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TWO DIMENSIONAL MATHEMATICAL MODEL TO STUDY THERMAL CHANGES AFTER AN ABNORMALITY DURING HEALING

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ABSTRACT

Skin is the largest organ of the integumentary system plays an important role to maintain the body core temperature (T_b) at 37°C. Any disturbance in the temperature regulation may cause lots of abnormality in the body. Paper deals with the study of thermal variation of tissues of human peripheral region using finite element method during wound healing process after plastic surgery. Study of complex biological system needs more accuracy to interpret results. Mathematical model is developed by incorporating physiological parameters like thermal conductivity, rate of metabolism, blood mass flow rate, latent heat, rate of evaporation etc as they are very important aspects during the process and play an important role. The temperature variations are noted for tissue of donor site (normal region) as well as tissues after surgery (abnormal region). The information obtained from this model can be of great use for biomedical scientists for application in treatment of various diseases as well as helpful to develop protocols for medical purpose

Key words: *Wound healing process, plastic surgery, skin graft, thermal conductivity, rate of metabolism, blood mass flow rate, latent heat, rate of evaporation and Finite element method.*

1. INTRODUCTION

Mathematical modeling of physical and biological systems for heat flow is very complex in nature. The parameters involved in the models are fully based on physical and chemical rules. Such models deal with initial and boundary value problem. Heat transfer in most of the biological and other non homogeneous composite media is concerned not only with rates of metabolism, thermal conductivity, evaporation, conduction, convection and radiation but also it is concerned with convective heat transfer (CHT) through perfusion. Convective heat exchange is the rate of net heat transfer per unit area between the surface and the moving fluidic medium per unit temperature difference between the surface and the medium [1].

A lot of work has been done to study thermal changes in the biological and non biological systems for different cases and conditions. Carslaw and Jaeger [2], gave the model for heat transfer, in which heat transfer through convection was not incorporated but Perl[3-4] gave the mathematical model by combining the term CHT using Fick's perfusion principle for convective heat and mass transfer in the invivo tissue.

Partial and ordinary differential equations governed from mathematical modeling need suitable mathematical techniques to obtain better results. There are various methods like Laplace Transformation, Fourier Transformation, Finite Difference Method, Finite Analysis Method and Finite Element Methods. A computational method for modeling biological systems based on the theory of stabilized finite element methods yield excellent solutions as compared to other methods. The Finite Element Method (FEM) is the most popular and well-developed method for analyzing and solving problems associated with imbedded geometrical singularities related to biological systems. This method is not only applicable for bounded region but deals with unbounded region also[5]. This method is suitable for analyzing heat variations in human body for normal and abnormal conditions. The heat equation (1.1) is extensively used by Saxena[6], Saxena and Arya[7] and Arya[8] to study the thermal changes in human with respect to different physiological parameters. Saxena and Pardasani[9-10] have studied thermal variation in skin and subcutaneous tissues for abnormalities like tumors. Shakya and Pardasani studied thermal changes due to uniformly perfused tumor[11] by using Finite element method. They also used similarity transformation to study heat flow in human peripheral region for unsteady state case [12].

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Study of temperature variation during wound healing process after plastic surgery is hot and new topic for researchers and work has been less detailed conducted in this regard. Adams *et al.* [13-14] have given mathematical models based on the Critical Size Defect (CSD), is defined as the smallest wound that does not heal within the lifetime of an animal or human being. Wound healing, is a complicated and combination of several processes like chemotaxis (movement of cells induced by a concentration gradient), neovascularization, synthesis of extracellular matrix proteins, and scar modeling [15]. Some models incorporate cell mitosis, cell proliferation, cell death, capillary density, oxygen supply and growth factor generation coupled to a cell density[16-17] and the advection dominated models[18] of wound healing using numerical techniques.

