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On Contra b$\hat{g}$ – Continuous functions in Topological Spaces

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ABSTRACT

In this paper we introduce a new class of weaker form of continuous function called Contra b$\hat{g}$–continuous function. Some characterization and several properties concerning Contra b$\hat{g}$–continuity are obtained.

Keywords: *b$\hat{g}$ – Closed sets, b$\hat{g}$–continuous, Contra b$\hat{g}$–continuous, Contra b$\hat{g}$–irresolute.*

AMS subject classification: *54C08, 54C10.*

1. INTRODUCTION

Levine [4] introduced generalized closed sets in topological spaces. Veerakumar [2] introduced $\hat{g}$–closed sets in topological spaces. Andrijevic[1] introduced a new class of generalized open sets called b–open sets. R. Subasree and M.Maria Singam [3] introduced b$\hat{g}$–closed sets in topological spaces.

Dontchev[6] introduced and investigated a new notion of continuity called Contra continuity. The purpose of this paper is to introduce and investigate some of the properties of contra b$\hat{g}$ –continuous functions, contra b$\hat{g}$ –irresolute functions and we obtain some of its characterization.

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2. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. Let us recall the following definitions.

Definition 2.1: A subset A of a space (X, τ) is called a

i) Semi-open set [6] if A ⊑ cl[Int (A)]
ii) α-open set[9] if A ⊑ Int[cl(Int (A))]
iii) b-open set[7] if A ⊑ cl[Int (A)] U Int [cl(A)]

The complement of a semi-open (resp. α-open, b-open) set is called semi-closed (resp.α-closed, b-closed) set.

The intersection of all semi-closed (resp. α-closed, b-closed) sets of X containing A is called the semi-closure (resp. α-closure, b-closure) and is denoted by scl(A) (resp. αcl(A), bcl(A)). The family of all semi-open (resp. α-open, b-open) subsets of a space X is denoted by SO(X) (resp. αO(X), bO(X)).

Definition 2.2: A subset A of a topological space (X, τ) is called

i) generalized closed(briefly g-closed) set[4] if cl(A)⊑U whenever A⊑U and U is open set in (X, τ).
ii) $\hat{g}$ -closed set[2] if cl(A) ⊑U whenever A⊑U and U is a semi-open set in (X, τ).
iii) b$\hat{g}$ -closed set[3] if bcl(A) ⊑U whenever A⊑U and U is $\hat{g}$-open set in (X, τ).

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The complement of a g-closed (resp. ĝ –closed and bĝ–closed) set is called g-open (resp. ĝ –open and bĝ –open) set .

Definition 2.3: A space (X, τ) is called a Tbĝ – space [3] if every bĝ – closed set in it is b–closed.

Definition 2.4: A function f: (X,τ) $\longrightarrow$ (Y, σ) is called

- **i)** bĝ – continuous [5] map if f^{-1}(V) is bĝ–closed in (X, τ) for every closed set V in (Y, σ).
- **ii)** bĝ – irresolute [5] map if f^{-1}(V) is bĝ–closed in (X, τ) for every bĝ –closed set V in (Y, σ).

Definition 2.5: A function f: (X, τ) $\longrightarrow$ (Y, σ) is called

- **i)** contra continuous[10] map if f^{-1}(V) is closed in (X, τ) for every open set V in (Y, σ).
- **ii)** contra semi–continuous[6] map if f^{-1}(V) is semi–closed in (X, τ) for every open set V in (Y, σ).
- **iii)** contra α –continuous[9] map if f^{-1}(V) is α–closed in (X, τ) for every open set V in (Y, σ).
- **iv)** contra b–continuous[12] map if f^{-1}(V) is b–closed in (X, τ) for every open set V in (Y, σ).
- **v)** contra g–continuous[11] map if f^{-1}(V) is g–closed in (X, τ) for every open set V in (Y, σ).

3. Contra bĝ – Continuous function

Definition 3.1: A function f: (X, τ) $\longrightarrow$ (Y, σ) is called contra bĝ –continuous if f^{-1}(V) is bĝ–closed in (X, τ) for every open set V in (Y, σ).

Example 3.2: Let X=Y= {a, b, c}
τ = {X, Φ, {a}, {b}, {a, b}} and σ = {Y, Φ, {a, b}, {c}}

Define a function f: (X, τ) $\longrightarrow$ (Y, σ) by f(a) = a, f(b) = c, f(c) = b.
bĝ–C(X) = {X, Φ, {a},{b},{c},{a, c},{b, c}}

Here the inverse images of open sets {a, b} and {c} in Y are {a, c} and {b} respectively which are bĝ – closed sets in X. Hence f is contra bĝ – continuous.

