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MHD FLOW THROUGH DARCY-BRINKMAN–FORCHHEIMER EXTENDED POROUS MEDIA OVER A NONLINEARSTRETCHING SHEET

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ABSTRACT

In this paper we have studied numerically as well as graphically, the MHD flow through Darcy-Brinkman-Forchheimer extended porous media over a nonlinear stretching sheet, the effect on the flow of magnetic, permeability and Forchheimer parameters respectively (keeping other parameters constant) and the effect of various parameters on skin friction.

Keywords: *MHD flow, Hartmann (magnetic), permeability, Forchheimer parameters.*

I INTRODUCTION:

The origin of the flow through porous media relies heavily upon Darcy's Experimental laws. The validity of this law subject to several limitations. It is shown that it can be possibly valid only in a certain ''seepage'' velocity domain outside which more general flow equations must be used to describe the flow velocity correctly. This happens because inertial effect becomes important (Gulab Ram, R.S.Mishra [1]).

Magnetohydrodynamic (MHD) fluid flow through porous media (Naji Qatanani and Mai Musmar[9]) has wide spread application in industry and environment.

Some examples of its application:
(1). Flow of ground water through soil and rocks (porous medium) is important for agriculture and pollution control.
(2). Extraction of oil and natural gas from rocks is important in oil and natural gas industry.
(3). (a) Functioning of tissues in body (bone, cartilage, and muscle etc being porous media) and flow of blood and nutrients through them;
(b) Understanding various medical conditions (such as tumour growth, a formation of porous medium) and their treatment (such as injection, a flow through porous medium) in medical science.

Gulab Ram, R.S.Mishra [1] studied Unsteady flow through magnetohydrodynamic porous media. C. L. Varshney[2] studied Fluctuating flow of Viscous fluid through a porous medium bounded by a porous plate. Adel.A Megahed[3] studied Unsteady MHD flow through Porous medium bounded by a porous plate. Komal kumar and C.L Varshney [4] studied Viscous flow through a porous medium past an oscillating plate in a rotating system. T.Y.Na, *et al.* [5] studied MHD flow over a moving flat plate with a step change in the magnetic field. O.D. Makinde, E. Osalusi [6] studied MHD steady flow in a channel with slip at the permeable boundaries. Fadzilah Md ALI, *et al.* [7] investigated numerically the problem of unsteady boundary layer flow caused by an impulsively stretching surface with constant viscous flow. Basant K. Jha, M.L. Kaurangini [8] studied New approximate analytical solutions for steady flow in parallel-plates channels filled with porous materials governed by non-linear Brinkman-Forchheimer extended Darcy mode. Naji Qatanani and Mai Musmar[9] studied The Mathematical Structure and Analysis of an MHD Flow in Porous Media. Saeid Abbasbandy, *et al.* [10] studied An approximate solution of the MHD flow over a non-linear stretching sheet by rational Chebyshev Collocation Method. Fadzilah Md Ali, *et al.* [11] studied Dual solutions in MHD flow on a nonlinear porous shrinking sheet in a viscous fluid.

Recently, S. S. Motsa and P. Sibanda [12] studied On the solution of MHD flow over a nonlinear stretching sheet' by an efficient semi-analytical homotopy analysis method and by bvp4c method.

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In the above study, Darcy–For chhiemer effect and Darcy Brinkman effect in porous media are not simultaneously considered

Presently, we study MHD flow through Darcy-Brinkman-Forchheimer extended porous media over a nonlinear stretching sheet, and the effect of magnetic, permeability and Forchheimer parameters respectively (keeping other parameters constant).With it, the effect of various parameters on skin friction is also studied numerically as well as graphically.

II. FORMULATION OF THE PROBLEM

Assumptions [1]:

1. There is steady two-dimensional boundary layer flow from a horizontal impermeable nonlinear stretching sheet through a viscous, incompressible, and electrically conducting fluid.
2. A magnetic field *B(x)* is applied normal to the direction of the flow.
   The induced magnetic field produced by the motion of the electrically conducting fluid is negligible in comparison with the applied magnetic field (this assumption is valid for small magnetic Reynolds numbers).
3. There is no applied electric field so that Hall effects are negligible.
4. The stretching sheet coincides with the plane y=0 as shown in
5. There is a constant pressure, and negligible magnetic pressure.