2. STATEMENT OF THE PROBLEM

Perl's Bio heat partial differential equation in two dimensional transient state case for heat distribution in the tissues of SST region of human body can be written as [2-3]

$$\left(\frac{\partial}{\partial x}\right)\left(K\frac{\partial T}{\partial x}\right)+\left(\frac{\partial}{\partial y}\right)\left(K\frac{\partial T}{\partial y}\right)+m_b c_b\left(T_b-T\right)+S=\rho c\frac{\partial T}{\partial t} \tag{1.1}$$

Here the effect of metabolic heat generation and blood mass flow are given by the terms S and $m_b c_b(T_b\text{-}T)$ respectively. T_b, K, ρ, c, m_b and c_b are body core temperature, thermal conductivity, density and specific heat of tissue, blood mass flow rate and specific heat of blood respectively. Right hand side of eq.(1) shows the storage of heat in tissues. The first two terms of the left hand side represents conduction of heat in the tissues, caused by the temperature gradient and third term is for heat transport between the tissues and microcirculatory blood perfusion. The last term represent heat generation due to metabolism. The outer surface of the body is exposed to the environment and heat loss at this surface takes place due to conduction, convection, radiation and evaporation. Thus the boundary conditions at the outer surface

$$-K\frac{\partial T}{\partial n}=h(T-T_a)+LE \quad for \quad t>0 \tag{1.2}$$

Where h heat transfer coefficient, T_a is atmospheric temperature, L and E are respectively, the latent heat and rate of evaporation and $\frac{\partial T}{\partial n}$ is the partial derivatives of T along the normal to the skin surface and for the inner surface

$$T=T_b \quad for \quad t\geq 0 \tag{1.3}$$

The outer surface of the skin assumed to be insulated at time *t=0* and hence the initial condition is given by

$$T(x,0)=T_b=37^oC \tag{1.4}$$

2.1 Physiological Facts

Wound healing or wound repair is the body's natural process of regenerating dermal and epidermal *tissue*. In normal skin, the epidermis (outermost layer) and dermis (inner or deeper layer) exist in a steady-state equilibrium. These layers form a protective barrier against the external environment. Once the protective barrier is broken i.e. wound appears, the normal physiologic process of wound healing immediately starts automatically. The entire wound healing process is a set of complex biochemical events and that begins at the moment of injury and can continue for months to years to repair the damaged tissues immediately after the wounding. Successful wound healing requires adequate blood and nutrients to be supplied to the site of damage. The overall health and nutritional status of the patient also influences the outcome of the damaged tissue [18].

The physiological properties depend on the different layers of the SST Region like they are almost uniform in subcutaneous tissues therefore numerical values of K, M and S are assumed for this region is constant. In the dermis layer, these values are variable, so numerical values are K, M and S are the function of time and space. In epidermis layer, due to no blood vessels, the values of M and S are taken zero and K as constant.

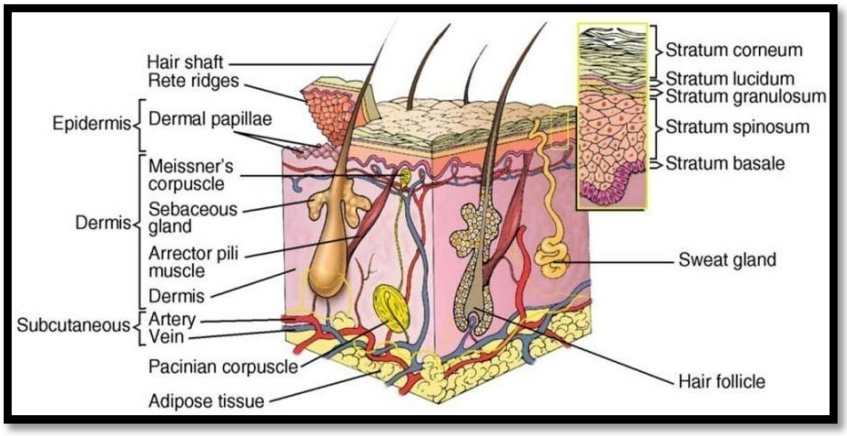

Fig. 1: Anatomy of human skin

The values of physiological parameters for unwounded and wounded sites are different from each other. These values are almost negligible in transplanted tissues just after the surgery and they increase gradually with respect to time, causing increase in tissue temperature of human body therefore mathematically it can be written as:

$$K(x,t)= \varsigma(t)\sum_{d=0}^{1}\alpha_d x^d,\ M(x,t)=\psi(t)\sum_{d=0}^{1}\beta_d x^d,\quad S(x,t)=\zeta(t)\sum_{d=0}^{1}\gamma_d x^d \tag{1.5}$$

Here the thickness of SST region is along x axis, therefore, changes in these parameters are the functions of x only. The values for α_d, β_d and γ_d are calculated layer wise.

