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EFFECT OF THERMAL RADIATION ON MHD FREE COVECTION FLOW PAST A FLAT PLATE WITH CONVECTIVE SURFACE BOUNDARY CONDITION

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ABSTRACT

The present paper analyzes the effect of thermal radiation on MHD free convective viscous incompressible boundary layer flow over a flat plate in the presence of internal heat generation and convective boundary condition. It is assumed that lower surface of the plate is in contact with a hot fluid while a stream of cold fluid flows steadily over the upper surface with a heat source that decays exponentially. The Rosseland approximation is used to describe radiative heat transfer as we consider optically thick fluids. The governing boundary layer equations are transformed into a system of ordinary differential equations using similarity transformations, which are then solved numerically by employing fourth order Runge-Kutta method along with shooting technique. The effects of the material parameters like the internal heat generation parameter, Prandtl number, local Biot number, magnetic parameter and radiation parameter on the velocity, temperature as well as the skinfriction coefficient, local Nusselt number and plate surface temperature are computed numerically and illustrated through graphs and tables. The present results are compared with the existing results and found to be a good agreement.

Key words: *Blasius flow, Convective boundary condition, Internal heat generation, Heat source, MHD, Radiation, Shooting method.*

I. INTRODUCTION

Laminar boundary layer about a flat-plate in a uniform stream of fluid continues to receive considerable attention because of its importance in many practical applications in a broad spectrum of engineering systems like geothermal reservoirs, cooling of nuclear reactors, thermal insulation, combustion chamber, rocket engine, etc. In a pioneering work, Blasius [1] presented a theoretical result for the boundary layer flow over a flat plate in a uniform stream and on a circular cylinder. Thereafter, several authors [2-6] have made significant advances in generalising his theoretical study to various situations of practical interest. Moreover, there are a number of physical circumstances where internal heat generation in an otherwise forced convective flow over a flat surface do occur. For instance, in the development of a metal waste form from spent nuclear fuel, phase change processes and thermal combustion processes, convection with internal heat generation plays an important role in the overall heat transfer process. Crepeau and Clarksean[7] considered the classical problem of natural convection from an isothermal vertical plate and added a heat generation term in the energy equation. They found that for a true similarity solution to exist, the internal heat generation must decay exponentially with the classical similarity variable. This type of model can be used in mixtures where a radioactive material is surrounded by inert alloys and in the electromagnetic heating of materials.

Convective heat transfer studies are very important in processes involving high temperatures such as gas turbines, nuclear plants, thermal energy storage, etc. Recently, Ishak [8] examined the similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition. Moreover, Aziz [9] studied a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition and also studied hydrodynamic and thermal slip flow boundary layers over a flat plate with constant heat flux boundary condition (see [10]).

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Magnetohydrodynamic flows have applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. In addition from the technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magnetospheres, aeronautical plasma flows, chemical engineering and electronics. An excellent summary of applications is to be found in Hughes and Young [11] Raptis[12] studied mathematically the case of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Helmy [13] analyzed MHD unsteady free convection flow past a vertical porous plate embedded in a porous medium. Elbashbeshy [14] studied heat and mass transfer along a vertical plate in the presence of magnetic field.

The effects of radiation are of vital importance. Recent development in hypersonic flights, missile re-entry, rocket combustion chambers, power plants for inter planetary flight and gas cooled nuclear reactors have focused attention on thermal radiation as a mode of energy transfer. Takhar *et al.* [15] presented one of the most robust studies of thermal and concentration boundary layers with MHD effects for the case of a point sink. Takhar *et al.* [16] extended this analysis to examine combined variable lateral mass flux (wall injection/suction), heat source effects and hall current effects on double-diffusive boundary layers under strong magnetic fields. Sahoo *et al.* [17] studied magnetohydrodynamic unsteady free convection flow past an infinite vertical plate with constant suction and heat sink. Recently, Bhaskar Reddy *et al.* [18] analyzed the radiation and mass transfer effects on MHD flow over a stretching surface with heat generation and suction/ injection. The interaction of radiation with laminar free convection heat transfer from a vertical plate was investigated by Cess [19] for an absorbing, emitting fluid in the optically thick region, using the singular perturbation technique. Arpaci [20] considered a similar problem in both the optically thin and optically thick regions and used the approximate integral technique and first-order profiles to solve the energy equation. Raptis [21] analyzed the thermal radiation and free convection flow through a porous medium bounded by a vertical infinite porous plate by using a regular perturbation technique. Ishak [22] studied the MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. The unsteady flow past a moving plate in the presence of free convection and radiation were presented by Mansour [23]. Sajid and Hayat [24] investigated the radiation effects on the mixed convection flow over an exponentially stretching sheet, and solved the problem analytically using the homotopy analysis method. The numerical solution for the same problem was then given by Bidin and Nazar [25]. Recently, Poornima and Bhaskar Reddy [26] presented an analysis of the radiation effects on MHD free convective boundary layer flow of nanofluids over a nonlinear stretching sheet.

