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EVALUATION OF RELIABILITY OF A PARALLEL SYSTEM WITH MULTI COMMON-CAUSE AND CRITICAL HUMAN ERRORS-GENERAL AND SPECIAL CASES

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ABSTRACT

In this paper, reliability and availability analyses of a parallel system with common-cause failures and critical human errors are discussed.

***Keywords**: Parallel system, multi common-cause, critical human errors and special cases.*

1. INTRODUCTION

Redundancy can generally be used to increase the reliability of a system without any change in the reliability of the individual units that form the system. The parallel configuration is one form of redundancy. This form is often used to improve system reliability. However, in conventional reliability and availability analyses of parallel systems, the occurrence of Common-cause failures and Critical human errors is not taken into consideration. Their impact on a system's reliability is much greater than many of us realize.

A Common-cause failure is defined as an instance where multiple units fail due to a single cause. Some of the Common-cause failures may due to [1] equipment design deficiencies, operations and maintenance errors, external normal environment, external catastrophe, functional deficiency, common manufacturer and common power source etc.

This paper separates Common-cause failures into two categories i.e, Common-cause failures resulting from human error and Common-cause failures resulting from non-human error. The Common-cause failures resulting from human error are called Critical human errors many studies [59-60] have indicated that a significant percentage of system failures are due to human error. Some of the similar models are presented in references [2-6]. The reliability and availability analyses of a parallel system with common-cause failures and critical human errors are discussed. The system may fail either due to a common-cause failure or a critical human error or when all units fail. It is assumed that when only one unit operates, a common-cause failure or a critical\human errors occurring (i,e, when one unit is operating) are categorized as common-cause failures\critical human error, the ones which may have led to a total system failure, if there have been more than one unit working normally in the system.

2. ASSUMPTIONS

1. Failures are statistically independent.
2. All systems units are active, identical and form a parallel network.
3. A unit failure rate is constant.
4. A Common Cause failure or a critical human error leads to a system failure.
5. A Common Cause failure or a critical human error can occur when one or more units are operating.
6. Critical human error and Common-cause failure rate are constant.
7. Failed system repair rates are constant
8. At least one unit must operate normally for the system's success.

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3. NOTATIONS

n- Number of units in the parallel system

λ- Constant failure rate of a unit

i−System upstate

i=0 (all n units operating normally)

i=1 (one unit failed, (n-1) operating)

i=2 (two units failed, (n-2) operating)

i=3 (three units failed, (n-3) operating)

i=k (k units failed, (n-k) operating)

k-Number of failed units in the system and corresponding upstate of the system for k=0, 1, 2,... (n-1).

λ_{c1i},λ_{c2i},λ_{c3i}- constant critical human error rate from system upstate i $for\ i$ =1,2,...k.

λ_{h1i},λ_{h2i},λ_{h3i} - constant critical human error rate from system up state i=1,2,3, ...k.

j- System down state.

j= n (all units failed due to a common–cause failure or a critical human error).

j=c_1,c_2,c_3, c_k - system failed due to common-cause failure.

j=h_1,h_2, h_3,h_k - system failed due to critical human error.

p_i(t)- Probability that the system is in upstate i at time t for i=0, 1, 2,.....k.

p_j(t)- Probability that the system is in downstate j at time t for j=n, c_1,c_2, c_3, $c_k$$h_1$,$h_2$, h_3,h_k

p_i- Steady state probability that the system is in upstate i for i=0, 1, 2, ...k.

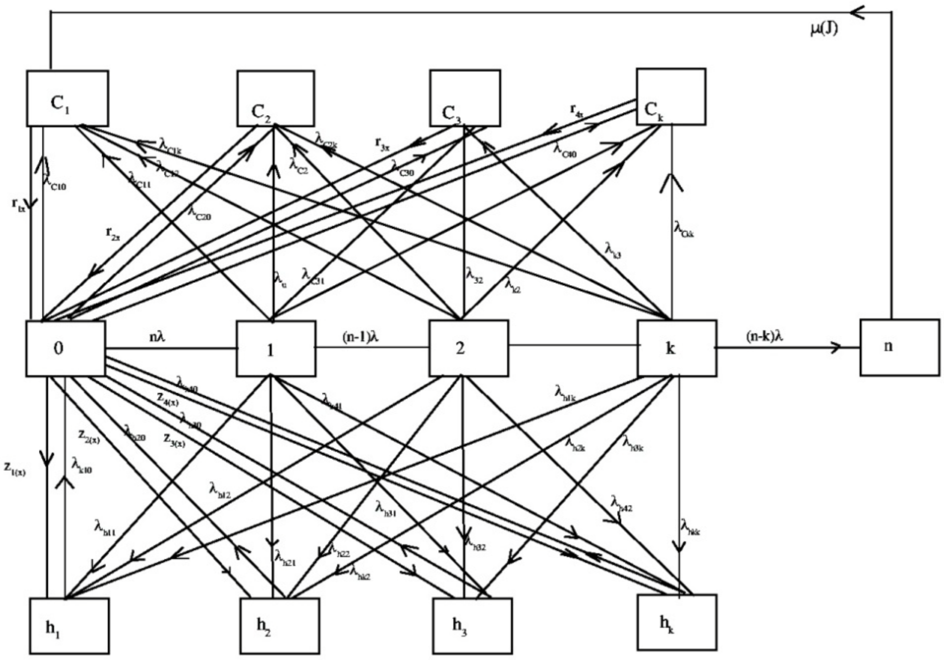
p_j -steady state probability that the system is in downstate j for j=n, c_1, c_2, c_3, c_k, h_1, h_2, h_3,h_k
$r_1(x)$, r_2(x),r_3(x),r_k(x),$p_{c1}(x)$,p_{c2}(x),p_{c3}(x),p_{ck}(x)-Repair rates and probability density functions of repair times respectively when the failed system is in states c_1,c_2, c_3, c_k and has an elapsed repair time of X.

$z(x)$, z(x),z_3(x),z(x),$h_{c1}(x)$,p_{h2}(x),p_{h3}(x),p_{hk}(x)-Repair rates and probability density functions of repair times respectively when the failed system is in states h_1,h_2, h_3,h_k and has an elapsed repair time of X.

μ(x),p_n(x)- Repair rates and probability density functions of repair times respectively, when the failed system is in state n and elapsed repair time of X.

s-Laplace Transform Variable.

μ,r_1,r_2,r_3,r_k,z_1,z_2,z_3,z_k- constant repair rates from failed system states n, c_1, c_2, c_3, $c_k$$h_1$,$h_2$, h_3,h_k respectively.



