International Journal of Mathematical Archive-6(1), 2015, 51-55
Available online through www.ijma.info ISSN 2229 – 5046

AN ACCURACY FUNCTION FOR INTERVAL-VALUED INTUITIONISTIC FUZZY NUMBERS

Bhagawati Prasad Joshi*[1] and Pushpendra Singh Kharayat[2]

*[1]Department of Applied Sciences,
Seemant Institute of Technology, Pithoragarh- 262 501, India.*

*[2]Deptment of Electrical Engineering,
Seemant Institute of Technology, Pithoragarh- 262 501, India.*

(Received On: 07-10-14; Revised & Accepted On: 31-01-15)

ABSTRACT

The theory of fuzzy sets (FSs) finds more and more attention for researchers since its appearance, so many generalizations are available in literature for various objectives. Atanassov [12] and Atanassov and Gargov [10] introduced the notions of intuitionistic fuzzy sets (IFSs) and interval-valued intuitionistic fuzzy sets (IVIFSs) respectively and are more suitable tools for dealing with imprecise information than FSs. IFSs and IVIFSs are very powerful tools in modeling real life problems such as: in decision-making, data analysis, artificial intelligence etc. However, the comparability of interval-valued intuitionistic fuzzy numbers (IVIFNs) is an important part for the interval-valued intuitionistic fuzzy decision-making problems. In this direction, Xu [25], Xu and Chen [21], Ye [9] and Nayagam, et al. [19] made efforts and proposed different accuracy functions for ranking of IVIFNs. In this study, a new novel accuracy function is introduced and studied for IVIFNs. The effectiveness and validity of the developed approach over existing methods are illustrated by numerical examples. Hence, the proposed approach would provide a powerful way to the decision-makers to make decisions under IVIFNs.

Keywords- *Interval-valued intuitionistic fuzzy numbers, Accuracy function, Multi-criteria decision-making.*

I. INTRODUCTION

After introducing the concept of fuzzy set (FSs) in [14], Atanassov [11], [12] introduced the notion of intuitionistic fuzzy sets (IFSs) by incorporating a hesitancy degree in FSs and is more powerful tool to deal with uncertainty and vagueness in real applications than FSs and received more and more attention since its appearance. [20] gave the concept of vague set but [7] proved that the notion of vague set is same as that of IFSs. [1] and [16] studied the notion of interval-valued fuzzy sets (IVFSs) and generalized by [10] by introducing the notion interval-valued intuitionistic fuzzy sets (IVIFSs), which is characterized by a membership degree range and a non-membership degree range. [13] defined some operators over IVIFSs. [3], [15], [8], [23], [24] and [2] developed some methods for multi-criteria fuzzy decision-making methods based on IFSs. The pioneering work [10] has been successfully applied by [25], [21], [22], [9] and [5] for multi-criteria fuzzy decision-making methods under IVIFSs.

In multi-criteria fuzzy decision-making problems under IVIFSs, the notion of ranking of interval-valued intuitionistic fuzzy numbers (IVIFNs) plays a vital role in real life applications. [25] and [21] proposed score function and accuracy function to rank IVIFNs. [9] and [19] also proposed a novel accuracy function to rank IVIFNs. One of main reason of the present study is that in some situations the existing techniques for ranking IVIFNs using a score function or an accuracy function do not give sufficient information about alternatives.

To do this, a new novel accuracy function is introduced to overcome this situation. The remaining part of this paper is as: in Section 2, we briefly studied the definitions of IFS, IVIFS and existing accuracy functions for IVIFNs. In Section 3, illustrative examples are given to show the inapplicability of the existing functions. Hence, a new novel accuracy function by taking into account the unknown degree (hesitancy degree) of IVIFNs is introduced in section 4. Section 5 deals with the applicability of the proposed approach over existing methods to show that the proposed function is more reasonable and effective to decision makers than the methods developed by [25], [9] and [19]. Finally the silent features are presented under the heading conclusion.