Theorem 3.3: Every contra continuous function is contra bĝ – continuous.

Proof: Let V be any open set in (Y, σ). Since f is contra continuous, f^{-1}(V) is closed in (X, τ). Since from [3] Remark 3.23 "Every closed set is bĝ – Closed". Then f^{-1}(V) is bĝ – Closed in (X, τ). Hence f is contra bĝ – continuous.

Remark 3.4: The converse of the above theorem need not be true.
(i.e) Every contra bĝ – continuous function need not be contra continuous as shown in the following example.

Example 3.5: Let X=Y= {a, b, c, d}
τ = {X, Φ, {a},{a, c},{a, b, d}} and σ = {Y, Φ, {b},{a, b},{b, c, d}}

Define a function f: (X, τ) $\longrightarrow$ (Y, σ) by f(a) = c, f(b) = b, f(c) = a, f(d) = d.
bĝ–C(X)={X,Φ,{b},{c},{d},{b,c},{c,d},{b,d},{a,b,c},{a,c,d},{a,b,d},{b,c,d}}

Here the inverse images of open sets {b},{a, b},{b, c, d} in (Y, σ) are {b},{b, c},{a, b, d} respectively which are bĝ – closed but not closed in (X, τ). Hence f is contra bĝ – continuous function but not contra continuous.

Theorem 3.6: Every contra g-continuous function is contra bĝ – continuous.

Proof: Let V be any open set in (Y, σ). Since f is contra g–continuous, f^{-1}(V) is g-closed in (X, τ). From [3] Proposition 3.6, "Every g–closed set is bĝ–Closed". Then f^{-1}(V) is bĝ –Closed in (X, τ). Hence f is contra bĝ–continuous.

Remark 3.7: The converse of the above theorem need not be true.
(i.e) Every contra bĝ – continuous need not be contra g–continuous function as shown in the following example.

Example 3.8: Let X=Y= {a, b, c, d}
τ = {X, Φ, {a},{a, c},{a, b, d}} and σ = {Y, Φ, {b},{a, b},{b, c, d}}

Define a function f: (X, τ) $\longrightarrow$ (Y, σ) by f(a) = c, f(b) = b, f(c) = a, f(d) = d.
g–C(X) = {X,Φ,{b},{c},{d},{b, c},{c, d},{b, d},{a, b, c},{a, c, d},{b, c, d}}
bĝ–C(X)={X,Φ,{b},{c},{d},{b,c},{c,d},{b,d},{a,b,c},{a,c,d},{a,b,d},{b,c,d}}

Here the inverse images of open set {b, c, d} in (Y, σ) is {a, b, d} which is bĝ–closed but not g–closed in (X, τ). Hence f is contra bĝ – continuous, but not contra g–continuous.

Theorem 3.9: Every contra b–continuous function is contra bĝ – continuous.

Proof: Let V be any open set in (Y, σ). Since f is contra b–continuous, f^{-1}(V) is b–closed in (X, τ). Since from [3] Proposition 3.3, "Every b–closed set is bĝ–Closed". Then f^{-1}(V) is bĝ –Closed in (X, τ). Hence f is contra bĝ–continuous.

Remark 3.10: The converse of the above theorem need not be true.
(i.e) Every contra bĝ – continuous need not be contra b–continuous map as shown in the following example.

Example 3.11: Let X=Y={a, b, c}
τ = {X, Φ, {a}} and σ = {Y, Φ, {a}, {b},{a, b},{a, c}}

Define a function f (X, τ)$\longrightarrow$(Y, σ) by f(a) = c, f(b) = a, f(c) = b.
bĝ–C(X)={X,Φ,{b},{c},{a, b},{a, c},{b, c}}
b–C(X)={X,Φ,{b},{c},{b, c}}

Here the inverse image of an open set {a, c} in (Y, σ) is {a, b} which is bĝ–closed but not b–closed in (X, τ). Hence f is contra bĝ–continuous, but not contra b–continuous.

Theorem 3.12: Every contra α–continuous function is contra bĝ – continuous.

Proof: Let V be any open set in (Y, σ). Since f is contra α–continuous, f^{-1}(V) is α–closed in (X, τ). From [3] Remark 3.23 "Every α–closed set is bĝ – Closed". Then f^{-1}(V) is bĝ –Closed in (X, τ). Hence f is contra bĝ-continuous.

Remark 3.13: The converse of the above theorem need not be true.
(i.e) Every contra bĝ – continuous need not be contra α–continuous function as shown in the following example.