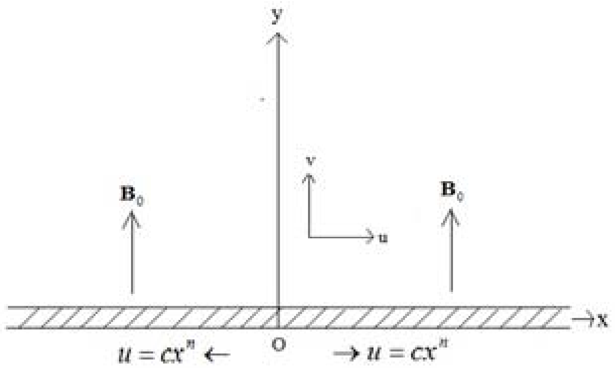

Figure-1. Physical model for MHD flow past over a porous substrate attached to the stretching sheet

Considering above assumptions and Darcy-Brinkman and Darcy-Forchhiemer effects the modified equations of continuity and momentum are

The equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

The Equation of Momentum:

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{K}u-\frac{b}{\sqrt{K}}u^2 \tag{2}$$

where u and υ are the velocity components in the x and y directions, respectively, with the x-axis chosen along the stretching sheet, ν is the kinematic viscosity, ρ is the fluid density, and σ is the electrical conductivity of the fluid. The strength of the spatially varying magnetic field *B(x)* is taken as

$$B(x)=B_0 x^{\frac{(n-1)}{2}} \tag{3}$$

where B_0 is the strength of the magnetic field.

The sheet is assumed to move with a power-law velocity so that the relevant boundary conditions are

$$\begin{aligned} &y=0: u(x,0)=cx^n\,(n>1), \upsilon(x,0)=0,\\ &y\to\infty: u(x,y)\to\infty \end{aligned} \tag{4}$$

where $n>0$ for an accelerated sheet and $n<0$ for a decelerated sheet.

III ANALYSIS

We define

$$\eta = \left(\frac{c(n+1)}{2\nu}\right)^{\frac{1}{2}} x^{\frac{n-1}{2}} y,$$

$$\psi = \left(\frac{2\nu c}{n+1}\right)^{\frac{1}{2}} x^{\frac{n+1}{2}} f(\eta),$$

$$u = \frac{\partial \psi}{\partial y}, \upsilon = -\frac{\partial \psi}{\partial x},$$

$$u = cx^n f'(\eta), (\eta > 1);$$

$$\upsilon = -\left(\frac{c\nu(n+1)}{2}\right)^{\frac{1}{2}} [f(\eta) + \frac{(n-1)}{(n+1)}\eta f'(\eta)] x^{(n-1)/2} \tag{5}$$

where η is similarity variable $f(\eta)$ is the dimensionless stream function u is velocity component parallel to x axis and v is velocity component parallel to y-axis, Using (5) equations (1) and (2) are reduced to the nonlinear differential equation

$$f''' + ff'' - \frac{2n}{n+1}(f')^2 - \frac{1}{n+1}\left(\left(M + \frac{1}{K1}\right)f' + Fs(f')^2\right) = 0 \tag{6}$$

with boundary conditions

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) = 0, \tag{7}$$

where $\beta = \frac{2n}{n+1}$ is the stretching parameter(For a linear stretching sheet that is $n = 1, \beta = 1$), M(the magnetic parameter), K1(Permeability parameter) and Fs (Forchheimer parameter) are defined by

$$M(MagneticParameter) = \frac{2\sigma B_0^2}{\rho c},$$

$$K1(PorosityParameter) = \frac{Kcx^{n-1}}{2\nu}, \tag{8}$$

$$Fs(ForchheimerParameter) = \frac{2bx}{\sqrt{K}},$$

The important physical quantity of interest in this work is the local skin friction coefficient, C_f which, in terms of the transformation variables, is given by

$$C_f = \left(\frac{2(n+1)}{\mathrm{Re}_x}\right)^{\frac{1}{2}} f''(0) \tag{9}$$

where $\mathrm{Re}\, ynoldNumber(\mathrm{Re}_x) = \frac{u_w(x)x}{\nu} = \frac{cx^{n+1}}{\nu}$, is the local Reynold number

IV RESULTS AND DISCUSSION

The numerical solutions are obtained using the above numerical scheme for some values of the governing parameters, namely, the magnetic parameter (M), the permeability parameter (K1), the Forchheimer Parameter (Fs), the stretching parameter (n). Effects of M, K1, Fs and n on the steady boundary layer flow over stretching sheet for some values of n are discussed in detail. The numerical computation is done using the MATLAB in-built numerical solver bvp4c. In the computation we have taken $\eta_\infty = 8$ and axis according to the clear figure-visuality.

In Figure 1 to Figure 5, we have considered graphs f'(η)(velocity profile) with respect to some values of M, K1, Fs and n respectively keeping other parameters constant.