4. GENERAL MODEL

Using supplementary variable method, the system of differential equations and boundary conditions associated with fig.1 are

$$\frac{d}{dt}p_0(t)(t)+(\lambda c_{10}+\lambda h_{10} + n\lambda + \lambda c_{20} + \lambda h_{20} + \lambda c_{30}+ \lambda h_{30})p_0(t)$$
$$=\int_0^\infty p_{c1}(x,t)r_1(x)dx + \int_0^\infty p_{c2}(x,t)r_2(x)dx + \int_0^\infty p_{c3}(x,t)r_3(x)dx$$
$$+ \int_0^\infty p_{ck}\,(x,t)r_k(x)dx + \int_0^\infty p_{h1}(x,t)z_1(x)dx$$
$$+ \int_0^\infty p\,h_2(x,t)z_2(x)dx + \int_0^\infty ph_3\,(x,t)z_3(x)dx + \int_0^\infty p_{hk}\,(x,t)z_k(x)dx. \quad (1)$$

$$\frac{d}{dt}p_1(t) + (\lambda c_{11}+\lambda h_{11}+ (n-1)\lambda+\lambda c_{21}+\lambda h_{21} +\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+ \lambda h_{k1})p_1(t)=p_0(t)n\lambda \quad (2)$$

$$\frac{d}{dt}p_2(t) + (\lambda c_{12}+\lambda h_{12}+ (n-2)\lambda+\lambda c_{22}+\lambda h_{22}+\lambda c_{32}+\lambda h_{32}+\lambda c_{k2}+\lambda h_{k2})p_2(t)= p_1(t)(n-1)\lambda \quad (3)$$

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$$\frac{d}{dt}p_k(t) + (\lambda c_{1k}+\lambda h_{1k}+ (n-k)\lambda+\lambda c_{2k}+\lambda h_{2k}+\lambda c_{3k}+\lambda h_{3k} +\lambda c_{kk}+\lambda h_{kk})p_k(t)=p_{k-1}(t)(n-k+1)\lambda \quad (4)$$

For k=1, 2, 3.................. (n-1).

$$d/dt\,p_n(x, t) + d/dx\,p_n(x, t) + \mu(x)p_n(x, t) = 0 \quad (5)$$

$$d/dt\,p_{c1}(x, t)+d/dx\,p_{c1}(x, t) + r_1(x)p_{c1}(x, t) = 0 \quad (6)$$

$$d/dt\,p_{c2}(x,t)+d/dx\,p_{c2}(x,t)+r_2(x)p_{c2}(x, t)=0 \quad (7)$$

$$d/dt\,p_{c3}(x,t)+d/dx\,p_{c3}(x,t)+r_3(x)p_{c3}(x, t)=0 \quad (8)$$

$$d/dt\,p_{ck}(x,t)+ d/dx\,p_{ck}(x,t)+ r_k\,(x)\,p_{ck}(x, t)=0 \quad (9)$$

$$d/dt\,p_{h2}(x,t)+ d/dx\,p_{h2}(x,t)+ z_2\,(x)p_{h2}(x, t)=0 \quad (11)$$

$$d/dt\,p_{h3}(x,t)+ d/dx\,p_{h3}(x,t)+ z_3\,(x)p_{h3}(x, t)=0 \quad (12)$$

$$d/dt\,p_{hk}(x,t)+d/dx\,p_{hk}(x,t)+z_k(x)p_{hk}(x, t)=0 \quad (13)$$

$$p_n(0, t)= (n-k)\lambda\, p_k(t) \quad (14)$$

$$p_{c1}(0, t)=\sum_{i=0}^{k} p\text{i}\,(t)\lambda\, c_{1i} \quad (15)$$

$p_{c2}(0, t)=\sum_{i=0}^{k} p\text{i}(t) \curlywedge c_{2i}$ (16)

$p_{c3}(0,t) = \sum_{i=0}^{k} pi \curlywedge c_{3i}$ (17)

$p_{ck}(0, t)=\sum_{i=0}^{k} pi \curlywedge c_{ck}$ (18)

$p_{h1}(0, t)=\sum_{i=0}^{k} pi \curlywedge h_{1i}$ (19)

$p_{h2}(O,t)=\sum_{i=0}^{k} pi \curlywedge h_{2i}$ (20)

$p_{h3}(0, t)=\sum_{i=0}^{k} pi \curlywedge h_{3i}$ (21)

$p_{hk}(0, t)=\sum_{i=0}^{k} pi \curlywedge h_{ki}$ (22)

$p_i(0)=1$ for $i=0$

$p_i(0)=0$ for $i=0$

$p_j(x,0)=0$ for $j= n, c_1, c_2, c_3, c_k, h_1, h_2, h_3, h_k$.

Solving above Equations using Laplace transforms leads to the following Laplace transforms of the probabilities.

$$sp_0(s) - 1 + (\curlywedge c_{10} + \curlywedge h_{10} + \curlywedge c_{20} + \curlywedge h_{20} + \curlywedge c_{30} + \curlywedge h_{30} + \curlywedge c_{k0} + \curlywedge h_{k0})p_0(s)$$
$$= \int_0^\infty pc_1(x,s)r_1(x)\,dx + \int_0^\infty p\, c_2(x,s)r_2(x)dx + \int_0^\infty p\, c_3(x,s)r_3(x)dx + \int_0^\infty p\, c_k(x,s)r_k(x)$$
$$+ \int_0^\infty p\, h_1(x,s)z_1(x)dx + \int_0^\infty p\, h_2(x,s)z_2(x) + \int_0^\infty p\, h_3(x,s)z_3(x) + \int_0^\infty p\, h_k(x,s)z_k(x)dx + \int_0^\infty p\, n(x,s)\mu\, dx \quad (23)$$

$Sp_1(s) + (\curlywedge c_{11} + \curlywedge h_{11} + (n-1)\curlywedge + \curlywedge c_{21} + \curlywedge h_{21} + \curlywedge c_{31} + \curlywedge h_{31} + \curlywedge c_{k1} + \curlywedge h_{k1})\, p_1(s) = p_0(s)n\curlywedge$ (24)

$Sp_2(s) + (\curlywedge c_{12} + \curlywedge h_{12} + (n-2)\curlywedge + \curlywedge c_{22} + \curlywedge h_{22} + \curlywedge c_{32} + \curlywedge h_{32} + \curlywedge c_{k2} + \curlywedge h_{k2})p_2(s) = p_1(s)(n-1)\curlywedge$ (25)

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$Sp_k(s) + (\curlywedge c_{1k} + \curlywedge h_{1k} + (n-k)\curlywedge c_{2k} + \curlywedge h_{2k} + \curlywedge c_{3k} + \curlywedge h_{3k} + \curlywedge c_{kk} + \curlywedge h_{kk})p_k(s)=p_{k-1}(s)(n-k+1)\curlywedge$ (26)

$[S + d/dx + \mu(x)]p_n(x, s)=0$ (27)

$[S + d/dx + r_1(x)]p_{c1}(x, s)=0$ (28)

$[S + d/dx + r_2(x)]p_{c2}(x, s)=0$ (29)

$[S + d/dx + r_3(x)]p_{c3}(x, s)=0$ (30)

$[S + d/dx + r_k(x)]p_{ck}(x, s)=0$ (31)

$[S + d/dx + h_1(x)]p_{h1}(x, s)=0$ (32)

$[S + d/dx + h_2(x)]p_{h2}(x, s)=0$ (33)

$[S + d/dx + h_3(x)]p_{h3}(x, s)=0$ (34)

$[S + d/dx + h_k(x)]p_{hk}(x, s)=0$ (35)

$p_n(0,s)=(n-k)\curlywedge p_k(s) = 0$ (36)

$p_{c1}(0, s) = \sum_{i=0}^{k} pi(s) \curlywedge c_{1i}$ (37)

$p_{c2}(0, s) = \sum_{i=0}^{k} p\text{i}(s) \curlywedge c_{2i}$ (38)

$p_{c3}(0,s) = \sum_{i=0}^{k} pi(s) \curlywedge c_{3i}$ (39)