For normal region **(**tissues of donor site) K, M and S depend on position only.

$$\varsigma(t)=1,\ \psi(t)=1\ and\ \ \zeta(t)=1 \tag{1.6}$$

and for abnormal region (transplanted tissues**)** K, M and S depend on position and time i.e.

$$\varsigma(t)=\left(\nu_0+\nu_1 e^{-\nu t}\right),\ \ \psi(t)=\left(\mu_0+\mu_1 e^{-\mu t}\right),\ \ \zeta(t)=\left(\theta_0+\theta_1 e^{-\theta t}\right) \tag{1.7}$$

In transplanted tissues initially values of physiological parameters M and S are very less or almost negligible as compared to the tissues of host site (normal region) and then after some time, they become normal and equal to that of the normal region. The thermal conductivity K is assumed half of the normal value in the beginning or just after the surgery.

3. METHODOLOGY

The bio-heat equation (1.1) can be descretized using the finite element method (FEM). The finite element equation is developed using the variational method can be written for the e^{th} element

$$I^{\,e}=\frac{1}{2}\int_{x_i}^{x_j}\left[K^e\left(\frac{\partial T^e}{\partial x}\right)^2+M^e(T_b-T^e)^2-2S^eT^e+\rho\overline{C}\ \frac{\partial(T^e)^2}{\partial t}\right]dx+\frac{1}{2}\left[h(T^e-T_a)^2+2LET^e\right] \tag{1.8}$$

for epidermis only

Here second term of the equation (1.8) is valid for the elements *e* adjoining the outermost surface of the skin and taken equal to zero for remaining elements.

The equation (1.8) can be written as

$$I^e=I_K^e+I_M^e-I_S^e+I_\rho^e+I_{\delta1}^e+I_{\delta2}^e \tag{1.9}$$

where

$$\left.\begin{aligned} &I_{K}^{e}=\frac{1}{2}\int_e K^e\left(\frac{\partial T^e}{\partial x}\right)^2 dx,\ I_{M}^{e}=\frac{1}{2}\int_e\left[M^e(T_b-T^e)^2\right]dx\\ &I_{S}^{e}=\int_e\left[S^eT^e\right]dx,\ \ I_{\rho}^{e}=\frac{1}{2}\int_e\rho c\ \frac{\partial(T^e)^2}{\partial t}dx\\ &I_{\delta1}^{e}=\frac{1}{2}\left[h(T^e-T_a)^2\right],\ I_{\delta2}^{e}=LET^e \end{aligned}\right\} \tag{1.10}$$

Differentiating I^e (eq. 1.10) with respect to each nodal temperature of e^{th} element and we get the matrix form as follows

$$\frac{dI^e}{d\overline{T}^e} = \left[A_1^e\right]\left[\overline{T}^e\right] + \left[A_2^e\right]\left[\overline{T}^e\right] - \left[A_3^e\right] + [A_4^e]\left\{\frac{\partial \overline{T}^e}{\partial t}\right\} + [A_5^e]\left[\overline{T}^e\right] + [A_6^e] \tag{1.11}$$

where

$$\left[A_1^e\right]_{pxp} = \int_e K^e [B^e]'[B^e]dx, \left[A_2^e\right]_{pxp} = \int_e M^e [N^e]'[N^e]dx$$

$$\left[A_3^e\right]_{px1} = \int_e (M^e T_b + S^e)[N^e]'dx, \left[A_4^e\right]_{pxp} = \int_e \rho c [N^e]'[N^e]dx$$

$$[A_5^e]_{pxp} = h[N^e]'[N^e], [A_6^e]_{px1} = (LE - hT_a)[N^e]'$$

$$\left[B^e\right] = \left[\frac{\partial N_i}{\partial x} \quad \frac{\partial N_j}{\partial x}\right]$$

Assembly of Elements after differentiating the region under study has been divided into n_e elements using n_n nodes. All the elements are assembled to get integral I as follows

$$I = \sum_{e=1}^{n_e} I^e \tag{1.12}$$

Extremising I , we get

$$\left[\frac{dI}{d\overline{\overline{T}}}\right]_{n_n x1} = \sum_{e=1}^{n_e} \left[D^e\right]'_{n_n \times p} \left[\frac{dI^e}{d\overline{T}^e}\right]_{p\times 1} = 0 \tag{1.13}$$

$$\frac{dI}{d\overline{\overline{T}}} = \left[\frac{\partial I}{\partial T_1} \quad \frac{\partial I}{\partial T_2} \quad . \quad . \quad . \quad . \quad . \quad \frac{\partial I}{\partial T_{n_n}}\right]', \overline{\overline{T}} = \left[T_1 \ldots\ldots T_{n_n}\right]'$$