Corresponding Author: Bhagawati Prasad Joshi[1], [1]Department of Applied Sciences,
Seemant Institute of Technology, Pithoragarh- 262 501, India.*

II. PRELIMINARIES

In this section, some definitions related to the present study are studied, summarized and presented.

A. IFS [11]

Let *X* be a non empty set called a universe of discourse. An IFS *A* in *X* is of the form $A=\{\langle x,t_A(x), f_A(x)\rangle : x\in X\}$, where the functions $t_A : X\rightarrow[0,1]$ and $f_A : X\rightarrow[0,1]$ define the "degree of membership" and the "degree of non-membership" of the element $x\in X$ respectively, and for every element *x* of *X*, $0\le t_A(x)+f_A(x)\le 1$. The degree of non-determinacy (uncertainty) for each element *x* of *X* in the IFS *A*, is defined by $\pi_A(x)=1-t_A(x)-f_A(x)$, where $\pi_A(x)\in[0,1]$.

B. IVIFS [10]

Let D[0,1] be the set of all closed subintervals of the interval [0,1] and *X* be ordinary finite non-empty sets. An IVIFS in *X* is of the form $A=\{\langle x,t_A(x), f_A(x)\rangle : x\in X\}$, where $t_A : X\rightarrow D[0,1]$, and $f_A : X\rightarrow[0,1]$, with the condition $0\le \sup_x t_A(x)+\sup_x f_A(x)\le 1$.

The intervals $t_A(x)$ and $f_A(x)$ denote, the degree of belongingness and the degree of non-belongingness respectively of the element *x* to the set *A*. Thus for every $x\in X$, $t_A(x)$ and $f_A(x)$ are closed intervals and their lower and upper end points are, respectively, denoted by $t_{AL}(x)$, $t_{AU}(x)$ and $f_{AL}(x)$, $f_{AU}(x)$. Hence, we can write an interval-valued intuitionistic fuzzy set by $A=\{\langle x,[t_{AL}(x),t_{AU}(x)],[f_{AL}(x), f_{AU}(x)]\rangle : x\in X\}$, where $0\le t_{AU}(x)+f_{AU}(x)\le 1$, $t_{AL}(x)\ge 0$, $f_{AL}(x)\ge 0$.

For the simplicity and our convenience, we shall denote the set of all IVIFSs in *X* by IVIFS(*X*) and an IVIFN by $A=([a,b],[c,d])$, where $[a,b]\subset[0,1]$, $[c,d]\subset[0,1]$ and $0\le b+d\le 1$. Let $A,B\in \text{IVIFS}(X)$. *A* is subset of *B*, *iff* $t_{AL}(x)\le t_{BL}(x), t_{AU}(x)\le t_{BU}(x)$ and $f_{AL}(x)\ge f_{BL}(x)$, $f_{AU}(x)\ge f_{BU}(x), \forall x\in X$, denoted by $A\subseteq B$. Equality of *A* and *B* holds and defined by $A=B\Leftrightarrow A\subseteq B$ and $B\subseteq A$ i.e. $t_{AL}(x)=t_{BL}(x), t_{AU}(x)=t_{BU}(x)$ and $f_{AL}(x)=f_{BL}(x), f_{AU}(x)=f_{BU}(x), \forall x\in X$.

C. Existing Accuracy Functions

Let $A=([a,b],[c,d])$ be an IVIFN. Then

1) The accuracy function *H* of an IVIFN developed in [25], based on the unknown degree can be represented as
$$H(A)=\frac{a+b+c+d}{2}. \tag{1}$$
2) The novel accuracy function *M* of an IVIFN developed in [9], based on the unknown degree is given by the following formula
$$M(A)=\frac{a-(1-a-c)+b-(1-b-d)}{2}=a+b-1+\frac{c+d}{2} \tag{2}$$
3) The novel accuracy function *L* of an IVIFN introduced in [19], based on the unknown degree is defined by
$$L(A)=\frac{a+b-d(1-b)-c(1-a)}{2}. \tag{3}$$

III. MOTIVATION TO THE DEVELOPMENT OF NEW NOVEL ACCURACY FUNCTION

This section deals with the inapplicability of the existing accuracy functions for IVIFNs. The invalidity of the existing accuracy functions are illustrated with examples.