Example 3.14: Let X=Y={a, b, c, d}
τ = {X, Φ, {a}, {b, c},{a, b, c}} and σ = {Y, Φ, {a},{a, c},{a, b, d}}

Define a function f: (X, τ)$\longrightarrow$ (Y, σ) by f(a) = b, f(b) = a, f(c) = c, f(d) = d.
α–C(X) = {X,Φ,{d},{a, d},{b, c, d}}
bĝ–C(X)={X,Φ,{a},{b},{c},{d},{b,c},{c,d},{a,d},{b,d},{a,c,d},{a,b,d},{b,c,d}}

Here the inverse images of open sets {a},{a, c},{a, b, d} in (Y, σ) are {b},{b, c},{a, b, d} respectively which are bĝ – closed but not α–closed in (X, τ). Hence f is contra bĝ – continuous, but not contra α–continuous.

Theorem 3.15: Every contra semi–continuous function is contra bĝ– continuous.

Proof: Let V be any open set in (Y, σ). Since f is contra semi–continuous, f^{-1}(V) is semi–closed in (X, τ). From [3] Remark 3.23 "Every semi–closed set is bĝ–Closed". Then f^{-1}(V) is bĝ –Closed in (X, τ). Hence f is contra bĝ-continuous.

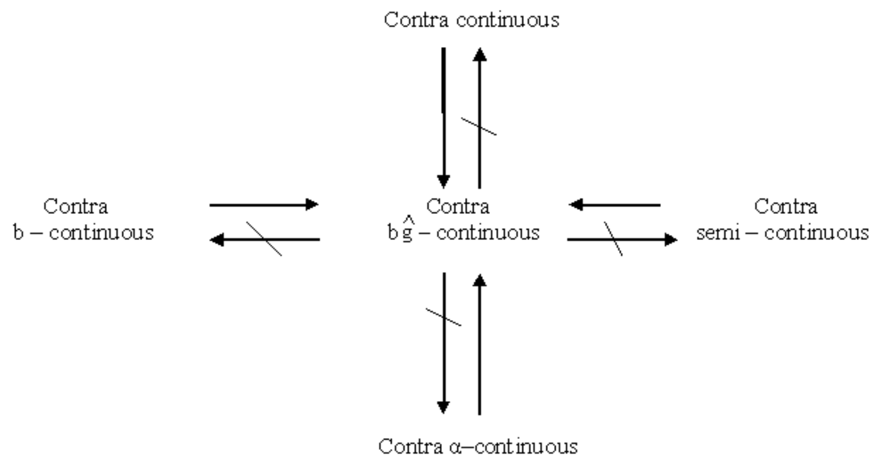
Remark 3.16: The converse of the above theorem need not be true.
(i.e) Every contra bĝ – continuous need not be contra semi–continuous function as shown in the following example.

Example 3.17: Let X=Y={a, b, c, d}
τ = {X, Φ, {b},{a, b},{b, c, d}} and σ = {Y, Φ, {a},{a, c},{a, b, d}}

Define a function f: (X, τ)$\longrightarrow$ (Y, σ) by f(a) = a, f(b) = b, f(c) = c, f(d) = d.
semi–C(X)={X,Φ,{a},{c},{d},{a, c},{a, d},{c, d},{a, c, d}}
bĝ–C(X)={X,Φ, {a},{c},{d},{a, c},{a, d},{c, d},{a, c, d},{a, b, c},{a, b, d}}

Here the inverse image of an open set {a, b, d} in (Y, σ) is {a, b, d} which is bĝ – closed but not semi–closed in (X, τ). Hence f is contra bĝ – continuous, but not contra semi–continuous.

Remark 3.18: From the above discussions and known results, we have the following diagram.



Remark 3.19: The Composition of two contra bĝ – continuous function need not be contra bĝ – continuous.

Example 3.20: Let X=Y=Z={a, b, c, d}
τ = {X, Φ, {a},{a, c},{a, b, d}}, σ = {Y, Φ, {b},{a, b},{b, c, d}} and η = {Z, Φ, {a},{b, c},{a, b, c}}

Define a function f: (X, τ)⟶(Y, σ) by f(a) =c, f(b) = b, f(c) = a, f(d) = d and
g: (Y, σ)⟶(Z, η) by g(a) =a, g(b) = d, g(c) = c, g(d) = b.

Clearly f and g are contra bĝ – continuous. But their composition is not contra bĝ – continuous function, since $(fog)^{-1}$ of an open set {b, c} in (Z, η) is {a, d} which is not bĝ – closed in (X, τ). Hence f*o*g is not contra bĝ-continuous.

