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ON REVERSE SUPER EDGE-MAGIC n-STARS

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ABSTRACT

A (V, E) graph G is reverse edge-magic if there exists a bijection f: $V \cup E \rightarrow \{1,2,3,\ldots, v+\varepsilon\}$ such that $\forall$ e= (u,v)∈ E , f(e) – { f(u) + f(v) } = constant . A reverse edge- magic graph is a reverse super edge- magic if f(V(G)) = {1,2,3, --- V} and f (E(G)) = {V+1, V+2 , V+3, ………. V+ ε} . For $n \geq 2$, let $a_1, a_2, a_3, \ldots, a_n$ be a sequence of increasing non-negative integers. A n- star St($a_1, a_2, a_3, \ldots, a_n$) is a disjoint union of n-stars St(a_1) , St(a_2) ,……,St(a_n) . In this paper , we investigate several classes of n-stars that are reverse super edge-magic.

1. INTRODUCTION

In this paper, we consider graphs with no loops or multiple edges. For undefined concepts we refer the reader to [1]. A (V,E) – graph G is with v vertices and ε edges is called reverse edge- magic if there is a bijection f : $V \cup E \rightarrow \{1,2,3,\ldots, v+\varepsilon\}$ such that f(e)- { f(u) + f(v)}= constant. A reverse edge- magic graph is a reverse super edge- magic graph if f(V(G))= {1,2,3,….. v}and f(E(G))= {v+1,v+2,v+3……..v+ε}. This concept of reverse super edge-magic graphs was introduced by Venkata Ramana *et al.* in 2007 [4]. An example of unicyclic graph with 6 vertices and its reverse super edge-magic labeling is shown in Fig 1.

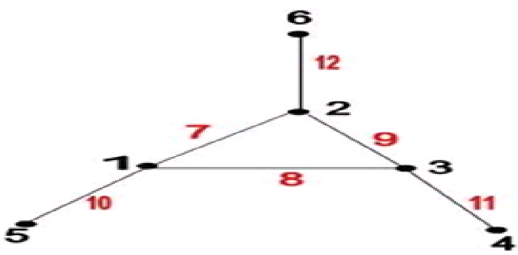

FIG-1

The original concept of reverse super edge-magic graph is due to Venkata ramana *et.al* [4]. They called it reverse super edge-magic graph. They proved the following results:
(1). If a non trivial graph G is reverse super edge-magic, then $|E(G)| \leq 2|V(G)|-3$
(2). A cycle Cn is reverse super edge-magic if and only if n is odd.
(3). A complete bipartite graph Km, n is reverse super edge-magic if and only if m=1 or n=1.
(4). The fan $f_n = P_n + K_1$ is reverse super edge-magic if and only if $1 \leq n \leq 6$.
(5). The ladder $L_n \cong P_n \times P_2$ is reverse super edge-magic where n is odd.
(6). The generalized prism Cm × Pn is reverse super edge-magic if m is odd and $n \geq 2$
(7). Let G = (n, 2)-kite. The graph G is reverse super edge-magic if and only if n is even.
(8). Let G = $K_2 \cup C_n$. The graph G is reverse super edge-magic if n is even ($n \neq 10$).

For $n \geq 2$, let $a_1, a_2, a_3, \ldots, a_n$ be a sequence of increasing non-negative integers. We will use St($a_1, a_2, a_3, \ldots, a_n$) to denote a n-Star, which is a disjoint union of n-stars K(1, a_1), K(1, a_2),……..,K(1,a_n). The graph St($a_1, a_2, a_3, \ldots, a_n$) is shown in Figure 2.

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2. REVERSE SUPER EDGE-MAGIC 2-STARS

By applying the result of Venkata Ramana *et al.* several classes of n-stars are shown to be reverse super edge-magic.

Theorem 1: The 2-star St(n,n+1) is reverse super edge-magic for all $n \geq 1$.

Proof: We will give two different reverse super edge-magic labelings for St(n,n+1).