However, the interaction of radiation with mass transfer due to a stretching sheet has received little attention. Hence, in this present study is to analyze the effect of thermal radiation on MHD steady free convective viscous incompressible boundary layer flow over a flat plate in the presence of internal heat generation and convective boundary condition. Using a similarity approach, the transport equations are transformed to nonlinear ordinary differential equations and solved numerically using a shooting iteration technique together with fourth order Runge-Kutta integration scheme. The pertinent results are displayed graphically and discussed quantitatively.

II. MATHEMATICAL ANALYSIS

A steady two-dimensional steady free convective flow of viscous incompressible, electrically conducting and radiating fluid adjacent to flat plate with heat transfer and heat generation is considered. The flow is assumed to be in the direction of x-axis along the plate and y-axis normal to it, with the fluid at temperature T_∞ over the upper surface of the flat plate with a uniform velocity U_∞ while the lower surface of the plate is heated by convection from a hot fluid at temperature T_f which provides heat transfer coefficient h_f as shown in Fig.1. The cold fluid is in contact with the upper surface of the plate generates heat internally at the volumetric rate q. A uniform magnetic field is applied in the direction perpendicular to the plate. The fluid is assumed to be slightly conducting, so that the magnetic Reynolds number is much less than unity, and hence the induced magnetic field is negligible in comparison with applied magnetic field. It is assumed that there is no applied voltage which implies the absence of electrical field. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the heat flux in the energy equation. The density variation in this fluid is taken into account using Boussinesq approximation.

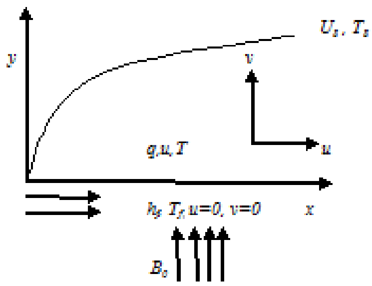

Fig.1 Physical model of the problem

The governing boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{1.2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+q-\frac{\partial q_r}{\partial y} \tag{1.3}$$

where u and v are the x (along the plate) and the y (normal to the plate) components of the velocities, respectively, T is the local temperature, ν is the kinematics viscosity of the fluid, σ is the electrical conductivity, B_0 is the magnetic induction, ρ is the fluid density, c_p is the specific heat at constant pressure and k is the thermal conductivity of the fluid and q_r is the radiative heat flux. The velocity boundary conditions can be expressed as

$$u(x,0)=v(x,0)=0, u(x,\infty)=U_\infty \tag{1.4}$$

The thermal boundary conditions at the plate lower surface and far into the cold fluid at the plate upper surface are

$$-k\frac{\partial T}{\partial y}(x,0)=h_f\left[T_f-T(x,0)\right] \tag{1.5}$$

$$T(x,\infty)=T_\infty \tag{1.6}$$

By using the Rosseland approximation (Brewster [27]), the heat flux q_r is given by

$$q_r=\frac{4\sigma^*}{3k'}\frac{\partial T^4}{\partial y} \tag{1.7}$$

where σ^* is the Stephen-Boltzmann constant and k' – the mean absorption. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences within the flow are sufficiently small, then equation (1.3) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms takes the form

$$T^4=4T_\infty^3 T-3T_\infty^4 \tag{1.8}$$

In view of the equations (1.5) and (1.6), the equation (1.3) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{q}{\rho c_p}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{1.9}$$

In order to transform the governing equations (1.2) and (1.9) into a set of ordinary differential equations, and the boundary conditions in the dimensionless form the following similarity transformations and dimensionless variables are introduced (Sajid and Hayat [28]).