Theorem 3.21: The following are equivalent for a function f: (X, τ)⟶(Y, σ)
(a) f is contra bĝ – continuous.
(b) For every closed subset F of Y, f^{-1}(F) is bĝ –open in X.
(c) For each xεX and each closed subset F of Y with f(x) ε F, there exists a bĝ –open set U of X with xεU, f(U) ⊑F.

Proof:
(a) ⟹(b) Obviously follows from the definition.

(b) ⟹ (c) Let F be any Closed subset of Y and let f(x) ε F where xεX. Then by (b), f^{-1}(F) is bĝ –open in X. Also xεf^{-1}(F). Take U = f^{-1}(F). Then U is a bĝ –open set containing x and f(U) ⊑F.

(c) ⟹ (b) Let F be any Closed subset of Y. If xεf^{-1}(F) then f(x) ε F. By (c), there exists a bĝ –open set U_x of X with xε U_x such that f(U_x) ⊑F. Then f^{-1}(F) =∪{U_x: xεf^{-1}(F)}. Hence f^{-1}(F) is bĝ –open in X.

Theorem 3.22: If a function f: (X, τ)⟶ (Y, σ) is contra bĝ – continuous and X is Tbĝ –space, then f is contra b – continuous.

Proof: Let V be any open set in Y. Since f is contra bĝ – continuous, f^{-1}(V) is bĝ –closed in X. Also since X is Tbĝ –space, f^{-1}(V) is b–closed in X. Hence f is contra b – continuous.

Corollary 3.23: If X is Tbĝ –space, then for the function f: (X, τ)⟶(Y, σ), the following statements are equivalent.
i) f is contra b – continuous.
ii) f is contra bĝ – continuous.

Proof:

(i) $\Longrightarrow$ (ii) Let V be any open set in Y. Since f is contra b –continuous, f^{-1}(V) is b –closed in X. From [3] proposition 3.3, "Every b –closed is bĝ –closed ". Therefore, f^{-1}(V) is bĝ –closed in X. Hence f is contra bĝ – continuous.

(ii) $\Longrightarrow$ (i) Let V be any open set in Y. Since f is contra bĝ –continuous, f^{-1}(V) is bĝ –closed in X. Since X is Tbĝ–space, f^{-1}(V) is b –closed in X. Hence f is contra b – continuous.

Theorem 3.24: Let f: (X, τ) $\longrightarrow$(Y, σ) be a function, then the following statements are equivalent.

1. f is bĝ – continuous.
2. For each point x ε X and each open set of Y with f(x) ε V, there exists a bĝ –open set U of X such that x ε U, f(U) ⊑V.

Proof:

(i)$\Longrightarrow$(ii) Let f(x) ε V, then x ε f^{-1}(V). Since f is bĝ – continuous, f^{-1}(V) is bĝ –open in X. Let U=f^{-1}(V), then x ε U and f(U) ⊑V.

(ii)$\Longrightarrow$(i) Let V be any open set in Y and x ε f^{-1}(V), then f(x) ε V. From (ii), there exists a bĝ – open set U_x of X such that x ε U_x⊑ f^{-1}(V) and f^{-1}(V) = ∪{U_x}. Then f^{-1}(V) is bĝ –open in X. Hence f is bĝ – continuous.

Theorem 3.25: If a function f: (X, τ) $\longrightarrow$ (Y, σ) is contra bĝ – continuous and Y is regular, then f is bĝ – continuous.

Proof: Let x ε X and V be an open set in Y with f (x) ε V. Since Y is regular, there exists an open set W in Y such that f(x) εW and Cl(W) ⊑V. Since f is contra bĝ – continuous and Cl(W) is a closed subset of Y with f (x) ε Cl(W). By theorem 3.21, there exists a bĝ –open set U of X with x ε U such that f(U) ⊑ Cl(W). That is f(U) ⊑ V. By theorem 3.24, f is bĝ – continuous.

Definition 3.26: A function f: (X, τ)$\longrightarrow$(Y, σ) is said to be strongly bĝ –continuous if f^{-1}(V) is closed in (X, τ) for every bĝ –closed set V in (Y, σ).

Definition 3.27: A function f: (X, τ) $\longrightarrow$(Y, σ) is said to be perfectly bĝ –continuous if f^{-1}(V) is clopen in (X, τ) for every bĝ –closed set V in (Y, σ).

Definition 3.28: A topological space (X, τ) is said to be bĝ –Hausdorff if for each pair of distinct points x and y in X, there exists bĝ –open subsets U and V of X containing x and y respectively such that $U \cap V = \emptyset$.