$p_{ck}(0, s) = \sum_{i=0}^{k} p\text{i}(s) \curlywedge c_{ki}$ (40)

p_{h1}(o, s) $=\sum_{i=0}^{k} pi(s) \lambda\, h_{1i}$ (41)

p_{h2}(0, s) $=\sum_{i=0}^{k} pi\,(s) \lambda\, h_{2i}$ (42)

p_{h3}(0, s) $=\sum_{i=0}^{k} pi$(s)$\lambda\, h_{3i}$ (43)

p_{hk}(s) $=\sum_{i=0}^{k} pi$(s)$\lambda\, h_{ki}$ (44)

From equation (24)
(s $+\lambda\, c_{11}+\lambda\, h_{11}+$ (n-1)$\lambda+\lambda\, c_{21}+\lambda\, h_{21}+\lambda\, c_{31}+\lambda\, h_{31}+\lambda\, c_{k1}+\lambda\, h_{k1})p_1$(s) $=p_0$(s)

nλ p_1(s) $=\frac{p_0(s)n\lambda}{s+\lambda_{c11}+\lambda_{h11}+(n-1)\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1}}$ (45)

From equation (25)
p_2(s)(s $+\lambda\, c_{12}+\lambda\, h_{12}+$(n-2)$\lambda+\lambda\, c_{22}+\lambda\, h_{22}+\lambda\, c_{32}+\lambda\, h_{32}+\lambda\, c_{k2}+\lambda\, h_{k2}$)$=p_1$(s)(n-1)$\lambda$

p_2(s)$=\frac{p_1(s)(n-1)\lambda}{s+\lambda_{c11}+\lambda_{h11}+(n-2)\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1}}$ (46)
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$p_k(s) = \frac{\prod_{i=0}^{k}(n-i)\lambda^k p_0(s)}{\prod_{i=0}^{k}(s+\lambda_{c1i}+\lambda_{h1i}+(n-i)+\lambda_{c2i}+\lambda_{h2i}+\lambda_{c3i}+\lambda_{h3i}+\lambda_{ck1i}+\lambda_{hk1i}}$ (47)

For k=1, 2, 3,.......(n-1)

From equation (27)
$d/dx\, p_n$(x,s) = - (s+μ(x))p_n(x,s)

$\frac{d/dx\, p_n(x,s)}{p_n(x,s)}$ = - (s+μ(x))

Integrating both sides
$\log[p_n(x,s)] = -sx - \int_0^x \mu$(x)dx

$\log\left[\frac{p_n(x,s)}{p_n(0,s)}\right] = -sx - \int_0^x \mu$(x)dx

$\frac{p_{c1}(x,s)}{p_{c1}(0,s)} = e^{-sx-\int_0^x \mu(x)dx}$

p_n(x, s) = p_n(0,s)$e^{-sx-\int_0^x \mu(x)dx}$

p_n(x, s) = (n-k)λ p_k(s)$e^{-sx-\int_0^x \mu(x)dx}$ (48)

From (36)

From equation (28)
$d/dx p_{c1}$(x, s) = - (s+r_1(x))p_{c1}(x,s)

$\frac{d/dx\, p_{c1}(x,s)}{p_{c1}(x,s)}$ = - (s+r_1(x))

Integrating both sides
$\log[p_{c1}(x,s)] = -sx - \int_0^x r_1$(x)dx

$\log\left[\frac{p_{c1}(x,s)}{p_{c1}(0,s)}\right] = -sx - \int_0^x r1$(x) dx

$\frac{p_{c1}(x,s)}{p_{c1\,(x,s)}} = e^{-sx-\int_0^x r1(x)dx}$

$p_{c1}(x,s) = p_{c1}(0,s)e^{-sx-\int_0^x r1(x)dx}$

$p_{c1}(\mathrm{x, s}) = \sum_{i=0}^{k} p_i(\mathrm{s})\lambda_{c1i}\, e^{-sx-\int_0^x r1(x)dx}$ (49)

$p_{c2}(x,s) = \sum_{i=0}^{k} pi(s)\, \lambda_{c2i}\, e^{-sx-\int_0^x r2(x)dx}$ (50)

$p_{c3}(\mathrm{x, s}) = \sum_{i=0}^{k} pi(s) \lambda\, c_{3i}\, e^{-sx-\int_0^x r3(x)dx}$ (51)

$p_{ck}(\mathrm{x, s}) = \sum_{i=0}^{k} pi(s) \lambda\, c_{ki}\, e^{-sx-\int_0^x rk(x)dx}$ (52)

$p_{h1}(\mathrm{x, s}) = \sum_{i=0}^{k} pi(s) \lambda\, h_{1i}\, e^{-sx-\int_0^x z1(x)dx}$ (53)

$p_{h2}(\mathrm{x. s}) = \sum_{i=0}^{k} pi(s) \lambda\, h_{2i}\, e^{-sx-\int_0^x z2(x)dx}$ (54)

$p_{h3}(\mathrm{x, s}) = \sum_{i=0}^{k} pi(s) \lambda\, h_{3i}\, e^{-sx-\int_0^x z3(x)dx}$ (55)

$p_{hk}(\mathrm{x, s}) = \sum_{i=0}^{k} pi(s) \lambda\, h_{ki}\, e^{-sx-\int_0^x zk(x)dx}$ (56)

Substituting in equation (23)

$$\begin{aligned}
&\mathrm{Sp_0(s)\text{-}1+ (\lambda\, c_{10}+\lambda\, h_{10}+n\lambda+\lambda\, c_{20}+\lambda\, h_{20}+\lambda\, c_{30}+\lambda\, h_{30}+\lambda\, c_{k0}+\lambda\, h_{k0})p_0(s)}\\
&=\int_0^\infty \sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{c_{1i}}\, e^{-sx-\int_0^x r1(x)dx} \mathrm{r_1(x)dx}+\int_0^\infty \sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{c_{2i}}\, e^{-sx-\int_0^x r2(x)dx} \mathrm{r_2(x)dx}\\
&+\int_0^\infty \mathrm{pi(s)} \lambda\, \mathrm{c_{3i}}\, e^{-sx-\int_0^x r3(x)dx} \mathrm{r_3(x)dx}+\int_0^\infty \sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{c_{ki}}\, e^{-sx-\int_0^x rk(x)dx} \mathrm{r_k(x)dx}\\
&+\int_0^\infty \sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{h_{1i}}\, e^{-sx-\int_0^x z1(x)dx} \mathrm{z_1(x)dx}+\int_0^\infty \sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{h_{2i}}\, e^{-sx-\int_0^x z2(x)dx} \mathrm{z_2(x)dx}\\
&+\int_0^\infty \sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{h_{3i}}\, e^{-sx-\int_0^x z3(x)dx} \mathrm{z_3(x)dx}+\int_0^\infty \sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{h_{ki}}\, e^{-sx-\int_0^x zk(x)dx} \mathrm{z_k(x)dx}\\
&+\int_0^\infty (\mathrm{n}-\mathrm{k}) \lambda\, \mathrm{p_k(s)} e^{-sx-\int_0^x \mu(x)dx} \mu(\mathrm{x})\mathrm{dx} \qquad (57)
\end{aligned}$$