$$\eta=\frac{y}{x}\sqrt{\mathrm{Re}_x},\quad u=U_\infty f',\quad v=\frac{\upsilon}{2x}\sqrt{\mathrm{Re}_x}\left(\eta f'-f\right),\ \theta=\frac{T-T_\infty}{T_f-T_\infty},$$
$$\lambda_x=\frac{qx^2e^n}{k\,\mathrm{Re}_x\left(T_f-T_\infty\right)},\quad M=\frac{\sigma B_0^2}{\rho U_\infty},\quad \mathrm{Pr}=\rho c_p\frac{\upsilon}{k},\quad R=\frac{4\sigma_s T_\infty^3}{k^* k} \tag{1.10}$$

where $f(\eta)$ is the dimensionless stream function, $f'(\eta)$ - the dimensionless velocity, $\theta(\eta)$ - the dimensionless temperature, λ_x - the local internal heat generation parameter, η - the similarity variable, M- the magnetic parameter, Pr - Prandtl number, and R- the radiation parameter.

In the view of the above transformations, the equations (1.2) and (1.9) reduce to

$$f'''+\frac{1}{2}ff'-Mf'=0 \tag{1.11}$$

$$\left(1+\frac{4}{3}R\right)\theta''+\frac{1}{2}\mathrm{Pr}\,f\theta'+\lambda_x e^{-n}=0 \tag{1.12}$$

The transformed boundary conditions can be written as

$$f(0) = f'(0) = 0 \quad \theta'(0) = -Bi\left[1 - \theta(0)\right] \tag{1.13}$$

$$f'(\infty) = 1 \quad \theta(\infty) = 0 \tag{1.14}$$

where,

$$Bi = \frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}}, \quad \Pr = \frac{\rho c_p \upsilon}{k} \tag{1.15}$$

where prime denotes differentiation with respect to η and $Re_x = U_\infty x/\upsilon$ is the local Reynolds number. The local internal heat generation parameter λ_x is defined so that the internal heat generation q decays exponentially with the similarity variable η as stipulated in (1.9).

The solutions generated whenever Bi_x and λ_x are defined as in equations (1.11) - (1.15) are the local similarity solutions. In order to have a true similarity solution the parameters Bi_x and λ_x must be constants and not depend on x. This condition can be met if the heat transfer coefficient h_f is proportional to $x^{1/2}$ and the internal heat generation q is proportional to x^{-1}. In this case, we assume

$$h_f = cx^{-\frac{1}{2}}, q = lx^{-1} \tag{1.16}$$

where c and l are constants but have the appropriate dimensions. Substituting equations (1.16) into equations (1.10) and (1.15), we obtain

$$Bi = \frac{c}{k}\sqrt{\frac{\upsilon}{U_\infty}} \tag{1.17}$$

The Biot number (Bi) lumps together the effects of convection resistance of the hot fluid and the conduction resistance of the flat plate. The parameter λ is a measure of the strength of the internal heat generation.

The main physical quantities of interest are the skin friction coefficient $f''(0)$, the local Nusselt number $\theta'(0)$, which represent the wall shear stress, the heat transfer rate and the plate surface temperature $\theta(0)$ respectively. Our task is to investigate how the values of $f''(0)$, $\theta'(0)$, and $\theta(0)$ vary with the radiation parameter R, magnetic parameter M and Prandtl number Pr, λ_x - internal heat generation, Bi_x = local biot number for hot fluid.

III. METHOD OF SOLUTION

The numerical solutions of the boundary value problem for system of ordinary differential equations were obtained by using Runge-Kutta fourth order technique along with shooting method. order technique along with shooting Using a similarity variable, the governing non-linear partial differential equations have been transformed into a set of coupled non- linear ordinary differential equations. First of all, higher order non-linear differential equations (1.11) and (1.12) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain *et al*. [29]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta$=0.001 is used to obtain the numerical solution with six decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the plate surface temperature, which are respectively proportional to $f''(0)$, $\theta'(0)$ and $\theta(0)$ are also sorted out and their numerical values are presented in a tabular form.

IV. RESULTS AND DISCUSSION

The aim of present study is to investigate the effect of thermal radiation on MHD steady free convective viscous incompressible boundary layer flow over a flat plate in the presence of internal heat generation and convective boundary condition.