Definition 3.29: A topological space (X, τ) is said to be bĝ –ultra Hausdorff if for each pair of distinct points x and y in X, there exists bĝ –clopen subsets U and V of X containing x and y respectively such that $U \cap V = \emptyset$.

Definition 3.30: A space (X, τ) is said to be locally indiscrete if every open subset of X is closed.

Definition 3.31: A Topological space (X, τ) is said to be bĝ –connected if X cannot be written as the disjoint union of nonempty bĝ –open sets.

Definition 3.32: A Topological space (X, τ) is said to be Urysohn space if for each pair of distinct points x and y in X, there exists two open sets U and V in X such that x ε U, y ε V and $cl(U) \cap cl(V) = \emptyset$.

Definition 3.33: A Topological space (X, τ) is said to be Tbĝ*–space if every bĝ –closed set in it is closed.

Example 3.34: Let X= {a, b, c}
τ = {X, Φ, {a}, {b}, {a, b}, {a, c}}
bĝ–C(X)={X,Φ,{b},{c},{a, c},{b, c}}
C(X)={X,Φ,{b},{c},{a, c},{b, c}}
Clearly X is Tbĝ*–space.

Theorem 3.35: Let f: (X, τ)$\longrightarrow$(Y, σ) be surjective, closed and contra bĝ–continuous. If X is Tbĝ*–space, then Y is locally indiscrete.

Proof: Let V be any open set in (Y, σ). Since f is contra bĝ –continuous, f^{-1}(V) is bĝ –closed in (X, τ). Also since X is Tbĝ*–space, f^{-1}(V) is closed in (X, τ). By hypothesis, f is closed and surjective, $f(f^{-1}$(V)) = V is closed in Y. Hence Y is locally indiscrete.

Theorem 3.36: If a function f: (X, τ) ⟶ (Y, σ) is continuous and (X, τ) is locally indiscrete space, then f is contra bĝ – continuous.

Proof: Let V be any open set in (Y, σ). Since f is continuous, f^{-1}(V) is open in (X, τ). Since X is locally indiscrete, f^{-1}(V) is closed in (X, τ). Now from [3] Remark 3.23, f^{-1}(V) is bĝ –closed in X. Hence f is contra bĝ –continuous.

Theorem 3.37: If a function f: (X, τ) ⟶(Y, σ) is contra bĝ –continuous, injective and Y is bĝ –ultra Hausdorff, then the topological space X is bĝ – Hausdorff.

Proof: Let x_1 and x_2 be two distinct points of X. Since f is injective, $f\,(x_1) \neq f(x_2)$ and since Y is bĝ –ultra Hausdorff, there exists bĝ –clopen sets U and V in Y such that $f\,(x_1)\varepsilon U$ and $f(x_2)\varepsilon V$ where $U \cap V = \emptyset$.Also $x_1\ \varepsilon\ f^{-1}(U)$ and $x_2\ \varepsilon\ f^{-1}(V)$ where $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence X is bĝ – Hausdorff.

Theorem 3.38: If a function f: (X, τ) ⟶ (Y, σ) is strongly bĝ –continuous function and g : (Y, σ) ⟶ (Z, η) is contra bĝ –continuous function then gof: (X, τ) ⟶ (Z, η) is contra continuous.

Proof: Let V be any open set in (Z, η). Since g is contra bĝ –continuous, then $g^{-1}(V)$ is bĝ –closed in (Y, σ). Since f is strongly bĝ –continuous, then $f^{-1}\big(g^{-1}(V)\big) = (gof)^{-1}(V)$ is closed in (X, τ). Hence gof is contra continuous.

Theorem 3.39: If a function f: (X, τ) ⟶ (Y, σ) is contra bĝ –continuous, injective and Y is Urysohn space, then the topological space X is bĝ – Hausdorff.

Proof: Let $x_1 and x_2$ be two distinct points of X. Suppose $y_1 = f\,(x_1)$ and $y_2 = f\,(x_2)$.Since f is injective, $x_1 \neq x_2$ then $y_1 \neq y_2$. Since Y is Urysohn, there exists open sets $V_1 and\ V_2$ containing y_1 and y_2 respectively in Y such that $Cl(V_1) \cap Cl(V_2) = \emptyset$. Since f is contra bĝ –continuous, by theorem 3.21 there exists bĝ –open sets U_1 and U_2 containing x_1 and x_2 respectively in X such that f(U_1) ⊑ Cl(V_1) and f(U_2) ⊑ Cl(V_2) . Since $Cl(V_1) \cap Cl(V_2) = \emptyset,\ then\ U_1 \cap\ U_2 = \emptyset$. Hence X is bĝ – Hausdorff.