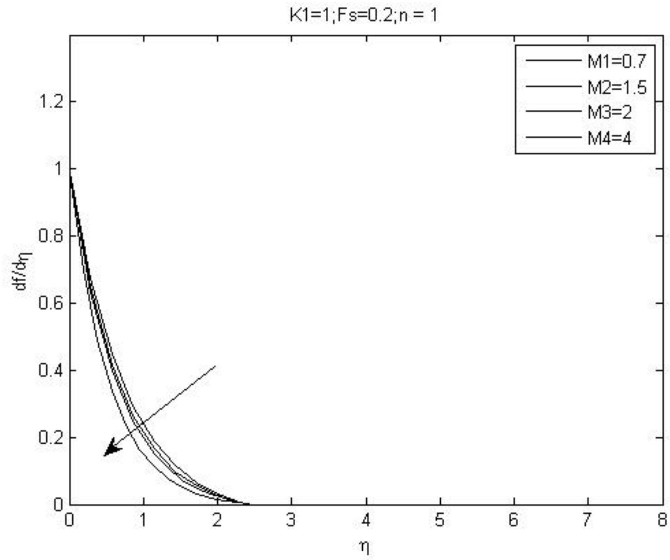

Figure-2. Velocity profile with respect to η for some values of M

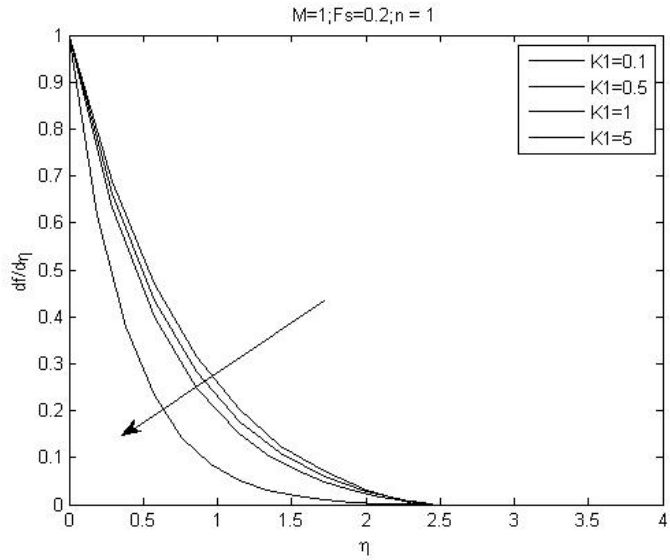

Figure-3. Velocity profile with respect to η for some values of K1

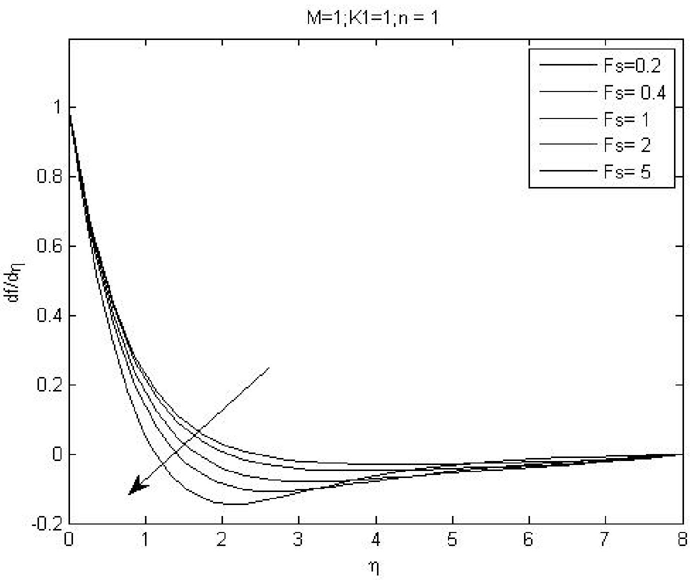

Figure-4. Velocity profile with respect to η for some values of Fs

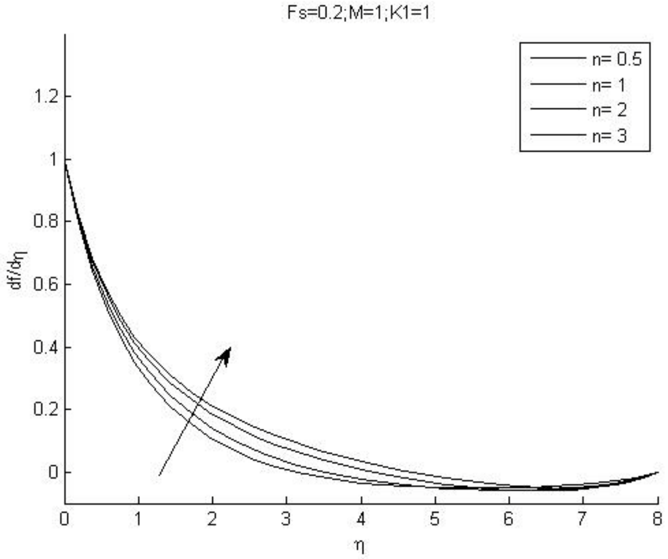

Figure-5. shows velocity profile increases as the value of n increases

Figure 2 shows velocity profile decreases as the value of M increases. Figure 3 shows velocity profile decreases as the value of K1 increases. Figure 4 shows velocity profile decreases as the value of Fs increases. Figure 5 shows velocity profile increases as the value of n increases.

Table-1.

M	f’’(0) n=0.5	f’’(0) ,n=1	f’’(0) ,n=2
0	-1.2413	-1.2555	-1.2685
0.5	-1.3610	-1.3456	-1.3288
1.0	-1.4736	-1.4317	-1.3875
1.5	-1.5795	-1.5140	-1.4446
2.0	-1.6796	-1.5927	-1.4998
2.5	-1.7745	-1.6680	-1.5534
3.0	-1.8650	-1.7404	-1.6054
3.5	-1.9514	-1.8100	-1.6559
