Let A = {a, c} is an αg-closed set but not a g^*-closed set. Therefore (X, τ) is not a $_{\alpha}T_c$ space.

Theorem 4.41: Every $_{\alpha}T_c^{**}$ space is an $_{\alpha}T_c^{***}$space.

Proof: Let A be a αg-closed set. Since (X, τ) is an $_{\alpha}T_c^{**}$space, A is a $^*g\alpha$-closed set. Since every $^*g\alpha$-closed set is $^{**}g\alpha$-closed set, A is $^{**}g\alpha$-closed. Therefore (X, τ) is a T_c^{***}space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.42: Let X = {a, b, c} with τ ={X, ϕ , {c}};τ^c={X, ϕ , {a, b},$^*g\alpha$-closed set={X, ϕ, {a, b}}, αg-closed set={ X, ϕ ,{a},{b},{a, b},{b, c}, {a, c},$^{**}g\alpha$- closed set={ X, ϕ , {a},{b}, {a, b},{b, c}, {a, c}}
Here (X, τ) is a $_{\alpha}T_c^{***}$ space. Let A = {a, c} is an αg-closed set but not a $^*g\alpha$-closed set. Therefore (X, τ) is not a $_{\alpha}T_c$** space.

Theorem 4.43: A space(X, τ) is an $_{\alpha}T_b$ space iff it is an $_{\alpha}T_c^{***}$and an $_{\alpha}T_{1/2}^{***}$

Proof: Necessity part: Let A be a αg-closed set of (X, τ) since (X, τ) is an $_{\alpha}T_b$ space, A is closed. Since every closed set is $^{**}g\alpha$-closed, A is $^{**}g\alpha$-closed set. Therefore (X, τ) is an $_{\alpha}T_c^{***}$space. Let A be a $^{**}g\alpha$-closed set of (X, τ). Since every $^{**}g\alpha$-closed set is αg-closed, A is a αg-closed. Since (X, τ) is an $_{\alpha}T_b$ space, A is closed. Therefore (X, τ) is an $_{\alpha}T_{1/2}^{***}$ space.

Sufficient part: Let A be a αg-closed set of (X, τ). Since (X, τ) is an $_{\alpha}T_c^{***}$ space, A is$^{**}g\alpha$-closed. Since (X, τ) is an $_{\alpha}T_{1/}2^{***}$space, A is closed set. Therefore (X, τ) is an $_{\alpha}T_b$ space.

Remark 4.44: $_{\alpha}T_c^{***}$ space and $T_{1/2}^{***}$ space are independent of each other. It can be seen from the following example.

Example 4.45: Let X = {a, b, c} with τ = {X, ϕ, {a, b}};τ^c={X, ϕ, {c}},$^{**}g\alpha$-closed set={ X , ϕ, { c}, {a, c}, {b, c}}

Here (X, τ) is an $_{\alpha}T_c^{***}$ space but not an $_{\alpha}T_{1/2}^{***}$ space. Since {a, c} is a $^{**}g\alpha$-closed set but not closed set.

Example 4.46: Let X ={a, b, c}with τ ={X, ϕ, {a}, {b, c}};τ^c={X, ϕ, {a}, {b, c}},αg-closed set={ X, ϕ, {a},{b},{c},{a, b}, {b, c}, {a, c}}$^{**}g\alpha$-closed={ X, ϕ, {a},{b, c}}

Here (X, τ) is a $_{\alpha}T_{1/2}^{***}$ space, but not an $_{\alpha}T_c^{***}$ space. Since {b} is an αg-closed set but not $^{**}g\alpha$-closed set.

Definition 4.47: A space (X, τ) is called a $^{***}{}_{\alpha}T_{1/2}$ space if every g-closed set is $^{**}g\alpha$-closed.

Theorem 4.48: Every $T_{1/2}$ space is an $^{***}{}_{\alpha}T_{1/2}$ space.

Proof: Let A be a g-closed set of (X, τ) since (X, τ) is an $T_{1/2}$ space, A is closed. Since every closed set is $^{**}g\alpha$-closed, A is $^{**}g\alpha$-closed set. Therefore (X, τ) is an $^{***}{}_{\alpha}T_{1/2}$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.49: Let X= {a, b, c} with τ ={X, ϕ, {a, b}};τ^c={X, ϕ , {c}},**gα-closed set={ X, ϕ, {c},{b, c}, {a, c}}, g-closed set={ X , ϕ, {c},{b, c}, {a, c}}

Here (X, τ) is a $^{***}{}_{\alpha}T_{1/2}$ space but not $T_{1/2}$ space. Since {b, c} is a g-closed set but not closed set.

Theorem 4.50: Every T_b space is a $^{***}{}_{\alpha}T_{1/2}$ space.

Proof: Let A be a g-closed set of (X, τ). Since every g-closed set is gs-closed, A is gs-closed. Since (X, τ) is a T_b space, A is closed set. Since every closed set is **gα-closed, A is **gα-closed set. Therefore (X, τ) is an ${}_{\alpha}T_c^{***}$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.51: Let X = {a, b, c} with τ ={X, ϕ, {b, c}};τ^c={X, ϕ, {a}},**gα-closed set={ X, ϕ , {a},{a, b}, {a, c}} g-closed set={X, ϕ, {a},{a, b}, {a, c}},gs- closed set={ X, ϕ, {a},{a, b}, {a, c}}

Here (X, τ) is a $^{***}{}_{\alpha}T_{1/2}$ space but not a T_b space. Since {a, b} is a gs-closed set but not closed set.

Theorem 4.52: Every ${}_{\alpha}T_b$ space is a $^{***}{}_{\alpha}T_{1/2}$ space.