$$\begin{aligned}
&Sp_0(s)\text{-}1+(\lambda\, c_{10}+\lambda\, h_{10}+n\lambda+\lambda\, c_{20}+\lambda\, h_{20}+\lambda\, c_{30}+\lambda\, h_{30}+\lambda\, c_{k0}+\lambda\, h_{k0})p_0(s)\\
&=\sum_{i=0}^{k} pi(s) \lambda\, c_{1i} \int_0^\infty e^{-sx-\int_0^x r1(x)dx}\, r_1(x)dx+\sum_{i=0}^{k} pi(s) \lambda\, c_{2i} \int_0^\infty e^{-sx-\int_0^x r2(x)dx}\, r_2(x)dx\\
&+\sum_{i=0}^{k} pi(s) \lambda\, c_{3i} \int_0^\infty e^{-sx-\int_0^x r3(x)dx}\, r_3(x)dx+\sum_{i=0}^{k} pi(s) \lambda\, c_{ki} \int_0^\infty e^{-sx-\int_0^x rk(x)dx}\, r_{ki}(x)dx\\
&+\sum_{i=0}^{k} pi(s) \lambda\, h_{1i} \int_0^\infty e^{-sx-\int_0^x z1(x)dx}\, z_1(x)dx+\sum_{i=0}^{k} pi(s) \lambda\, h_{2i} \int_0^\infty e^{-sx-\int_0^x z2(x)dx}\, z_2(x)dx\\
&+\sum_{i=0}^{k} pi(s) \lambda\, h_{3i} \int_0^\infty e^{-sx-\int_0^x z3(x)dx}\, z_3(x)dx+\sum_{i=0}^{k} pi(s) \lambda\, h_{ki} \int_0^\infty e^{-sx-\int_0^x zk(x)dx}\, z_k(x)dx.
\end{aligned}$$

$$\begin{aligned}
&\mathrm{(S+\lambda\, c_{10}+\lambda\, h_{10}+n\lambda+\lambda\, c_{20}+\lambda\, h_{20}+\lambda\, c_{30}+\lambda\, h_{30}+\lambda\, c_{k0}+\lambda\, h_{k0})p_0(s)}\\
&=1+\sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{c_{1i}} \int_0^\infty e^{-sx-\int_0^x r1(x)dx}\, \mathrm{r_1(x)dx}+\sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{c_{2i}} \int_0^\infty e^{-sx-\int_0^x r2(x)dx}\, \mathrm{r_2(x)dx}\\
&+\sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{c_{3i}} \int_0^\infty e^{-sx-\int_0^x r3(x)dx}\, \mathrm{r_3(x)dx}+\sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{c_{ki}} \int_0^\infty e^{-sx-\int_0^x rk(x)dx}\, \mathrm{r_{ki}(x)dx}\\
&+\sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{h_{1i}} \int_0^\infty e^{-sx-\int_0^x z1(x)dx}\, \mathrm{z_1(x)dx}+\sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{h_{2i}} \int_0^\infty e^{-sx-\int_0^x z2(x)dx}\, \mathrm{z_2(x)dx}\\
&+\sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{h_{3i}} \int_0^\infty e^{-sx-\int_0^x z3(x)dx}\, \mathrm{z_3(x)dx}\\
&+\sum_{i=0}^{k} \mathrm{pi(s)} \lambda\, \mathrm{h_{ki}} \int_0^\infty e^{-sx-\int_0^x zk(x)dx}\, \mathrm{z_k(x)dx+(n\text{-}k)}\lambda\, \mathrm{p_k(s)}\int_0^\infty e^{-sx-\int_0^x \mu(x)dx} \mu(\mathrm{x})\mathrm{dx} \qquad (58)
\end{aligned}$$

5. PARTICULAR CASES

CASE-I: Put n=2 in equation (45)

$$p_1(\mathrm{s})=\frac{2p_0(s)\lambda}{s+\lambda_{c11}+\lambda_{h11}+\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1}} \qquad (59)$$

From equation (49)

$$\begin{aligned}
\sum_{i=0}^{k} pi(s) \lambda\, c_{1i} \int_o^\infty e^{-sx-\int_0^x r1(x)dx}\, dx &= \sum_{i=0}^{k} pi(s) \lambda\, c_{1i} \frac{1}{s+r_1}\\
&=\left[p_o(s) \lambda\, c_{10} + \sum_{i=1}^{k} p_i(s) \lambda\, c_{1i}\right]\frac{1}{s+r_1}\\
&=\left[p_0(s) \lambda\, c_{10} + \sum_{i=1}^{k} p_1(s) \lambda\, c_{11}\right]\frac{1}{s+r_1}
\end{aligned}$$

$$=\left[p_0(s)\,\lambda_{c10}+\frac{2\lambda p_0(s)\lambda_{c11}}{s+\lambda c_{11}+\lambda h_{11}+n\lambda+\lambda c_{21}+\lambda_{h21}+\lambda c_{31}+\lambda 1+\lambda_{k1}+\lambda h_{k1}}\right]\frac{1}{s+r_1}$$
$$=\left[\lambda\, c_{10}+\frac{2\lambda\lambda_{c11}}{s+\lambda c_{11}+\lambda h_{11}+n\lambda+\lambda c_{21}+\lambda_{h21}+\lambda c_{31}+\lambda 1+\lambda_{k1}+\lambda h_{k1}}\right]\frac{p_0(s)}{s+r_1}$$
$$=p_{c1}(\mathrm{s}) \tag{60}$$

Again from equation (50)

$$\sum_{i=0}^{k} pi(s)\,\lambda\, c_{2i}\int_0^\infty e^{-sx-\int_0^x r2(x)dx}\,\mathrm{dx}=\left[\lambda_{c20}+\frac{2\lambda\lambda_{c21}}{s+\lambda c_{11}+\lambda h_{11}+n\lambda+\lambda c_{21}+\lambda_{h21}+\lambda c_{31}+\lambda 1+\lambda_{k1}+\lambda h_{k1}}\right]\frac{p_0(s)}{s+r_1}$$
$$=p_{c2}(\mathrm{s}) \tag{61}$$

From equation (48)

$$p_n(\mathrm{x,\,s})=(\mathrm{n-k})\lambda\, p_k(\mathrm{s})\int_0^\infty e^{-sx-\int_0^x \mu(x)dx}\,\mathrm{dx}$$
$$=(\mathrm{n-k})p_k(\mathrm{s})\frac{1}{s+\mu}$$
$$=\frac{(\mathrm{n-k})\lambda\prod_{i=0}^{k-1}(2-i)\lambda^k p_0(\mathrm{s})}{\prod_{i=0}^{k}(s+\lambda c_{1i}+\lambda h_{1i}+(2-\mathrm{i})\lambda+\lambda c_{2i}+\lambda h_{2i}+\lambda c_{3i}+\lambda h_{3i}+\lambda c_{ki}+\lambda h_{ki})}\,\frac{1}{s+\mu}$$
$$=\frac{2\lambda^2 p_0(\mathrm{s})}{(s+\lambda c_{11}+\lambda h_{11}+\lambda+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1})}\,\frac{1}{s+\mu}$$