Method 1: We label the vertices by

$f(x_{1,j}) = 3 + 2j, \quad 1 \leq j \leq n,$ $\qquad f(c_1) = 1$

$f(x_{2,j}) = 2j, \quad 1 \leq j \leq n+1,$ $\qquad f(c_2) = 3.$

Then we see that the edges in K(1,n) has labels {2n+5,2n+7,….,4n+3} and the edges in K(1,n+1) has labels {2n+4,2n+6,…..,4n+4}.Thus St(n,n+1) is reverse super edge-magic with reverse edge-magic number 2n-1.

Method-2: We label the vertices by

$g(x_{1,j}) = 2j-1, \quad 1 \leq j \leq n,$ $\qquad g(c_1) = 2n+3$

$g(x_{2,j}) = 2j, \quad 1 \leq j \leq n+1,$ $\qquad g(c_2) = 2n+1.$

Then we see that the edges in K(1,n) has labels {2n+5,2n+7,….,4n+3} and the edges in K(1,n+1) has labels {2n+4,2n+6,…..,4n+4}.Thus St(n,n+1) is reverse super edge-magic with reverse edge-magic number 1.

Example 1: Reverse super edge – magic labelings for 2-stars St(1,2), St(2,3) and St(3,4) using the above two different methods.

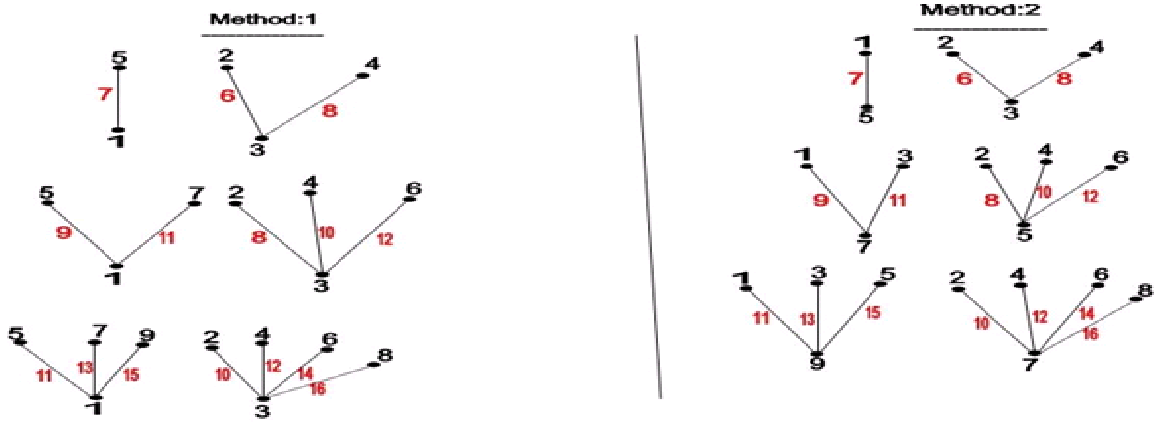

FIG-2

Theorem 2: The 2-star St(m,n) is reverse super edge-magic for all $n \equiv 0(\text{mod } m+1)$.

Proof: Assume n = (m+1)k . The 2- star St(m,(m+1)k) has (m+1)(k+1) +1 vertices. We define a labeling
$f : V(St(m.(m+1)k)) \rightarrow \{1, 2, 3, \ldots(m+1)(k+1)+1\}$ as follows.

$f(c_1)= (m+1)(k+1)+1$ $\qquad f(c_2) = (m+1)(k+1) - k$

$f(x_{1,j}) = 1 +(j-1)(k+1), \; 1 \leq j \leq m.$ $\qquad f(x2,i) = 1+i, 1 \leq i \leq k;$

$f(x2,i) = i +2, \; k + 1 \leq i \leq 2k.$

Hence $f^+(E(St(1,2k)) = \{k+5,k+6,k+7,\ldots\ldots..,2k+6\}$.