Here $[D^e]'$ shows the transpose of matrix $[D^e]$

$$\left[D^e\right] = \begin{bmatrix} 0 & 0 & 0 & . & . & 1 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & 0 & . & . & 0 & 1 & . & . & . & 0 & 0 \end{bmatrix}_{2xn_n}$$

4. NUMERICAL RESULTS AND DISCUSSIONS

Each layer has different physiological values, layer wise values for α_d, β_d and γ_d (eq. 6) has been assigned as follows:

The following values have been assigned to α^e, β^e *and* γ^e

i) For sub-dermis region: $\alpha_o^e = K_1, \alpha_1^e = 0, \beta_0^e = M^e, \beta_1^e = 0, \gamma_0^e = S^e, \gamma_1^e = 0,$

ii) For dermal region : $\alpha_0^e = K_2, \alpha_1^e = 0, \beta_0^e = M^e, \beta_1^e = 0, \gamma_0^e = S^e, \gamma_1^e = 0,$

iii) For epidermis region : $\alpha_0^e = K_3, \alpha_1^e = 0, \beta_0^e = 0, \beta_1^e = 0, \gamma_0^e = 0, \gamma_1^e = 0,$

The numerical results have been computed for different atmospheric temperatures T_a=15°C, 23°C and 33°C and different rates of evaporation E=0, $0.24\text{x}10^{-3}$ and $0.48\text{x}10^{-3}$ gm/cm^2-min. The values of physical and physiological parameters are taken from (Table-1 and 2) [11].

Thermal Conductivity (cal/cm min °C)	**Heat Transfer Coefficient h (cal/cm^2 min °C)**	**Specific Heat of Tissues c (cal/gm°C)**
K_1=0.060, K_2=0.045, K_3=0.030	0.009	0.830
Blood Density of Tissues ρ (gm/cm^3)	**Latent Heat L (cal/gm)**	**Body Core Temperature T_b (°C)**
1.090	579.0	37

Table-1: Values for physical and physiological parameters

Atmospheric Temperature T_a (^oC)	**Rate of Evaporation E (gm/ cm^2min)**	**Blood Mass Flow Rate M (cal/ cm min. ^oC)**	**Rate of metabolism S (cal/cm^3 min^1)**
15	0	0.0030	0.0357
23	0, $0.24x10^{-3}$, $0.48x10^{-3}$	0.0180	0.0180
33	$0.24x10^{-3}$,$0.48x10^{-3}$	0.0315	0.0180

Table-2: M, S and E for different atmospheric temperature

5. RESULTS AND DISCUSSIONS

All the graphs are plotted between temperature T and time t for different atmospheric temperature (Table-2) for normal region as well as for abnormal region. In both the models it is assumed that normal region consists of tissues of donor sites having normal physical and physiological values whereas the abnormal region of surgical or transplanted tissues with disturbed physiological parameters. Experimental studies show that body core temperature, skin temperature, ambient temperature and temperature of wound bed play very important role in wound healing process [19-21]. All the experimental studies are based on the effect of cleansing solution warmed at atmospheric temperatures used during the dressing of wound, these show that atmospheric temperature play an important role during the wound healing [21]. Since, present study is based on the theoretical prediction therefore different atmospheric temperatures are taken into consideration to analyze temperature of the region instead of taking cleansing solution warmed at different atmospheric temperature. Thermal conductivity, Metabolism and blood mass flow are taken into consideration because skin temperature affects the local blood flow and a small rise in temperature can increase metabolic activity and oxygen demand [19-21]

Initially it is assumed that the whole SST region is insulated therefore temperature of tissues at $t=0$ is 37 ^oC. When insulation is removed more heat loss at the upper most boundary on the y axis takes place through radiation, conduction, convection and evaporation whereas across the boundaries on x axis. The gradient in temperature is almost negligible along x axis. In the present study the temperature variation is noted for normal region (tissues of donor site) as well as abnormal region (transplanted tissues). For the abnormal region the physiological functioning is not same as that of in normal region and these are the increasing function of time. Therefore, for abnormal region $\nu=\mu=\theta=0.01$ and for normal region $\varsigma(t)= \psi(t)= \zeta(t)=1$ is considered.