Substituting these values in (58)

$$(\mathrm{S}+\lambda\, c_{10}+\lambda\, h_{10}+2\lambda+\lambda\, c_{20}+\lambda\, h_{20}+\lambda\, c_{30}+\lambda\, h_{30}+\lambda\, c_{k0}+\lambda\, h_{k0})p_0(\mathrm{s})$$
$$=1+\left[\frac{2\lambda^2}{s+\lambda c_{11}+\lambda h_{11}+\lambda c_{21}+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1}}\right]\frac{\mu p_0(s)}{s+\mu}$$
$$+\left[\lambda\, c_{10}+\frac{2\lambda\lambda c_{11}}{s+\lambda c_{11}+\lambda h_{11}+\lambda c_{21}+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1}}\right]\frac{r_1 p_0(s)}{s+r_1}+\left[\lambda\, c_{20}+\frac{2\lambda\lambda c_{21}}{s+\lambda c_{11}+\lambda h_{11}+\lambda c_{21}+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1}}\right]\frac{r_2 p_0(s)}{s+r_2}+\left[\lambda\, c_{30}+\frac{2\lambda\lambda c_{31}}{s+\lambda c_{11}+\lambda h_{11}+\lambda c_{21}+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1}}\right]\frac{r_3 p_0(s)}{s+r_3}+\left[\lambda\, c_{k0}+\frac{2\lambda\lambda c_{k1}}{s+\lambda c_{11}+\lambda h_{11}+\lambda c_{21}+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1}}\right]\frac{r_k p_0(\mathrm{s})}{\mathrm{s}+r_k}+\left[\lambda\, c_{h10}+\frac{2\lambda\lambda c_{h11}}{s+\lambda c_{11}+\lambda h_{11}+\lambda c_{21}+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1}}\right]\frac{z_1 p_0(\mathrm{s})}{\mathrm{s}+z_1}+\left[\lambda\, c_{h20}+\frac{2\lambda\lambda c_{h21}}{s+\lambda c_{11}+\lambda h_{11}+\lambda c_{21}+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1}}\right]\frac{z_2 p_0(\mathrm{s})}{\mathrm{s}+z_2}+\left[\lambda\, c_{h30}+\frac{2\lambda\lambda c_{h31}}{s+\lambda c_{11}+\lambda h_{11}+\lambda c_{21}+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1}}\right]\frac{z_3 p_0(\mathrm{s})}{\mathrm{s}+z_3}+\left[\lambda\, c_{hk0}+\frac{2\lambda\lambda c_{hk1}}{s+\lambda c_{11}+\lambda h_{11}+\lambda c_{21}+\lambda h_{21}+\lambda c_{31}+\lambda h_{31}+\lambda c_{k1}+\lambda h_{k1}}\right]\frac{z_k p_0(\mathrm{s})}{\mathrm{s}+z_k}.$$

$$(\mathrm{s}+\lambda_{c10}+\lambda_{h10}+2\,\lambda+\lambda_{c20}+\lambda_{h20}+\lambda_{h30}+\lambda_{c30}+\lambda_{ck0}+\lambda_{hk0})$$

$$p_0(\mathrm{s})=1+\mu p_2(\mathrm{s})+r_1 p_{c1}(\mathrm{s})+r_2 p_{c2}(\mathrm{s})+r_3 p_{c3}(\mathrm{s})+r_k p_k(s)+z_1 p_{h1}(\mathrm{s})+z_2 p_{h2}(\mathrm{s})+z_3 p_{h2}(\mathrm{s})+z_k p_{hk}(\mathrm{s})$$

$$p_0(\mathrm{s})=\frac{1+\mu p_2(\mathrm{s})+r_1 p_{c1}(\mathrm{s})+r_2 p_{c2}(\mathrm{s})+r_3 p_{c3}(\mathrm{s})+r_k p_k(s)+z_1 p_{h1}(\mathrm{s})+z_2 p_{h2}(\mathrm{s})+z_3 p_{h2}(\mathrm{s})+z_k p_{hk}(\mathrm{s})}{(\mathrm{s}+\lambda_{c10}+\lambda_{h10}+2\lambda+\lambda_{c20}+\lambda_{h20}+\lambda_{c30}+\lambda_{h30}+\lambda_{ck0}+\lambda_{ck0})} \tag{62}$$

Availability of the system AV(s) = p_0(s) + p_1(s)

$$=\frac{1+\mu p_2(\mathrm{s})+r_1 p_{c1}(\mathrm{s})+r_2 p_{c2}(\mathrm{s})+r_3 p_{c3}(\mathrm{s})+r_k\mu p_2 p_k(s)+z_1 p_{h1}(\mathrm{s})+z_2 p_{h2}(\mathrm{s})+z_3 p_{h2}(\mathrm{s})+z_k p_{hk}(\mathrm{s})}{(s+\lambda_{c10}+\lambda_{h10}+2\lambda+\lambda_{c20}+\lambda_{h20}+\lambda_{c30}+\lambda_{h30}+\lambda_{ck0}+\lambda_{ck0})}+\frac{p_0(s)2\lambda}{s+\lambda_{c11}+\lambda_{h11}+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{ck1}+\lambda_{hk1}} \tag{63}$$

and unavailability of the system

s=p_2(s) +p_{c1}(s) +p_{c2}(s) +p_{c3}(s) +p_{ck}(s) +p_{h1}(s)+p_{h2}(s)+p_{h3}(s)+p_{hk}(s).

CASE-II:

For n=2 and exponentially distributed failed system repair times, s=0

Let $A=\lambda+\lambda_{c11}+\lambda_{h11}+\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1}$

Then

$$p_1=\frac{2\lambda p_0}{A} \tag{64}$$

$$p_2=\frac{2\lambda^2 p_0}{A\mu} \tag{65}$$

$$p_{c1}=\left[\frac{\lambda_{c10}}{r_1}+\frac{2\lambda\lambda_{c11}}{Ar_1}\right]p_0 \tag{66}$$

$$p_{c2}=\left[\frac{\lambda_{c20}}{r_2}+\frac{2\lambda\lambda_{c21}}{Ar_2}\right]p_0 \tag{67}$$

$$p_{c3}=\left[\frac{\lambda_{c30}}{r_3}+\frac{2\lambda\lambda_{c31}}{Ar_3}\right]p_0 \tag{68}$$

$$p_{ck}=\left[\frac{\lambda_{ck0}}{r_k}+\frac{2\lambda\lambda_{ck1}}{Ar_k}\right]p_0 \tag{69}$$

$$p_{h1}=\left[\frac{\lambda_{h10}}{z_1}+\frac{2\lambda\lambda_{h11}}{Az_1}\right]p_0 \tag{70}$$

$$p_{h2}=\left[\frac{\lambda_{h20}}{z_2}+\frac{2\lambda\lambda_{h21}}{Az_2}\right]p_0 \tag{71}$$

$$p_{h3}=\left[\frac{\lambda_{h30}}{z_3}+\frac{2\lambda\lambda_{h31}}{Az_3}\right]p_0 \tag{72}$$

$$p_{hk}=\left[\frac{\lambda_{hk0}}{z_k}+\frac{2\lambda\lambda_{hk1}}{Az_k}\right]p_0 \tag{73}$$