Example 3.1: Comparability of two different IVIFNs $A_1 = ([0.2,0.4],[0.3,0.5])$ and $A_2 = ([0.3,0.5],[0.2,0.4])$.

By using eq. (1), we can obtain $H(A_1) = 0.7$ and $H(A_2) = 0.7$. Here, we do not know which alternative is best and no conclusion can be drawn about the alternatives. If we apply eq. (2) to Example 3.1, we get $M(A_1) = 0.0$ and $M(A_2) = 0.1$, which shows that alternative A_2 is better than alternative A_1. It can be verified that $A_1 \subset A_2$ using the order relation. So, the accuracy function proposed in [9] is more efficient than that of [25].

Example 3.2: If corresponding to two different IVIFNs $A_1 = ([0.2,0.3],[0.3,0.4])$ and $A_2 = ([0.1,0.4],[0.1,0.6])$.

By applying eq. (1) and eq. (2), we obtain $H(A_1) = 0.6$, $H(A_2) = 0.6$ and $M(A_1) = -0.15$, $M(A_2) = -0.15$ respectively. In this case, both definitions fail and we can not select the best alternative.

Example 3.3: For different IVIFNs $A_1 = ([0.2,0.3],[0.5,0.6])$ and $A_2 = ([0.3,0.4],[0.2,0.4])$.

By using eq. (1) and eq. (2), we can find $H(A_1) = 0.8$, $H(A_2) = 0.65$ and $M(A_1) = 0.05$, $M(A_2) = 0.0$ respectively. Both definitions give the result that the alternative A_1 is better than alternative A_2. But using the order relation, it is clear that $A_1 \subset A_2$. Hence both definitions give absurd conclusion.

Example 3.4: For two different IVIFNs $A_1 = ([0.25,0.35],[0.15,0.45])$ and $A_2 = ([0.27,0.33],[0.05,0.55])$.

By using eq. (1) and eq. (2), we have $H(A_1) = 0.6$, $H(A_2) = 0.6$ and $M(A_1) = -0.1$, $M(A_2) = -0.1$ respectively. By taking eq. (3), we get $L(A_1) = 0.0975$ and $L(A_2) = 0.0975$. In this case eq. (1), eq. (2) and eq. (3) seems to be inapplicable to compare the IVIFNs.

In such situations, the accuracy functions proposed in [25], [9] and [19] did not tell us about the superiority of an IVIFN over others and leads to give difficult even absurd decision for decision makers. Therefore it is necessary to account this issue and needs to develop another function which is helpful for decision maker for taking decisions under IVIFNs.

IV. A NEW NOVEL ACCURACY FUNCTION FOR AN IVIFN

In this section, we defined a new novel accuracy function and proposed. Further, some properties of the developed approach are studied and illustrated with example.

Let $A = ([a,b],[c,d])$ be an IVIFN, a new approach to the accuracy function P of an IVIFN, based on the unknown degree is proposed and defined by

$$P(A) = \frac{a + b + ab - cd}{3}. \tag{4}$$

Let $A = ([a_1,b_1],[c_1,d_1])$ and $B = ([a_2,b_2],[c_2,d_2])$ be two IVIFNs. Using eq. (4), it is easy to prove that the accuracy function P has some properties as follows:

1) $P(A) = 0$ iff $a_1 + b_1 + a_1b_1 = c_1d_1$
2) $P(A) = 1$ iff $a_1 + b_1 + a_1b_1 = 3 + c_1d_1$.

Theorem 4.1: Let $A, B \in \text{IVIFS}(X)$ and for the comparable of A and B, if $A \le B$ then $P(A) \le P(B)$.