In all the graphs (Fig.2-7), the fall in temperature is more in transplanted tissue than that of the tissues of normal region because rate of metabolism, thermal conductivity and blood mass flow in transplanted tissues are not same as that of tissues of donor side. The fall in tissue temperature is noted more at the skin surface (epidermis) in comparison to the interior tissues (dermal and subcutaneous) because more heat loss occurs at the surface due to conduction, convection, radiation and evaporation.

For the same rates of evaporation the decline in tissue temperature is more at lower atmospheric temperature (Fig. 2&3; for E=0 at T_a=15 and 23^{o}C; Fig. 4&6, for E=$0.24x10^{-3}$ at T_a=23^{o}C and 33^{o}C; Fig. 5&7, for E=$0.48x10^{-3}$ at T_a=23^{o}C and 33^{o}C). This is because more temperature gradient occurs due to low atmospheric temperature.

For higher rates of evaporation at the same atmospheric temperature, the fall in tissue temperature of epidermis, dermis and subcutaneous is noted more i.e. at T_a=23^{o}C, the fall in tissue temperature is more for E=$0.48x10^{-3}$than that of E=0. It shows that the rate of evaporation significantly effects on temperature profile in the region. The results obtained in the model are agreement to the physiological facts.

In the present study, it is observed that with the release of insulation of normal as well as abnormal region more heat loss takes place from the abnormal region during cleansing and dressing of wound (open wound) than that of the normal region. Hence more and fast temperature fall is noted about 20 minutes for abnormal region and slow temperature fall about 10 minutes for normal region.

At the time of surgery the values of physiological parameters in transplanted tissues are negligible and with the increase of time the value of these physiological parameters also increases [19-21]. In this period of wound healing process, biological and chemical processes take place gradually. These processes are responsible to increase tissues temperature[21]. Cell division takes place gradually resulting in slow increase in temperature. This mitotic division needs normal body temperature. Therefore very small and gradual increment for about 100 minutes is noted for transplanted tissues and tends to become steady after about 3 hours and 20 minutes and approaching towards normal values. Due to normal cell division in the tissues of normal region temperature becomes almost steady after 20 minutes. The steady state occurs when the physiological parameters of abnormal region starts attaining the values equal to that of the normal tissues. For different atmospheric temperature and different rate of evaporation, temperature profile varies layer wise for both the cases. Steady state nearer to the normal values shows the proper functioning of physiological parameters thus fast healing. The results obtained from the model are near to experimental facts.