Since $p_0+p_1+p_2+p_{c1}+p_{c2}+p_{c3}+p_{ck}+p_{h1}+p_{h2}+p_{h3}+p_{hk}=1$

$$p_0+\frac{2\lambda p_0}{A}+\frac{2\lambda^2 p_0}{A\mu}+\left[\frac{\lambda_{c10}}{r_1}+\frac{2\lambda\lambda_{c11}}{Ar_1}\right]p_0+\left[\frac{\lambda_{c20}}{r_2}+\frac{2\lambda\lambda_{c21}}{Ar_2}\right]p_0+\left[\frac{\lambda_{c30}}{r_3}+\frac{2\lambda\lambda_{c31}}{Ar_3}\right]p_0+\left[\frac{\lambda_{ck0}}{r_k}+\frac{2\lambda\lambda_{ck1}}{Ar_k}\right]p_0+\left[\frac{\lambda_{h10}}{z_1}+\frac{2\lambda\lambda_{h11}}{Az_1}\right]p_0$$
$$+\left[\frac{\lambda_{h20}}{z_2}+\frac{2\lambda\lambda_{h21}}{Az_2}\right]p_0+\left[\frac{\lambda_{h30}}{z_3}+\frac{2\lambda\lambda_{h31}}{Az_3}\right]p_0+\left[\frac{\lambda_{hk0}}{z_k}+\frac{2\lambda\lambda_{hk1}}{Az_k}\right]p_0=1$$

$$p_0=\begin{bmatrix} 1+\frac{2\lambda}{A}+\frac{2\lambda^2}{Au}+\frac{\lambda_{c10}}{r_1}+\frac{2\lambda\lambda_{c11}}{Ar_1}+\frac{\lambda_{c20}}{r_2}+\frac{2\lambda\lambda_{c21}}{Ar_2}+\frac{\lambda_{c30}}{r_3}+\frac{2\lambda\lambda_{c31}}{Ar_3}+\frac{\lambda_{ck0}}{r_k} \\ +\frac{2\lambda\lambda_{ck1}}{Ar_k}+\frac{\lambda_{h10}}{z_1}+\frac{2\lambda\lambda_{h11}}{Az_1}+\frac{\lambda_{h20}}{z_2}+\frac{2\lambda\lambda_{21}}{Az_2}+\frac{\lambda_{h30}}{z_3}+\frac{2\lambda\lambda_{31}}{Az_3}+\frac{\lambda_{hk0}}{z_k}+\frac{2\lambda\lambda_{hk1}}{Az_k}\end{bmatrix} \tag{74}$$

The Systems Steady State Availability AV(s s) and Unavailability UAV(s s) are given by
AV(s s) = $\sum_{i=0}^{1} p_i = p_0 + p_1$

UAV(s s) = $p_2+p_{c1}+p_{c2}+p_{c3}+p_{ck}+p_{h1}+p_{h2}+p_{h3}+p_{hk}$.

For u=r=z=15, λ=10, $\lambda_c=\lambda_{c11}=\lambda_{c20}=\lambda_{c21}=\lambda_{c30}=\lambda_{ck0}=\lambda_{ck1}$ and
$\lambda_h=\lambda_{h10}=\lambda_{h11}=\lambda_{h20}=\lambda_{h21}=\lambda_{h30}=\lambda_{h31}=\lambda_{hk0}=\lambda_{hk1}$=2, 3, 4

Table: 1

S.No.	λ_h	λ_c	P_o	P_1	Availability = $P_0 + P_1$
1	3	0	0.2510462	0.31380779	0.56485399
2	3	1	0.2506496	0.26384168	0.514490
3	3	2	0.24737639	0.2248876	0.472264
4	3	3	0.2419357	0.1935485	0.43548426
5	3	4	0.235426	0.1681614	0.4035874

Table: 2

S.No.	λ_h	λ_c	P_o	P_1	Availability = $P_0 + P_1$
1	4	0	0.25088	0.26408494	0.5149649
2	4	1	0.247377	0.224888	0.472265
3	4	2	0.24193	0.193544	0.435474
4	4	3	0.2354	0.16816	0.40354
5	4	4	0.228389	0.147347	0.375736

Table: 3

S.No.	λ_h	λ_c	P_o	P_1	Availability = $P_0 + P_1$
1	5	0	0.24737639	0.2248876	0.472264
2	5	1	0.241935	0.193548	0.435483543
3	5	2	0.235426	0.16816144	0.40358744
4	5	3	0.2283891	0.1473478	0.3757369
5	5	4	0.2211639	0.1300964	0.3512603

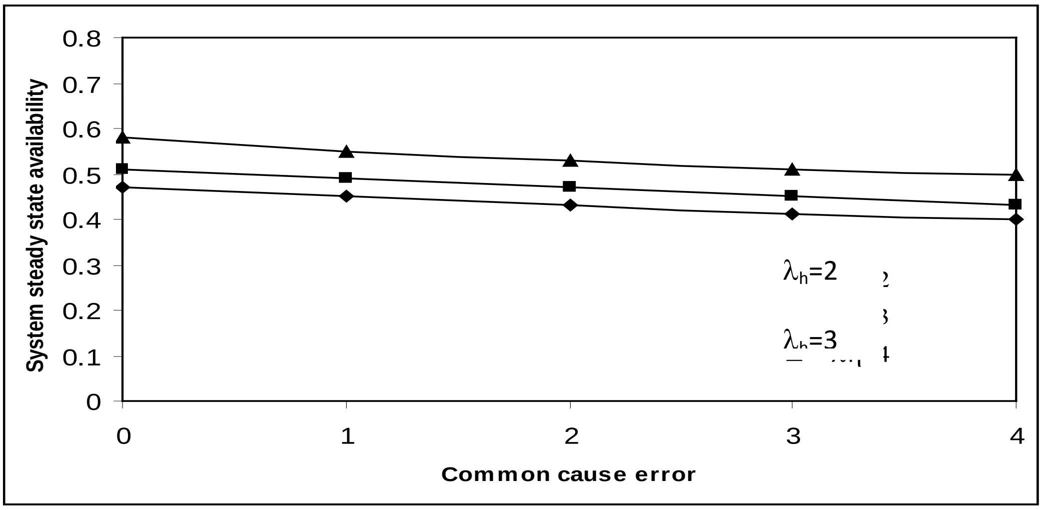

Fig.-1: System Steady State Availability

CASE-III:

By taking $\mu=r_1=r_2=r_3=r_k=z_1=z_2=z_3=z_k=0$ in General Model.

The system of differential equations reduces to

$$\frac{d}{dt}p_0(t)+(\lambda_{c10}+\lambda_{h10}+n\lambda+\lambda_{c20}+\lambda_{h20}+\lambda_{c30}+\lambda_{h30}+\lambda_{cko}+\lambda_{hk0})p_0(t)=0 \quad (75)$$

$$\frac{d}{dt}p_1(t)+(\lambda_{c11}+\lambda_{h11}+(n-1)\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1})p_1(t)=p_0(t)n\lambda \quad (76)$$

$$\frac{d}{dt}p_2(t)+(\lambda_{c12}+\lambda_{h12}+(n-2)\lambda+\lambda_{c22}+\lambda_{h22}+\lambda_{c32}+\lambda_{h32}+\lambda_{ck2}+\lambda_{hk2})\,p_2(t)=p_1(t)(n-1) \quad (77)$$

$$\frac{d}{dt}p_k(t)+(\lambda_{c1k}+\lambda_{h1k}+(n-k)\lambda+\lambda_{c2k}+\lambda_{h2k}+\lambda_{c3k}+\lambda_{h3k}+\lambda_{ckk}+\lambda_{hkk})p_k(t)=p_{k-j}(n-k+1)\lambda \quad (78)$$

For k=1, 2, 3, ... , (n-1).