Corollary 1: The 2-Star St(1,n) is reverse super edge-magic if n is even.

Example 2: A reverse super edge-magic labeling of the 2-star St(1,n) .

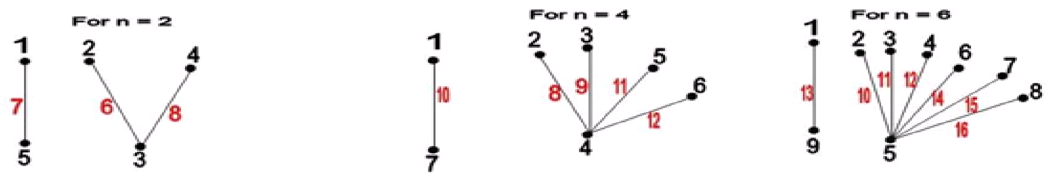

FIG:3

Corollary 2: The 2-Star St(2,n) is reverse super edge-magic if n is a multiple of 3.

Example 3: Reverse super edge-magic labeling for 2-star St(2,n), where n=3,6.

FIG-4

3. REVERSE SUPER EDGE-MAGIC 3-STARS

Theorem 3: The 3-star St(1,1,n) is reverse super edge-magic for all n ≥ 1.

Proof: A reverse super edge-magic labeling of St(1,1,n) is given as follows: Define f:V(St(1,1,n))→ { 1,2,3,….,n+5} as follows:

f(c1) = 1, f(c2) =3; f(c3) = 2

f(x1,1)= 5; f(x2,1) =4; f(x3,i) = 5+i, $1 \le i \le n$.

It can be easily verified that f induces a reverse super edge-magic labeling.

Example 4: A reverse super edge-magic labeling for 3-star St(1,1,6) .

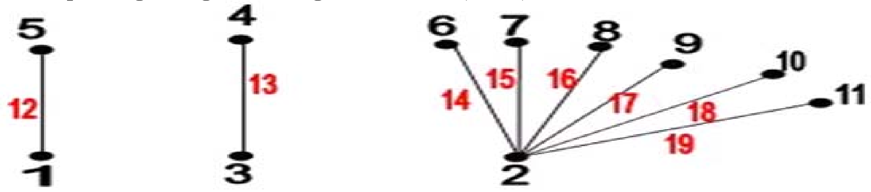

FIG-5

Theorem 4: The 3-star St (1, 2, n) is reverse super edge-magic for all n ≥ 2.

Proof: A reverse super edge-magic labeling of St(1,2,n) is given in figure 6.

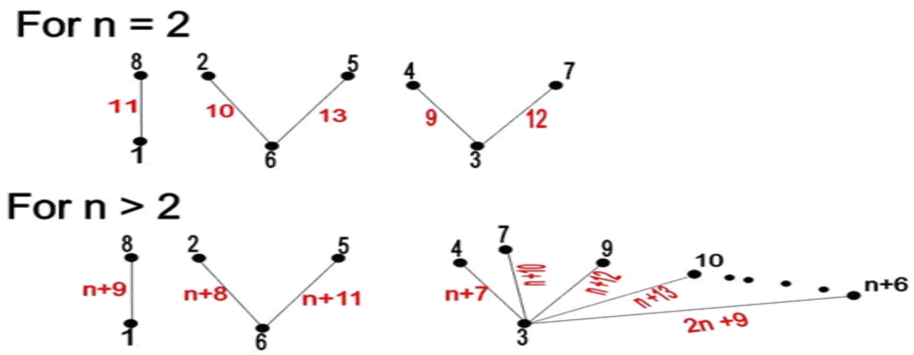

FIG-6

Theorem 5: The 3-star St(1,n,n) is reverse super edge-magic for all n ≥ 1.

Proof: A reverse super edge-magic labeling of St(1,2,n) is given in figure 7.