4.0	-2.0343	-1.8772	-1.7050
4.5	-2.1141	-1.9421	-1.7528

Table-2.

Fs	f’’(0) n=0.5	f’’(0) n=1	f’’(0) n=2
0.5	-1.4977	-1.4537	-1.4064
1.0	-1.5310	-1.4838	-1.4322
1.5	-1.5593	-1.5092	-1.4539
2.0	-1.5841	-1.5316	-1.4730
2.5	-1.6066	-1.5518	-1.4901
3.0	-1.6272	-1.5703	-1.5059
3.5	-1.6462	-1.5875	-1.5205
4.0	-1.6640	-1.6035	-1.5341
4.5	-1.6808	-1.6186	-1.5470
5.0	-1.6967	-1.6329	-1.5591

Table-3.

K1	f''(0), n=0.5	f''(0), n=1	f''(0) ,n=2
0.5	-1.6796	-1.5927	-1.4998
1.0	-1.4736	-1.4317	-1.3875
1.5	-1.3993	-1.3747	-1.3485
2.0	-1.3610	-1.3456	-1.3288
2.5	-1.3377	-1.3279	-1.3168
3.0	-1.3219	-1.3160	-1.3088
3.5	-1.3106	-1.3074	-1.3031
4.0	-1.3021	-1.3010	-1.2988
4.5	-1.2954	-1.2960	-1.2954
5.0	-1.2901	-1.2920	-1.2927

The values of f''(0) for some values of n,K1,Fs,and M are tabulated in Table 1, Table 2, Table 3. These are used to draw the graphs for f''(0) with respect to some values of M,K1,and Fs (keeping values of n=0.5, 1, 2 as constant) in Figures 6, 7and 8 respectively.