$$\frac{d}{dt}p_n(x,t)+\frac{d}{dx}p_n(x,t)=0 \quad (79)$$

$$\frac{d}{dt}p_{c1}(x,t)+\frac{d}{dx}p_{c1}(x,t)=0 \quad (80)$$

$$\frac{d}{dt}p_{c2}(x,t)+\frac{d}{dx}p_{c2}(x,t)=0 \quad (81)$$

$$\frac{d}{dt}p_{c3}(x,t)+\frac{d}{dx}p_{c3}(x,t)=0 \quad (82)$$

$$\frac{d}{dt}p_{ck}(x,t)+\frac{d}{dx}p_{ck}(x,t)=0 \quad (83)$$

$$\frac{d}{dt}p_{h1}(x,t)+\frac{d}{dx}p_{h1}(x,t)=0 \quad (84)$$

$$\frac{d}{dt}p_{h2}(x,t)+\frac{d}{dx}p_{h2}(x,t)=0 \quad (85)$$

$$\frac{d}{dt}p_{h3}(x,t)+\frac{d}{dx}p_{h3}(x,t)=0 \quad (86)$$

$$\frac{d}{dt}p_{hk}(x,t)+\frac{d}{dx}p_{hk}(x,t)=0 \quad (87)$$

$$p_n(0,t)=(n-k)\lambda\, p_k(t) \quad (88)$$

$p_{c1}(0, t)= \sum_{i=0}^{k} p_1(t)\lambda_{c1i}$ (89)

$p_{c2}(0, t)= \sum_{i=0}^{k} p_2(t)\lambda_{c2i}$ (90)

$p_{c3}(0, t)= \sum_{i=0}^{k} p_3(t)\lambda_{c3i}$ (91)

$p_{ck}(0, t)= \sum_{i=0}^{k} p_k(t)\lambda_{cki}$ (92)

$p_{h1}(0, t)= \sum_{i=0}^{k} p_1(t)\lambda_{c1i}$ (93)

$p_{h2}(0, t)= \sum_{i=0}^{k} p_2(t)\lambda_{c2i}$ (94)

$p_{h3}(0, t)= \sum_{i=0}^{k} p_3(t)\lambda_{c3i}$ (95)

$p_{hk}(0, t)= \sum_{i=0}^{k} p_k(t)\lambda_{cki}$ (96)

$p_i(0)=1$ for $i=0$

$p_i(0)=0$ for $i=1, 2, 3, \ldots\ldots\ldots\ldots\ldots k$

$p_j(0)=0$ for $j=n, c1, c2, c3, ck, h1, h2, h3, hk$

Solving above equations using Laplace Transforms leads to the following Laplace Transforms of the Probabilities

$sp_0(s)-1+ (\lambda_{c10}+\lambda_{h10}+ (n-1)\lambda+\lambda_{c20}+\lambda_{h20}+\lambda_{c30}+\lambda_{h30}+\lambda_{ck0}+\lambda_{hk0})p_0(s)=0$ (97)

$sp_1(s)-1+ (\lambda_{c11}+\lambda_{h11}+ (n-1)\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1})p_1(s)=p_0(s)n\lambda$ (98)

$sp_2(s)-1+ (\lambda_{c12}+\lambda_{h12}+ (n-2)\lambda+\lambda_{c22}+\lambda_{h22}+\lambda_{c32}+\lambda_{h32}+\lambda_{ck2}+\lambda_{hk2})p_2(s)=p_1(s)(n-1)\lambda$ (99)

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$sp_k(s)-1+ (\lambda_{c1k}+\lambda_{h1k}+ (n-k)\lambda+\lambda_{c2k}+\lambda_{h2k}+\lambda_{c3k}+\lambda_{h3k}+\lambda_{ckk}+\lambda_{hkk})p_k(s) = p_{k-1}(s)(n-k+1)\lambda$ (100)

$\left[s+\frac{d}{dx}\right]p_n(x, s)=0$ (101)

$\left[s+\frac{d}{dx}\right]p_{c1}(x, s)=0$ (102)

$\left[s+\frac{d}{dx}\right]p_{c2}(x, s)=0$ (103)

$\left[s+\frac{d}{dx}\right]p_{c3}(x, s)=0$ (104)

$\left[s+\frac{d}{dx}\right]p_{ck}(x, s)=0$ (105)

$\left[s+\frac{d}{dx}\right]p_{h1}(x, s)=0$ (106)

$\left[s+\frac{d}{dx}\right]p_{h2}(x, s)=0$ (107)

$\left[s+\frac{d}{dx}\right]p_{h3}(x, s)=0$ (108)

$\left[s+\frac{d}{dx}\right]p_{hk}(x, s)=0$ (109)

$p_n(0, s)= (n-k)\lambda\, p_k(s)$ (110)

$p_{c1}(0, s)=\sum_{t=0}^{k} p_i(s)\lambda_{c1i}$ (111)

$p_{c2}(0, s)=\sum_{t=0}^{k} p_i(s)\lambda_{c2i}$ (112)

$p_{c3}(0, s)=\sum_{t=0}^{k} p_i(s)\lambda_{c3i}$ (113)

$p_{ck}(0, s)=\sum_{t=0}^{k} p_i(s)\lambda_{cki}$ (114)

$p_{h1}(0, s)=\sum_{t=0}^{k} p_i(s)\lambda_{h1i}$ (115)

$p_{h2}(0, s)=\sum_{t=0}^{k} p_i(s)\lambda_{h2i}$ (116)

$p_{h3}(0, s)=\sum_{t=0}^{k} p_i(s)\lambda_{h3i}$ (117)

$p_{hk}(0, s)=\sum_{t=0}^{k} p_i(s)\lambda_{hki}$ (118)

From equation (82)
$(s+\lambda_{c11}+\lambda_{h11}+(n-1)\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1})p_1(s)=p_0(s)\, n\lambda$

$p_1(s)=\frac{p_0(s)n\lambda}{s+\lambda_{c11}+\lambda_{h11}+(n-1)\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1}}$ (119)

$sp_2(s)-1+(\lambda_{c12}+\lambda_{h12}+(n-2)\lambda+\lambda_{c22}+\lambda_{h22}+\lambda_{c32}+\lambda_{h32}+\lambda_{ck2}+\lambda_{hk2})p_2(s)=p_1(s)(n-1)\lambda$

$p_2(s)=\frac{p_1(s)(n-1)\lambda}{s+\lambda_{c12}+\lambda_{h12}+(n-2)\lambda+\lambda_{c22}+\lambda_{h22}+\lambda_{c32}+\lambda_{h32}+\lambda_{ck2}+\lambda_{hk2}}$ (120)

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$p_k(s)=\frac{\prod_{i=0}^{k-1}(n-i)\lambda^k p_0(s)}{\prod_{i=0}^{k}(s+\lambda_{c1i}+\lambda_{h1i}+\lambda_{c2i}+\lambda_{h2i}+\lambda_{c3i}+\lambda_{h3i}+\lambda_{cki}+\lambda_{hki})}$ (121)

For k=1, 2, 3, 4,........ (n-1)

From equation (101)
$[s+d/dx]p_n(x,s)=0$

$S+p_n(x,s)+d/dx\, p_n(x,s)=0$

$d/dx\, p_n(x,s)=-sp_n(x,s)$

$\frac{d/dx\, p_n(x,s)}{p_n(x,s)}=-s$

Integrating both sides
$\int \frac{d/dx\, p_n(x,s)}{p_n(x,s)}\,dx=\int -s\,dx$

$\log[p_{c1}(x,s)]_0^x=-sx$

$\log\left[\frac{p_n(x,s)}{p_n(0,s)}\right]=-sx$

$p_n(x,s)=p_n(o,s)e^{-sx}$

$p_n(x,s)=(n-k)p_n(s)e^{-sx}$ (122)