In order to get a physical insight of the problem, a representative set of numerical results are shown graphically in Figs.1-8, to illustrate the influence of physical parameters viz., the internal heat generation parameter, Prandtl number, local Biot number, magnetic parameter and radiation parameter on the velocity, temperature as well as the skinfriction coefficient, local Nusselt number and plate surface temperature are computed numerically and presented in table. The effects of various parameters on the velocity are depicted in Fig.1 and Fig.2. The effects of various parameters on the temperature are depicted in Figs. 3-7. Fig. 1 shows the sample of velocity profile. The fluid velocity is zero at the plate surface, increases rapidly to and attains its free stream velocity values far away from the plate satisfying the boundary condition. There is no significant variation in velocity profile for other parameters except for magnetic parameter. Fig. 2 shows the dimensionless velocity for different values of magnetic parameter *(M)*.It is seen that, as expected, the velocity decreases with an increase of magnetic parameter. The magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter. The effect of Biot number on the temperature distribution is depicted in Fig. 3. Because of strong internal heat generation (λ_x=10), Fig.3 shows that the plate surface temperatures exceed the temperature of the fluid on the lower surface of the plate and the direction of heat flow is reversed as noted in the earlier discussion. The peak temperature occurs in the thermal boundary in a region close to the plate. Although the temperature reduces as the local Biot number increases but the back heat flow persists. The effect of Prandtl number on the temperature distribution is depicted in Fig.4. As the Prandtl number increases, the thermal boundary layer thickness decreases, leaving less energy for the back heat flow Fig.5 reveals that only for weak internal heat generation i.e λ_x= 0.1, the plate surface temperature is less 1 and heat is able to flow from the lower surface of plate into the fluid on the upper face of the plate. For all other values of λ_x, the heat flows back into the plate. It must be kept in mind that for Fig.5, the convection on the lower surface of the plate is rather weak (Bi_x= 0.1) and consequently unable to push heat through the plate to the fluid on the upper face of the plate except when the internal heat generation is weak, e.g. λ_x =0.1**.** The magnetic parameter(*M*) on the temperature is illustrated in Fig.6. It is observed that as the magnetic parameter increases, the temperature increases. Fig.7 illustrates the effect of the radiation parameter (*R*) on the temperature. It is noticed that as the radiation parameter increases, the temperature of the fluid decreases on the upper surface of the plate. This is because of the fact that the thermal boundary- layer thickness decreases with increasing thermal radiation.

Numerical results for the skin-friction, local Nusselt number and plate surface temperature are reported in the Tables 1 and 2. In Table3, the present results are compared with Olanrewaju in the absence of thermal radiation and found that there is a perfect agreement. For all values of thermophysical parameters embedded in the system, except for magnetic parameter the value of local skin friction coefficient represented by $f''(0)= 0.329629$. Moreover, the local Nusselt number coefficient represented by and $\theta'(0)$ the plate surface temperature $\theta(0)$ for different combination values of parameters are presented in Table 1 and Table 2. In table3, for the case of λ_x=0 (no internal heat generation) the present results are compared with those of Olanrewaju[30] are found good agreement.

In Table1, we highlight the effect of the internal heat generation and radiation on thermal boundary layer with respect to heat transfer characteristics of the flow. Note that we have tabulated the values of $\theta'(0)$ and not - $\theta'(0)$ as in Table3, because except for one case, $\theta'(0)$ is positive means heat flows into the flat plate. It is observed that for the conditions of weak plate heating ($Bi_{x=}$0.1),as the local internal heat generation increases i.e as λ_x increases from 1 to 10 the first four rows shows that local Nusselt number increases slightly and the plate surface temperature increases rapidly. As Bi_x increases from 0.1 to10 i.e. the plate heating becomes stronger next three rows in Table 1 shows that the local Nusselt number increases slightly with an increase in the back flow of heat into the plate while the plate upper surfaces temperature decreases. With data in the eighth, ninth, tenth and eleventh rows of Table1, one can see that as the *Pr* increases (from 0.72 Air to 7.1water) the plate surface temperature decreases curtailing the back heat flow into the plate. When *Pr* = 7.10, the normal heat flow direction (from the lower plate surface into the cold fluid) is restored. The data in the last four rows of Table 1, it is noticed that as the radiation parameter increases, local Nusselt number decreases slightly and the plate surface temperature increases rapidly. It is observed that in table 3, with an increase in the magentic parameter the skin friction decreases slightly and the local Nusselt number, the plate surface temperature increses slightly.

V. CONCLUSIONS

In the present paper the effect of thermal radiation on MHD free convective viscous incompressible fluid of exponentially decaying internal heat generation on boundary layer flow over the upper surface of a flat plate with a convective boundary condition on its lower surface is investigated numerically. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. The results are summarized as follows.