Proof: Let A be a g-closed set of (X, τ). Since every g-closed set is αg-closed, A is αg-closed. Since (X, τ) is an ${}_{\alpha}T_b$ space, A is closed set. Since every closed set is **gα-closed, A is **gα-closed set. Therefore (X, τ) is an $^{***}{}_{\alpha}T_{1/2}$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.53: Let X = {a, b, c}with τ ={X, ϕ ,{a, c}};τ^c={X, ϕ,{b}},g-closed set={ X, ϕ, {b},{a, b}, {b, c}} gs-closed set= {X, ϕ , {b},{a, b}, {b, c}},**gα- closed set={ X, ϕ,{b},{a, b}, {b, c}},αg-closed set={ X, ϕ , {b},{a, b}, {b, c}}

Here (X, τ) is a $^{***}{}_{\alpha}T_{1/2}$ space but not an ${}_{\alpha}T_b$ space. Since {b, c} is an αg-closed set but not closed set.

Theorem 4.54: Every T_c^{***} space is a $^{***}{}_{\alpha}T_{1/2}$ space.

Proof: Let A be a g-closed set of (X, τ). Since every g-closed set is gs-closed, A is gs-closed set. Since (X, τ) is a T_c^{***}space, A is **gα-closed set. Therefore (X, τ) is an $^{***}{}_{\alpha}T_{1/2}$ space.

The converse of the above theorem is not true. It can be seen by the following example.

Example 4.55: Let X = {a, b, c} with τ ={X,ϕ, {b}, {c}, {b, c}};τ^c={X, ϕ, {a}, {a, b}, {a, c}}
g-closed set={ X, ϕ ,{a}, {a, b}, {a, c}},gs-closed set={ X, ϕ , {a}, {b}, {c}, {a, c}, {a, b}}
**gα- closed set={X, ϕ ,{a},{a, b}, {a, c}}

Here (X, τ) is a $^{***}{}_{\alpha}T_{1/2}$ space but not an T_c^{***}space. Since {b} is a gs-closed set but not **gα-closed set.

Theorem 4.56: The space (X, τ) is a $T_{1/2}$ space if and only if it is a $^{***}{}_{\alpha}T_{1/2}$ space and an ${}_{\alpha}T_{1/2}^{***}$ space.

Proof: Necessity part: Let A be a g-closed set of (X, τ) since (X, τ) is an $T_{1/2}$ space, A is closed. Since every closed set is **gα-closed, A is **gα-closed set. Therefore (X, τ) is an $^{***}{}_{\alpha}T_{1/2}$ space. Let A be a **gα-closed set of (X, τ). Since every **gα-closed set is g-closed, A is a g-closed. Since (X, τ) is a $T_{1/2}$ space, A is closed. Therefore (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$ space.

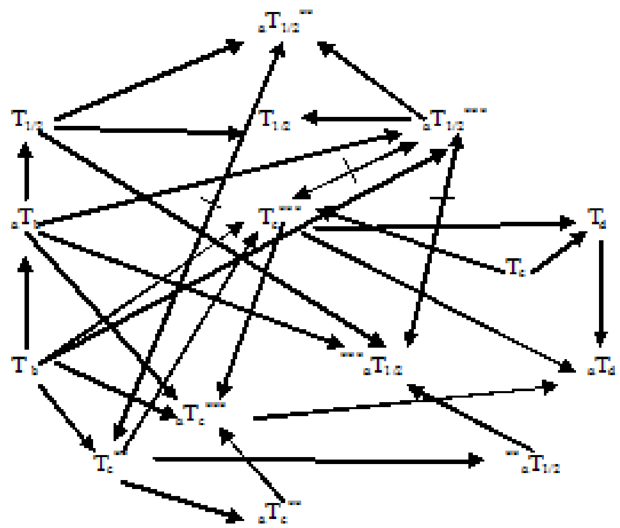
Sufficient part: Let A be a g-closed set of (X, τ). Since (X, τ) is an $^{***}{}_{\alpha}T_{1/2}$ space, A is **gα-closed. Since (X, τ)is an ${}_{\alpha}T_{1/2}^{***}$space, A is closed set. Therefore (X, τ) is a $T_{1/2}$ space.

Remark 4.57: $^{***}{}_{\alpha}T_{1/2}$ space and ${}_{\alpha}T_{1/2}^{***}$ space are independent of each other. It can be seen from the following example

Example 4.58: Let X= {a, b, c} with τ ={X, ϕ, {a},{b, c}};τ^c={X, ϕ , {a}, {b, c}}
g-closed set ={ X, ϕ ,{a},{b},{c},{a, b},{b, c}, {a, c}},**gα-closed set ={ X, ϕ ,{a}, {b, c}}
Here (X, τ) is an ${}_{\alpha}T_{1/2}^{***}$ space but not an$^{***}{}_{\alpha}T_{1/2}$ space. Since {c} is a g-closed set but not **gα-closed set.

Example 4.59: Let X = {a, b, c} with τ ={X, ϕ, {a, c}};τ^c={X, ϕ , {b}},**gα-closed set={ X, ϕ ,{b},{a, b}, {b, c}}
Here (X, τ) is a $^{***}{}_{\alpha}T_{1/2}$ space but not an ${}_{\alpha}T_{1/2}^{***}$space. Since {a, b} is a **gα-closed set but not closed set.

Remark 4.60: The following diagram shows them relationship among the separation axioms considered in this paper and reference [23], [24]. A→B (A↮B) represents A implies B but B need not imply A always. (A and B are independent of each other).



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