From equation (102)
$[s+d/dx]p_{c1}(x, s)=0$

$d/dx\,p_{c1}(x,s)=-sp_{c1}(x,s)$

$\frac{d/dx\, p_{c1}(x,s)}{p_{c1}(x,s)}=-s$

Integrating both sides
$\log[p_{c1}(x,s)]_0^x=-sx$

$\frac{p_{c1}(x,s)}{p_{c1}(0,s)}=e^{-sx}$

$p_{c1}(x,s)=\sum_{i=0}^{k} p_i(s)\, \lambda_{c1i}\, e^{-sx}$ (123)

Similarly

$p_{c2}(x,s)=\sum_{i=0}^{k} p_i(s)\, \lambda_{c2i}\, e^{-sx}$ (124)

$p_{c3}(x,s)=\sum_{i=0}^{k} p_i(s)\, \lambda_{c3i}\, e^{-sx}$
(125)

$p_{ck}(x,s)=\sum_{i=0}^{k} p_i(s)\, \lambda_{cki}\, e^{-sx}$
(126)

$p_{h1}(x,s)=\sum_{i=0}^{k} p_i(s)\, \lambda_{h1i}\, e^{-sx}$
(127)

$p_{h2}(x,s)=\sum_{i=0}^{k} p_i(s)\, \lambda_{h2i}\, e^{-sx}$ (128)

$p_{h3}(x,s)=\sum_{i=0}^{k} p_i(s)\, \lambda_{h3i}\, e^{-sx}$
(129)

$p_{hk}(x,s)=\sum_{i=0}^{k} p_i(s)\, \lambda_{hki}\, e^{-sx}$
(130)

Substituting in equation (97)

$sp_0(s)-1+(\lambda_{c10}+\lambda_{h10}+(n-1)\lambda+\lambda_{c20}+\lambda_{h20}+\lambda_{c30}+\lambda_{h30}+\lambda_{ck0}+\lambda_{hk0})p_0(s)=0$

$(s+\lambda_{c10}+\lambda_{h10}+(n-1)\lambda+\lambda_{c20}+\lambda_{h20}+\lambda_{c30}+\lambda_{h30}+\lambda_{ck0}+\lambda_{hk0})p_0(s)=1$

$p_0(s)=\frac{1}{s+\lambda_{c10}+\lambda_{h10}+n\lambda\lambda_{c10}+\lambda_{c20}+\lambda_{h20}+\lambda_{c30}+\lambda_{h30}+\lambda_{ck0}+\lambda_{hk0}}$ (131)

$p_1(s)=\frac{np_0(s)\lambda}{s+\lambda_{c11}+\lambda_{h11}+(n-1)\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1}}$ (132)

$\because\ p_2(s)=\frac{p_1(s)(n-1)\lambda}{s+\lambda_{c12}+\lambda_{h12}+(n-2)\lambda+\lambda_{c22}+\lambda_{h22}+\lambda_{c32}+\lambda_{h32}+\lambda_{ck2}+\lambda_{hk2}}$

When n=2

$p_0(s)=\frac{1}{s+\lambda_{c10}+\lambda_{h10}+2\lambda+\lambda_{c20}+\lambda_{h20}+\lambda_{c30}+\lambda_{h30}+\lambda_{ck0}+\lambda_{hk0}}$

$=\frac{1}{s+k_1}$ (134)

where $k_1=s+\lambda_{c10}+\lambda_{h10}+2\lambda+\lambda_{c20}+\lambda_{h20}+\lambda_{c30}+\lambda_{h30}+\lambda_{ck0}+\lambda_{hk0}$

$p_1(s)=\frac{2p_0(s)\lambda}{s+\lambda_{c11}+\lambda_{h11}+\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1}}$

$=\frac{2\lambda p_0}{s+k_2}$

Where $k_2=s+\lambda_{c11}+\lambda_{h11}+\lambda+\lambda_{c21}+\lambda_{h21}+\lambda_{c31}+\lambda_{h31}+\lambda_{ck1}+\lambda_{hk1}$

$=\frac{2\lambda}{(s+k_1)(s+k_2)}$

$p_2(s)=\frac{2\lambda^2 p_0(s)}{(s+k_1)(s+k_2)}$ (135)

where $k_3=s+\lambda_{c12}+\lambda_{h12}+\lambda+\lambda_{c22}+\lambda_{h22}+\lambda_{c32}+\lambda_{h32}+\lambda_{ck2}+\lambda_{hk2}$

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Taking Inverse Laplace Transform to equation (134)

$p_0(s)=\frac{1}{s+k_1}$

$L^{-1}[p_0(s)]=l^{-1}\left[\frac{1}{s+k_1}\right]$

$p_0(t)=e^{-k_1 t}$ (136)

Similarly

$p_1(s)=\frac{2⋋}{k_1-k_2}\left[\frac{1}{s+k_2}-\frac{1}{s+k_1}\right]$

$L^{-1}p_1(s)=\frac{2⋋}{k_1-k_2}L^{-1}\left[\frac{1}{s+k_2}-\frac{1}{s+k_1}\right]$

$p_1(t)=\frac{2⋋}{k_1-k_2}[e^{-k_2 t}-e^{-k_1 t}]$ (137)

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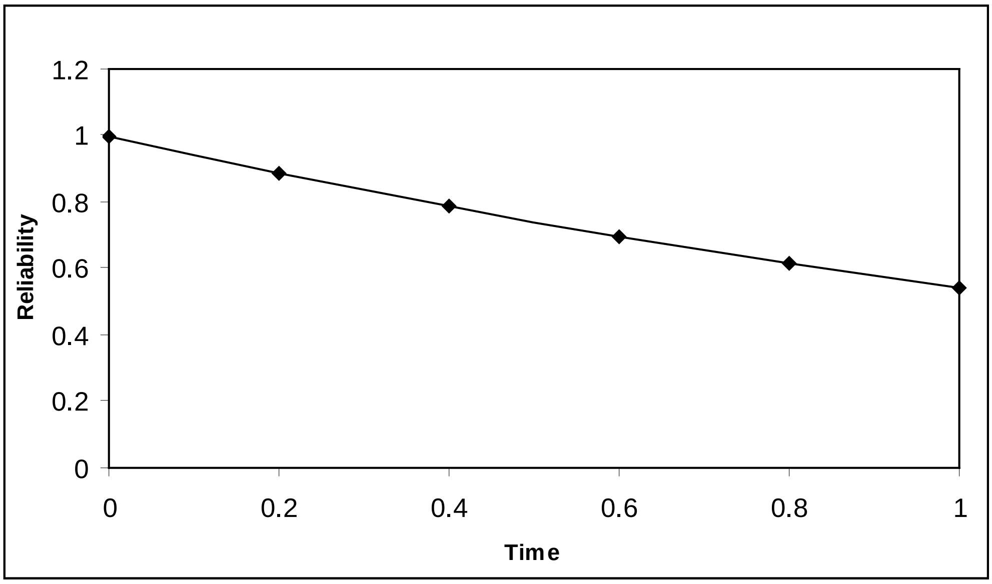
The system reliability is given by

R(t) = p_0(t)+p_1(t) = $e^{-k_1}+\frac{2⋋}{k_1-k_2}[e^{-k_2 t}-e^{-k_1 t}]$

Table: 4

S.No.	Time	$P_o(t)$	$P_1(t)$	Reliability of the system
1	0	1	0	1
2	0.2	0.8251437	0.03442906	0.88657276
3	0.4	0.72614903	0.05926934	0.78541837
4	0.6	0.6187833	0.076527	0.6953103
5	0.8	0.5272924	0.0878332	0.6151256
6	1	0.449328	0.0945146	0.5438426



Reliability Vs Time

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