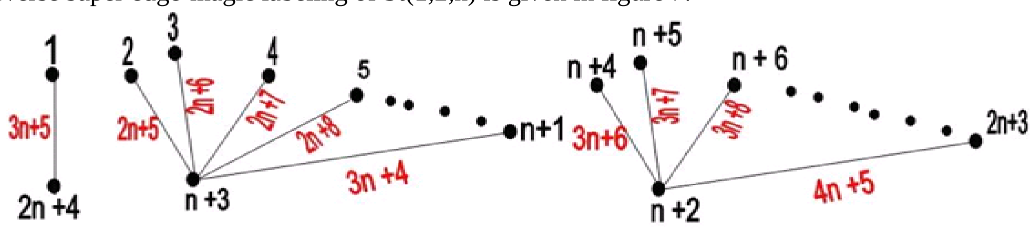

FIG-7

Theorem 6: The 3-star St(2,2,n) is reverse super edge-magic for all n ≥ 2.

Proof: A reverse super edge-magic labeling of St(2,2,n) is given in figure 8.

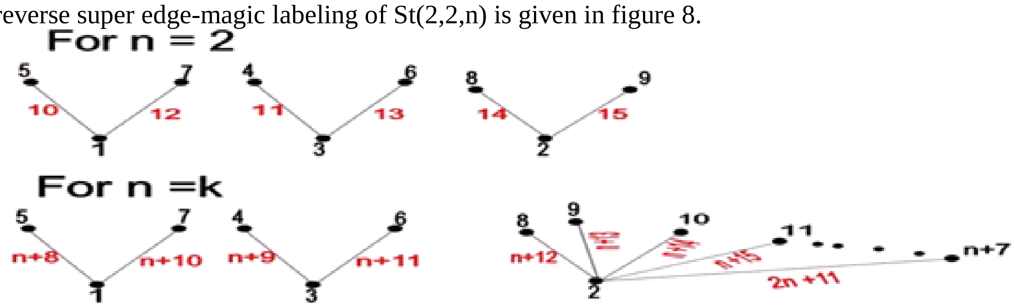

FIG- 8

Theorem 7: The 3-star St(2,3,n) is reverse super edge-magic for all n ≥ 3.

Proof: A reverse super edge-magic labeling of St(2,3,n) is given in figure 9.

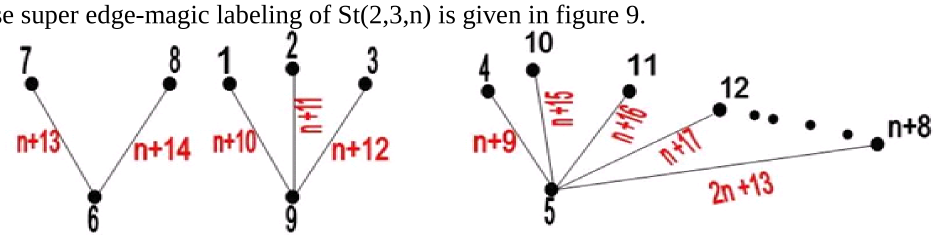

FIG-9

4. REVERSE SUPER EDGE-MAGIC 4-STARS

Theorem 8: The 4-star St(1,1,2,n) is reverse super edge-magic for all n ≥ 2.

Proof: A reverse super edge-magic labeling for St(1,1,2,n) for n ≥ 2 is shown in figure 10.

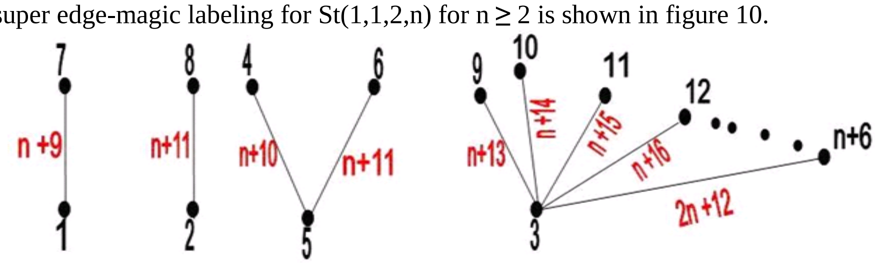

FIG-10

Theorem 9: The 4-star St(1,1,3,n) is reverse super edge-magic for all n ≥ 3.