Proof: Let $A = ([a_1,b_1],[c_1,d_1])$ and $B = ([a_2,b_2],[c_2,d_2])$ be two IVIFNs such that $A \subseteq B$. Using eq. (4), we have

$$P(A) = \frac{a_1 + b_1 + a_1b_1 - c_1d_1}{3} \quad \text{and} \quad P(B) = \frac{a_2 + b_2 + a_2b_2 - c_2d_2}{3}.$$

Now,
$3[P(B)-P(A)]=(a_2-a_1)+(b_2-b_1)+a_2b_2-a_1b_1+c_1d_1-c_2d_2$.

Given $A\subseteq B \Rightarrow a_1\le a_2$, $b_1\le b_2$, $c_1\ge c_2$, so $a_1b_1\le a_2b_2$ and $c_1d_1\ge c_2d_2$. Hence, $3[P(B)-P(A)]\ge 0$ $\Rightarrow P(A)\le P(B)$.

Example 4.1: Let $A_1=([0.2,0.4],[0.2,0.3])$ and $A_2=([0.3,0.5],[0.1,0.2])$ be IVIFNs for two different alternatives. Clearly $A_1\subset A_2$ and eq. (4), we have $P(A_1)=0.206667, P(A_2)=0.31 \Rightarrow P(A_1)<P(A_2)$
.

V. APPLICABILITY OF THE PROPOSED APPROACH OVER THE EXISTING METHODS

In this section, the proposed new novel accuracy function is used to the examples of Section 3, where the existing methods fail to compare two IVIFNs.

By using proposed new novel accuracy function given in eq. (4) to Example 3.1, we find $P(A_1)=0.176667$ and $P(A_2)=0.29$, which indicates that alternative A_2 is better than A_1. The proposed accuracy function is applicable where eq. (1) is not.

By using eq. (4) to Example 3.2, we find $P(A_1)=0.146667$ and $P(A_2)=0.16$, which indicates that alternative A_2 is better than A_1. The proposed accuracy function is applicable where eq. (1) and eq. (2) are not.

Using eq. (4) to Example 3.3, we find $P(A_1)=0.086667$ and $P(A_2)=0.246667$, which indicates that alternative A_2 is better than A_1. As $A_1\subseteq A_2$, hence the proposed approach leads us accurate decision but eq. (1) and eq. (2) give absurd decision.

Using eq. (4) to Example 3.4, we find $P(A_1)=0.206667$ and $P(A_2)=0.220533$, which indicates that alternative A_2 is better than A_1. The proposed accuracy function is applicable where eq. (1), eq. (2) and eq. (3) are not telling us about the superiority of an IVIFN.

Hence it is clear from the above section the proposed accuracy function compares all IVIFNs correctly. It may be useful to decision makers to compare all IVIFNs correctly which are not covered so far.

VI. CONCLUSION

In this paper, a new novel accuracy function is introduced and studied for IVIFNs. This work is come into existence due the large applicability of IVIFNs in decision making, data analysis, artificial intelligence and so many. But the comparability of IVIFNs is always an important task for every decision maker to deal with real life problems under intuitionistic fuzzy environment. So, the present study comes into picture due to the inapplicability and illogicality of the existing methods. Hence, a new novel accuracy function is introduced to compares IVIFNs and it compares all comparable IVIFNs correctly; this shows the superiority and wide applicability of the developed approach to decision making process over others.

ACKNOWLEDGMENT

The present study is partially supported by the facilities provided by Seemant Institute of Technology, Pithoragarh, INDIA.

REFERENCES

[1] B. Turksen, "Interval-valued fuzzy sets based on normal forms," Fuzzy Sets and Systems, vol. 20, pp. 191–210, 1986.
[2] C. Tan and X. Chen, "Intuitionistic fuzzy Choquet integral operator for multi-criteria decision making," Expert Systems with Applications, vol. 37, pp. 149–157, 2010.
[3] D. F. Li, "Multi-attribute decision making models and methods using intuitionistic fuzzy sets," Journal of Computer and System Sciences, vol. 70, pp. 73–85, 2005.