- The velocity boundary layer thickness decreases with an increase in the Magnetic parameter.
- The thermal boundary layer thickness decreases with an increase in the local Prandtl number and the local Biot number
- An increase in the internal heat generation prevents the rapid flow of heat from the lower surface to the upper surface of the plate.
- The radiation reduces temperature.
- The magnetic field elevates the temperature and reduces the velocity.

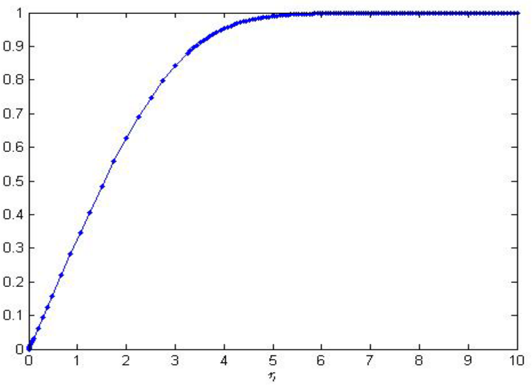

Fig.1. Velocity Distribution

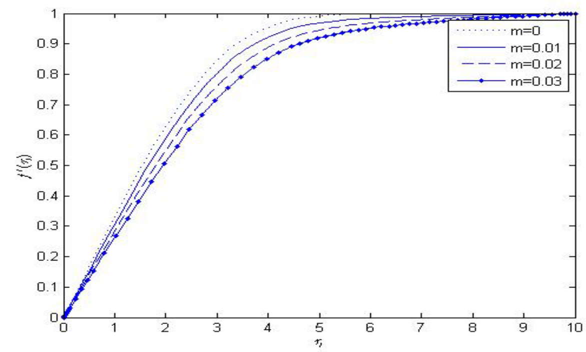

Fig. 2. Velocity distribution for different values of *M* (Pr=0.72, λ_x=10, Bi_x=0.1 , R=1.0)

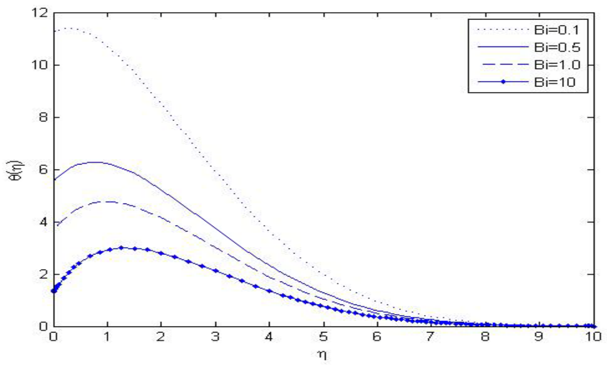

Fig.3. Temperature distribution for different values of *Bi* (Pr=0.72, λ_x=10, M=0.001, R=1.0)

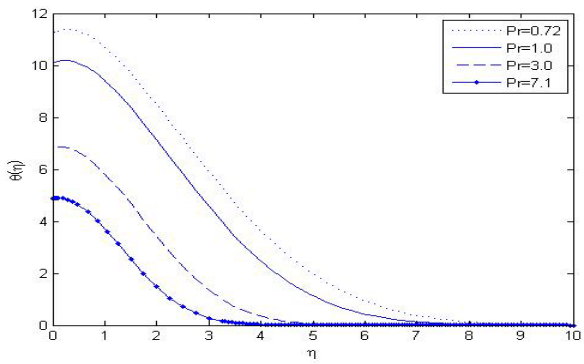

Fig.4. Temperature distribution for different values of *Pr* (Bi_x=0.1, λ_x=10, M=0.001, R=1.0)

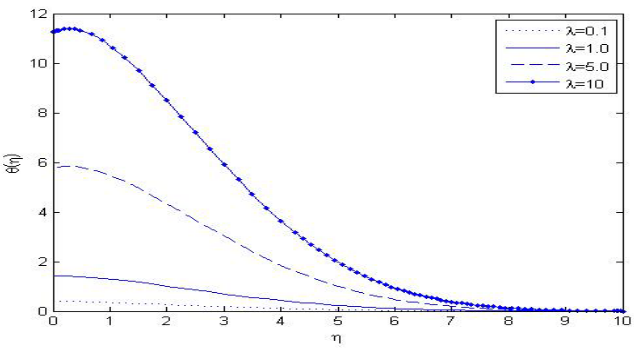

Fig.5 Temperature distribution for different values of *λ* (Pr=0.72, Bi_x=0.1, M=0.001, R=1.0)