Theorem 3.40: Let f: (X, τ)⟶ (Y, σ) be a function and g: X⟶ XxY be a graph function defined by $g(x) = (x, f(x))$ for every x ε X. If g is contra bĝ –continuous, then f is contra bĝ – continuous.

Proof: Let V be a closed subset of Y. Then X x V is a closed subset of XxY. Since g is contra bĝ –continuous, $g^{-1}(X\text{x}V)$ is bĝ –open subset of X. Also $g^{-1}(X\text{x}V) = f^{-1}(V)$. Hence f is contra bĝ – continuous.

4. Contra bĝ – Irresolute

Definition 4.1: A function f: (X, τ) ⟶(Y, σ) is called contra bĝ –irresolute if f^{-1}(V) is bĝ–closed in (X, τ) for every bĝ–open set V in (Y, σ).

Example 4.2: Let X=Y= {a, b, c, d}
τ = {X, Φ, {a}, {b, c}, {a, b, c}} and σ = {Y, Φ, {b},{a, b},{b, c, d}}

Define a function f: (X, τ) ⟶ (Y, σ) by f(a) = c, f(b) = a, f(c) = d, f(d) = b.
bĝ–C(X)={X,Φ,{a},{b},{c},{d},{b,c},{c,d},{b,d},{a,d},{a,c,d},{a,b,d},{b,c,d}}
bĝ–O(Y)={Y,Φ,{b},{c},{d},{b,c},{a,b},{b,d},{a,b,c},{a,b,d},{b,c,d}}

Clearly f is contra bĝ – irresolute.

Remark 4.3: The following example show that the concepts of bĝ – irresolute and contra bĝ –irresolute are independent of each other.

Example 4.4: Let X=Y= {a, b, c, d}
τ = {X, Φ, {a},{b, c},{a, b, c}} and σ = {Y, Φ, {b},{a, b},{b, c, d}}

Define a function f: (X, τ) ⟶ (Y, σ) by f(a) = c, f(b) = a, f(c) = d, f(d) = b.
bĝ–C(X)={X,Φ,{a},{b},{c},{d},{b,c},{c,d},{b,d},{a,d},{a,c,d},{a,b,d},{b,c,d}}
bĝ–O(Y)={Y,Φ,{b},{c},{d},{b,c},{a,b},{b,d},{a,b,c},{a,b,d},{b,c,d}}

Clearly f is contra bĝ – irresolute, but not bĝ–irresolute. Since the inverse images of bĝ – closed sets {c, d}, {a, c}, {a, c, d} in (Y, σ) are {a, c},{a, b},{a, b, c} respectively which are not bĝ – closed in (X, τ).

Example 4.5: Let X=Y={a, b, c}
τ = {X, Φ, {a},{b},{a, b},{a, c}} and σ = {Y, Φ, {a},{a, b},{a, c}}

Define a function f: (X, τ) ⟶ (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
bĝ–C(X) = {X,Φ,{b},{c},{a, c},{b, c}}
bĝ–C(Y) = {Y,Φ,{b},{c},{b, c}}

Clearly f is bĝ – irresolute, but not contra bĝ – irresolute. Since the inverse images of bĝ – closed sets {a} and {a, b} in (Y, σ) are {a} and {a, b} respectively which are not bĝ – closed in (X, τ).

Theorem 4.6: Every contra bĝ –irresolute function is contra bĝ – continuous.

Proof: Let V be any open set in (Y, σ). From [3] Remark 3.23, V is also bĝ–open in (Y, σ). Since f is contra bĝ – irresolute, f^{-1}(V) is bĝ –Closed in (X, τ). Hence f is contra bĝ–continuous.

Remark 4.7: The converse of the above theorem need not be true.
(i.e) Every contra bĝ – continuous function need not be contra bĝ –irresolute as shown in the following example.

Example 4.8: Let X=Y={a, b, c}
τ = {X, Φ, {a},{b},{a, b}} and σ = {Y, Φ, {a, b},{c}}

Define a function f: (X, τ) ⟶ (Y, σ) by f(a) = a, f(b) = c, f(c) = b.
bĝ–C(X)={X,Φ,{a},{b},{c},{a, c},{b, c}}
bĝ–O(Y)={Y,Φ,{a},{b},{c},{a, b},{a, c},{b, c}}

Clearly f is contra bĝ –continuous but not contra bĝ –irresolute. Since the inverse image of a bĝ –open set {a,c} in (Y, σ) is {a, b}which is not bĝ–closed in (X, τ).