[4] D. H. Hong and C. H. Choi, "Multicriteria fuzzy decision-making problems based on vague set theory," Fuzzy Sets and Systems, vol. 114, pp. 103–113, 2000.
[5] D. Yu, Y. Wu and T. Lu, "Interval-valued intuitionistic fuzzy prioritized operators and their application in group decision making," Knowledge-Based Systems, vol. 30, pp. 57–66, 2012.
[6] H. B. Mitchell, "Ranking intuitionistic fuzzy numbers," International Journal of Uncertainty, Fuzziness and Knowledge Based Systems, vol. 12(3), pp. 377–386, 2004.
[7] H. Bustine, and P. Burillo, "Vague sets are intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 79, pp. 403–405, 1996.
[8] H. W. Liu and G. J. Wang, "Multi-criteria decision-making methods based on intuitionistic fuzzy sets," European Journal of Operational Research, vol. 179, pp. 220–233, 2007.
[9] J. Ye, "Multi-criteria fuzzy decision-making method based on a novel accuracy function under interval-valued intuitionistic fuzzy environment," Expert Systems with Applications, vol. 36, pp. 6899–6902, 2009.
[10] K. Atanassov and G. Gargov, "Interval-valued intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 31(3), pp. 343–349, 1989.
[11] K. Atanassov, "Intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 20, pp. 87–96, 1986.
[12] K. Atanassov, "Intuitionistic fuzzy sets," VII ITKR's session, Sofia, June, 1983.
[13] K. Atanassov, "Operators over interval-valued intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 64(2), pp. 159–174, 1994.
[14] L. A. Zadeh, "Fuzzy sets," Information and Control, vol. 8(3), pp. 338–356, 1965.
[15] L. Lin, X. H. Yuan and Z. Q. Xia, "Multi-criteria decision-making methods based on intuitionistic fuzzy sets," Journal of Computer and System Sciences, vol. 73, pp. 84–88, 2007.
[16] M. B. Gorzaleczany, "A method of inference in approximate reasoning based on interval-valued fuzzy sets," Fuzzy Sets and Systems, vol. 21, pp. 1–17, 1987.
[17] S. M. Chen and J. M. Tan, "Handling multi-criteria fuzzy decision-making problems based on vague set theory," Fuzzy Sets and Systems, vol. 67, pp. 163–172, 1994.
[18] V. L. G. Nayagam, G. Venkateshwari and G. Sivaraman, "Ranking of intuitionistic fuzzy numbers," In Proceedings of the IEEE international conference on fuzzy systems (IEEE FUZZ 2008), pp. 1971–1974, 2008.
[19] V. L. G. Nayagam, S. Muralikrishnan and G. Sivaraman, "Multi-criteria decision-making method based on interval-valued intuitionistic fuzzy sets," Expert Systems with Applications, vol. 38, pp. 1464–1467, 2011
[20] W. L. Gau and D. J. Buehrer, "Vague sets," IEEE Transactions on Systems, Man and Cybernetics, vol. 23(2), pp. 610–614, 1993.
[21] Z. S. Xu and J. Chen, "An approach to group decision making based on interval-valued intuitionistic fuzzy judgment matrices," System Engineer – Theory and Practice, vol. 27(4), pp. 126–133, 2007.
[22] Z. S. Xu and J. Chen, "On geometric aggregation over interval-valued intuitionistic fuzzy information," In FSKD, 4th international conference on fuzzy systems and knowledge discovery (FSKD 2007), vol. 2, pp. 466–471, 2007.
[23] Z. S. Xu, "Intuitionistic fuzzy aggregation operators," IEEE Transactions on Fuzzy Systems, vol. 15(6), pp. 1179–1187, 2007.
[24] Z. S. Xu, "Intuitionistic preference relations and their application in group decision making," Information Science, vol. 177(11), pp. 2363–2379, 2007.
[25] Z. S. Xu, "Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making," Control and Decision, vol. 22(2), pp. 215–219, 2007.

Source of support: Seemant Institute of Technology, Pithoragarh, India.
Conflict of interest: None Declared