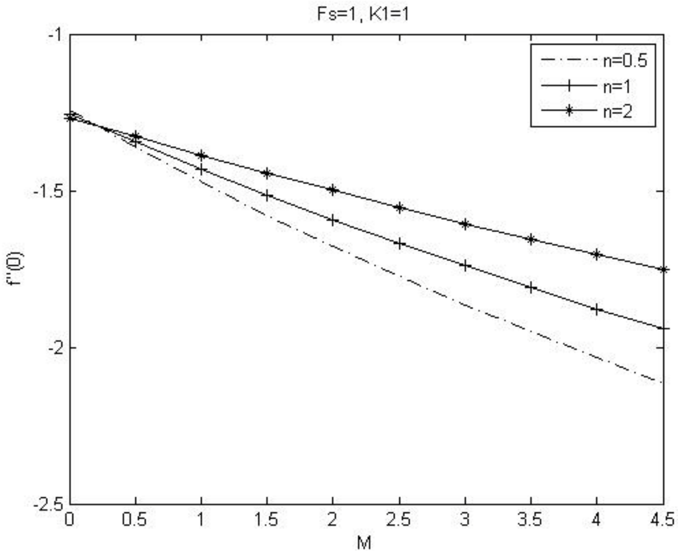

Figure-6. Variation of the skin friction *f* ''(0) with n for each value of M

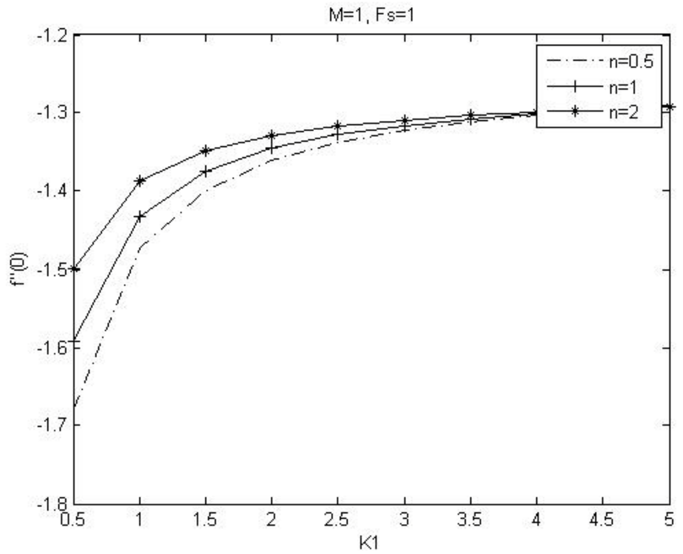

Figure-7. Variation of the skin friction *f* ''(0) with n for each value of K1

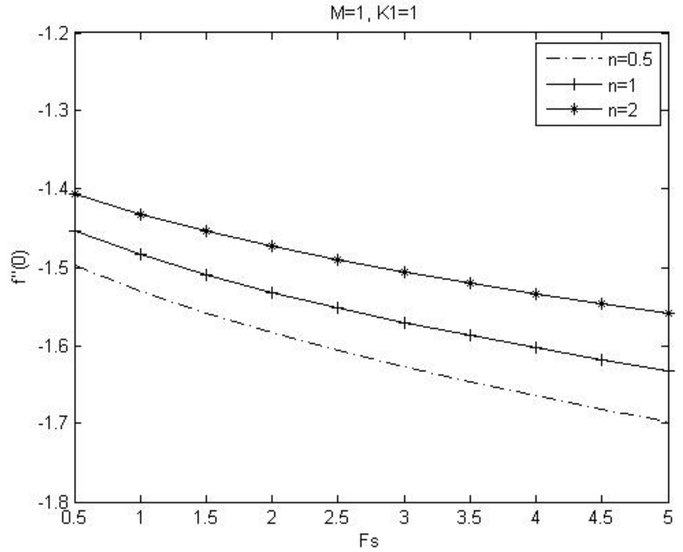

Figure-8. Variation of the skin friction *f* ''(0) with n for each value of Fs

Figures 6 shows the effects of the Hartmann number M and the stretching parameter n on the variation of the skin friction *f* ''(0) on the surface of the sheet. The skin friction is larger for small Hartmann numbers but for large and positive values of n the magnetic field appears to have decreasing effect on the skin friction values as the slop of the curves increase. For large and positive values of n (and for constant magnetic field) the value of skin friction increases.

Figures 7 shows the effects of the permeability parameter K1 and the stretching parameter n on the variation of the skin friction *f* ''(0) on the surface of the sheet. The skin friction is smaller for small permeability parameter but for large and positive values of n the permeability parameter appears to have decreasing effect on the skin friction values. For large and positive values of n (and for constant permeability parameter) the value of skin friction increases. As K1 approaches infinity, skin friction becomes constant.

Figures 8 shows the effects of the Forchheimer parameter Fs and the stretching parameter n on the variation of the skin friction *f* ''(0) on the surface of the sheet. The skin friction is larger for small Forchheimer parameter Fs and for large and positive values of n the Forchheimer parameter appears to have increasing effect on the skin friction values. For large and positive values of n (and for constant Forchheimer parameter Fs) the value of skin friction increases. As Fs approaches infinity skin friction becomes smaller and smaller.

V. CONCLUSION

In this we have studied MHD flow through Darcy-Brinkman–Forchheimer extended porous media over a nonlinear stretching sheet. We have found that numerically tabulated values and figures show same results: Velocity profile decreases as the value of M (K1, and Fs respectively) increases keeping other parameters as constant.
Velocity profile increases as the value of n increases keeping other parameters as constant.

PROPOSED WORK

In the equation of momentum we can consider
1. The term of porosity.
2. Variability in porosity.
3. Variability in viscosity.
4. Variability in magnetic parameter.
5. Variability in permeability.

COMPETING INTERESTS

The authors declare that they have no competing interests.

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