Remark 4.9: The following example show that the concepts of bĝ– continuous and contra bĝ –continuuous are independent of each other.

Example 4.10: Let X=Y={a, b, c, d}
τ = {X, Φ, {b},{a, b},{b, c, d}} and σ = {Y, Φ, {a},{a, c},{a, b, d}}

Define a function f: (X, τ) ⟶ (Y, σ) by f(a) = a, f(b) = b, f(c) = c, f(d) = d.
bĝ–C(X)={X,Φ,{a},{c},{d},{a,c},{c,d},{a,d},{a,c,d},{a,b,c},{a,b,d}}

Clearly f is contra bĝ – continuous, but not bĝ – continuous. Since the inverse images of closed sets {b, d} and {b, c, d} in (Y, σ) are {b, d} and {b, c, d} respectively which are not bĝ – closed in (X, τ).

Example 4.11: Let X=Y= {a, b, c, d}
τ = {X, Φ, {b},{a, b},{b, c, d}} and σ = {Y, Φ, {a},{a, c},{a, b, d}}

Define a function f: (X, τ) ⟶ (Y, σ) by f(a) = b, f(b) = a, f(c) = d, f(d) = c.
bĝ–C(X)={X,Φ,{a},{c},{d},{a,c},{c,d},{a,d},{a,c,d},{a,b,c},{a,b,d}}

Clearly f is bĝ – continuous, but not contra bĝ – continuous. Since the inverse images of open sets {a} and {a, c} in (Y, σ) are {b} and {b, d} respectively which are not bĝ – closed in (X, τ).

Theorem 4.12: If a function f: (X, τ) ⟶ (Y, σ) is contra bĝ –irresolute function and g : (Y, σ) ⟶ (Z, η) is bĝ –irresolute function then gof: (X, τ) ⟶ (Z, η) is contra bĝ –irresolute.

Proof: Let V be any bĝ –open set in (Z, η). Since g is bĝ –irresolute, then $g^{-1}(V)$ is bĝ –open in (Y, σ). Since f is contra bĝ –irresolute, then $f^{-1}\big(g^{-1}(V)\big) = (gof)^{-1}(V)$ is also bĝ –closed in (X, τ). Hence gof is contra bĝ –irresolute.

Theorem 4.13: If a function f: (X, τ) ⟶ (Y, σ) is contra bĝ –irresolute function and g : (Y, σ) ⟶(Z, η) is bĝ –continuous then gof: (X, τ)⟶ (Z, η) is contra bĝ –continuous.

Proof: Let V be any open set in (Z,η). Since g is bĝ – continuous, then $g^{-1}(V)$ is bĝ –open in (Y, σ). Since f is contra bĝ –irresolute, then $f^{-1}\big(g^{-1}(V)\big) = (gof)^{-1}(V)$ is bĝ –closed in (X, τ). Hence gof is contra bĝ – continuous.

5. Perfectly Contra bĝ – Irresolute

Definition 5.1: A function f: (X, τ)⟶(Y, σ) is called perfecty contra bĝ –irresolute if f^{-1}(V) is bĝ–clopen in (X, τ) for every bĝ–open set V in (Y, σ).

Example 5.2: Let X=Y={a, b, c}
τ = {X, Φ, {a}, {b, c}} and σ = {Y, Φ,{a, c}}

Define a function f: (X, τ)⟶(Y, σ) by f(a) = a, f(b) = b, f(c) = c.
bĝ–O(X)={X,Φ,{a},{b},{c},{a, b},{a, c},{b, c}}
bĝ–C(X)={X,Φ,{a},{b},{c},{a, b},{a, c},{b, c}}
bĝ–O(Y)={Y,Φ,{a},{c},{a, b},{a, c},{b, c}}

Clearly f is perfecty contra bĝ –irresolute, since the inverse images of all bĝ–open sets in Y are bĝ–clopen in X.

Theorem 5.3: Every perfecty contra bĝ –irresolute map is contra bĝ –irresolute

Proof: It directly follows from the definition.

Remark 5.4: The converse of the above theorem need not be true.
(i.e) Every contra bĝ – irresolute map need not be perfecty contra bĝ –irresolute map as shown in the following example.

Example 5.5: Let X=Y={a, b, c}
τ = {X, Φ, {a}} and σ = {Y, Φ,{a, c}}

Define a function f: (X, τ)⟶(Y, σ) by f(a) = b, f(b) = a, f(c) = c.
bĝ–O(X)={X,Φ,{a},{b},{c},{a, b},{a, c}}
bĝ–C(X)={X,Φ,{b},{c},{a, b},{a, c},{b, c}}
bĝ–O(Y)={Y,Φ,{a},{c},{a, b},{a, c},{b, c}}

Clearly f is contra bĝ –irresolute but not perfecty contra bĝ –irresolute. Since the inverse image of bĝ–open set {a, c} in Y is {b, c} which is not bĝ–clopen in X.