Proof: A reverse super edge-magic labeling for St(1,1,3,n) for n □ 3 is shown in figure11 .

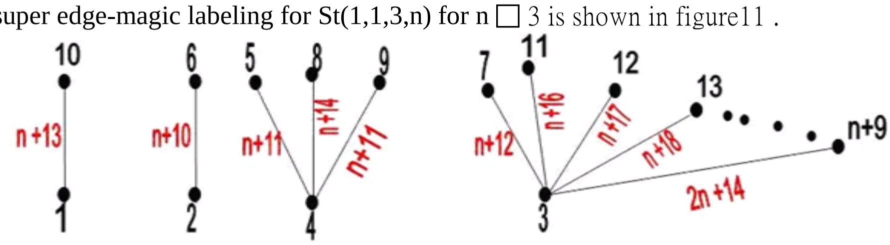

FIG-11

Theorem 10: The 4-star St(1,2,2,n) is reverse super edge-magic for all n ≥2.

Proof: A reverse super edge-magic labeling for St(1,2,2,n) for n ≥2 is shown in Figure12

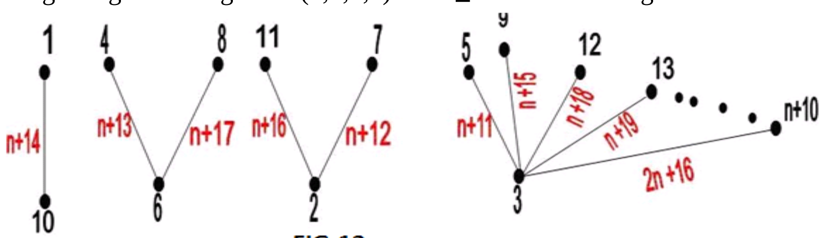

FIG-12

Theorem 11: The 4-star St(2,2,2,n) is reverse super edge-magic for all n≥2

Proof: A reverse super edge magic labeling for St(2,2,2,n) for n ≥ 2 is shown in figure 13.

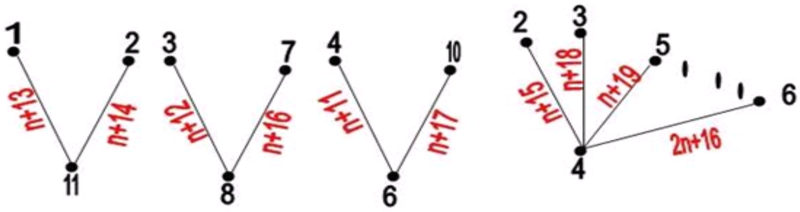

FIG-13

We propose the following

CONJECTURE

Given any odd integer n ≥ 2. Let $a_1, a_2, a_{3,} \ldots\ldots, a_n$ be a sequence of increasing non-negative integers. The n-star St($a_1, a_2, a_{3,} \ldots\ldots, a_n$) is reverse super edge-magic.

REFERENCES

1. G.Chartrand and L.Lesniak, Graphs and digraphs, 2nd edition, Wadsworth & Brooks /Cole, Monterey 1986.
2. H.Enomonoto,A.S.Llado, T.Nagamigana, A.Ringel, Super edge-magic graphs, SUT J. Math. Vol 34, No-2 (1998), 105-109.
3. J.A.Gallian, A dynamic survey of graph labeling ,Electron .J. Combin., 5, 2001 ed, 95 pages.
4. S.Venkata Ramana ,S.Shareefh basha, Reverse super edge-magic labeling of a graph, Ph.D. Thesis.

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