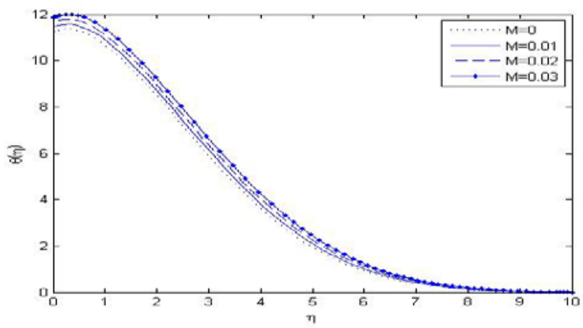

Fig.6 Temperature distribution for different values of *M* (Pr=0.72, Bi_x=0.1, λ_x=10, R=1.0)

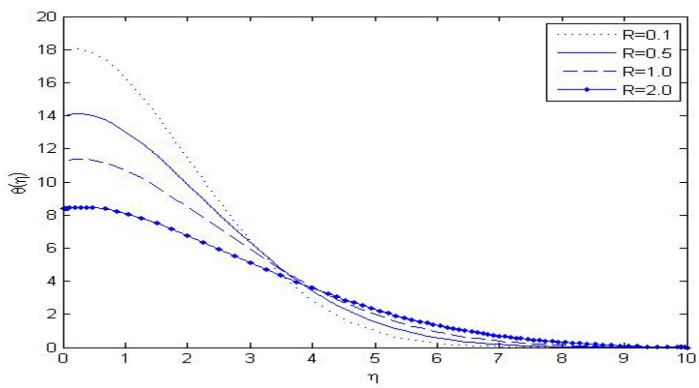

Fig.7 Temperature distribution for different values of *R* (Pr=0.72, Bi_x=0.1, λ_x=10, M=0.001)

Table-1: Numerical values of $f''(0)$, $\theta'(0)$ and $\theta(0)$ for different values of Bi_x, Pr, λ_x and R

Bi_x	Pr	λ_x	M	R	$\theta'(0)$	$\theta(0)$
0.1	0.72	0.1	0.001	1.0	0.057492	0.425082
0.1	0.72	1.0	0.001	1.0	0.060952	1.409514
0.1	0.72	5.0	0.001	1.0	0.478480	5.784798
0.1	0.72	10	0.001	1.0	1.025390	11.253895
0.5	0.72	10	0.001	1.0	2.265745	5.531490
1.0	0.72	10	0.001	1.0	2.669371	3.669371
10	0.72	10	0.001	1.0	3.179069	1.317907
0.1	0.72	10	0.001	1.0	1.025390	11.253895
0.1	1.0	10	0.001	1.0	0.908855	10.088547
0.1	3.0	10	0.001	1.0	0.581538	6.815276
0.1	7.1	10	0.001	1.0	0.387650	4.876580
0.1	0.72	10	0.001	0.1	1.684970	17.849700
0.1	0.72	10	0.001	0.5	1.296999	13.969988
0.1	0.72	10	0.001	1.0	1.025390	11.253895
0.1	0.72	10	0.001	2.0	0.736977	8.369773

Table-2: Numerical values of $f''(0)$, $\theta'(0)$ and $\theta(0)$ for different values of M

Bi_x	Pr	λ_x	M	R	$f''(0)$	$\theta(0)$	$\theta'(0)$
0.1	0.72	10	0	1.0	0.332034	11.233955	1.023395
0.1	0.72	10	0.01	1.0	0.308251	11.432255	1.043225
0.1	0.72	10	0.02	1.0	0.285178	11.636345	1.063635
0.1	0.72	10	0.03	1.0	0.262939	11.853831	1.085383

Table-3: Comparison of the present results with that of Olanrewaju

Bi_x	$\theta(0)$ present results	$-\theta'(0)$ Present results	$\theta(0)$ Olanrewaju	$-\theta'(0)$ Olanrewaju
0.05	0.1447	0.0428	0.1447	0.0428
0.10	0.2528	0.0747	0.2528	0.0747
0.20	0.4035	0.1193	0.4035	0.1193
0.40	0.5750	0.1700	0.5750	0.1700
0.60	0.6699	0.1981	0.6699	0.1981
0.80	0.7302	0.2159	0.7302	0.2159
1.00	0.7718	0.2282	0.7718	0.2282
5.00	0.9441	0.2791	0.9441	0.2791
10.00	0.9713	0.2871	0.9713	0.2871
20.00	0.9854	0.2913	0.9854	0.2913

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