Theorem 5.6: Every perfecty contra bĝ –irresolute map is bĝ –irresolute

Proof: It directly follows from the definition.

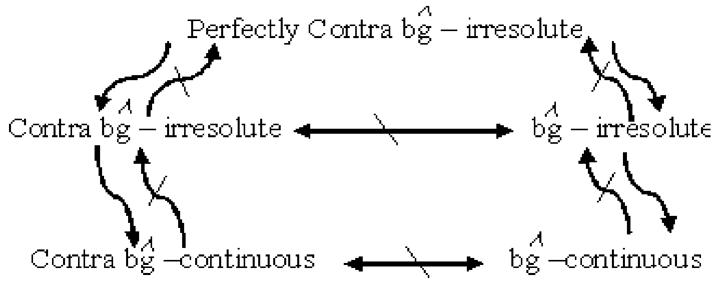
Remark 5.7: The converse of the above theorem need not be true.
(i.e) Every bĝ – irresolute map need not be perfecty contra bĝ –irresolute map as shown in the following example.

Example 5.8: Let X=Y={a, b, c, d}
τ = {X, Φ, {a},{a, c},{a, b, d}} and σ = {Y, Φ, {b},{a, b},{b, c, d}}

Define a function f: (X, τ)⟶ (Y, σ) by f(a) = b, f(b) = a, f(c) = c, f(d) = d.
bĝ–O(X) = {X,Φ,{a},{b},{c},{d},{a, b},{a, c},{a, d},{a, c, d},{a, b, c},{a, b, d}}
bĝ–C(X) = {X,Φ,{b},{c},{d},{b,c},{c,d},{b,d},{a,c,d},{a,b,c},{a,b,d},{b,c,d}}
bĝ–O(Y) = {Y,Φ,{b},{c},{d},{a, b},{b, c},{b, d},{b, c, d},{a, b, c},{a, b, d}}

Clearly f is bĝ–irresolute, but not perfectly contra bĝ–irresolute. Since the inverse images of bĝ –open sets {b}, {a, b}, {b, c} and {b, d} in Y are {a},{a, b},{a, c} and {a, d} respectively which are not bĝ – clopen in (X, τ).

Remark 5.9: From the above discussions and known results, we have the following diagram.



In this diagram, A ⟶ B means A implies B but not conversely. A ⟷ B means A and B are independent of each other.

6. REFERENCES

1. D. Andrijevic, "On b-open sets", *Mat. Vesnik 48(1996), no. 1-2, 59-64*.
2. M.K.R.S.Veera Kumar, "ĝ-closed sets and GLC functions", *Indian Journal Math.*, 43(2) (2001), 231-247.
3. R.Subasree, M. Maria singam, "On bĝ-closed sets in topological spaces", *International journal of Mathematical Archive*, 4(7), 2013, 168-173.
4. N Levine, "Generalized closed sets in topology", *Rend. Circ. Mat. Palermo*, (2) 19(1970) 89-96.
5. R.Subasree, M. Maria singam, "On bĝ-continuous maps and bĝ-open maps in topological spaces", *International journal of Engineering Research and Technology*, Vol 2, Issue 10, 2013, 3443-3451.
6. J. Dontchev , T.Noiri, "Contra Semi-continuous functions", *Math. Pannon 10(2), 1999, 159-168.*
7. Ahmad Al. Omari and Mohd.Salmi MD. cNoorani, "On Generalized b-closed sets", *Bull. Malaysian Mathematical Sciences Society (2) 32(1) (2009), 19-30.*
8. S. Willard, "General Topology", *Addison-Wesley, 1970.*
9. S. Jafari and T.Noiri, "Contra α-continuous functions between Topological spaces", *Iranian International Journal of Science, 2(2), 2001, 153-167.*
10. J. Dontchev , "Contra continuous functions and strongly S-closed spaces", *International Journal of Math Science, 19(1996), 303-310.*
11. M.Caldas, S. Jafari, T. Noiri, M. Simeos, "A generalization of contra continuity via Levines g-closed sets", *Chaos Solitans Fractals 2(2007), 1595-1603.*
12. J.H. Park, "Strongly $\theta - b$-continuous functions", *Acta Math. Hungar*, 110, No.4, 347-359, 2006.

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