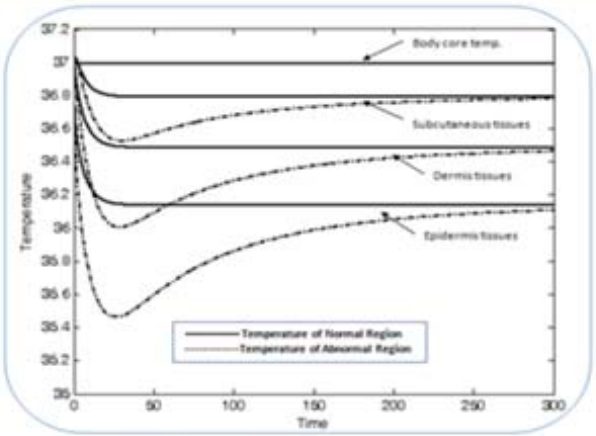

Fig.-2: T_a=15° C, E= 0.0 gm/cm²-min, v=μ = θ = 0.01

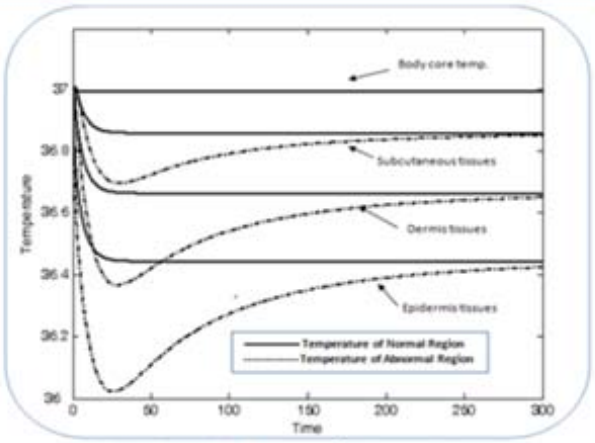

Fig.-3: T_a=23° C, E= 0.0 gm/cm²-min, v=μ = θ = 0.01

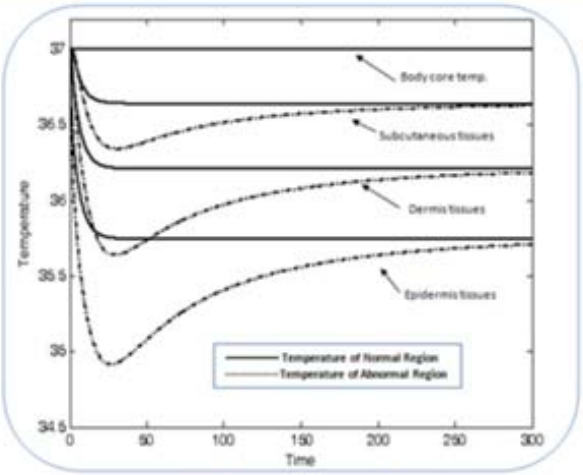

Fig.-4: T_a=23° C, E= 0.24x10⁻³ gm/cm²-min, v=μ = θ = 0.01

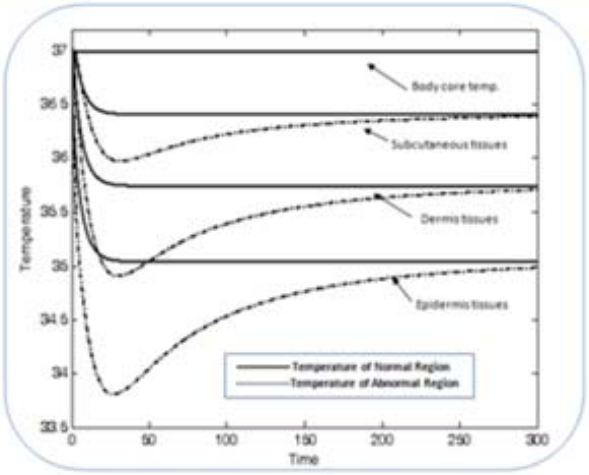

Fig.-5: T_a=23° C, E= 0.48x10⁻³ gm/cm²-min, v=μ = θ = 0.01

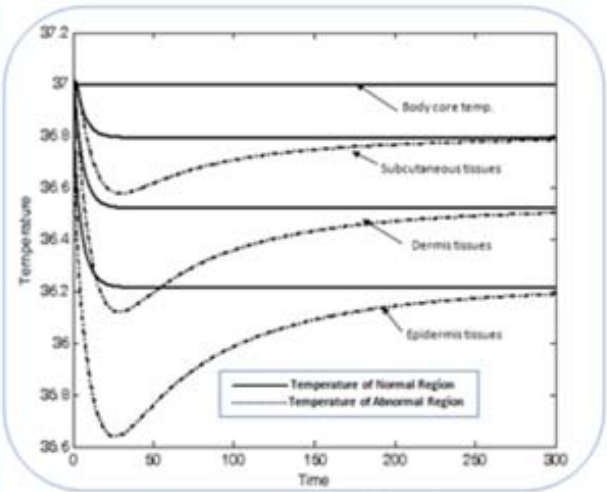

Fig.-6: T_a=33° C, E= 0.24x10⁻³ gm/cm²-min, v=μ = θ = 0.01

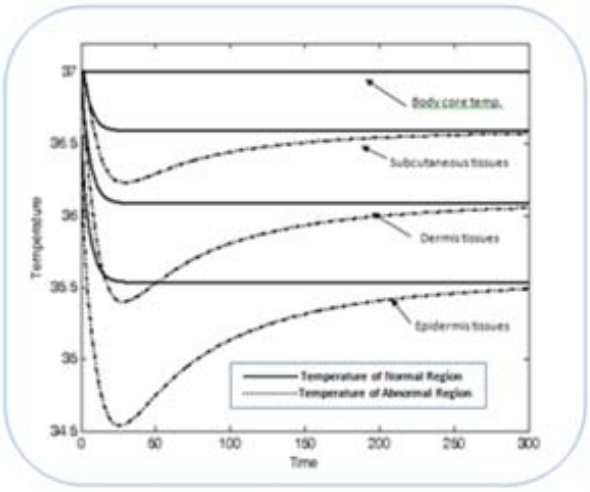

Fig.- 7: T_a=33° C, E= 0.48x10⁻³ gm/cm²-min, v=μ = θ = 0.01

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