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PERISTALTIC FLOW OF A FRACTIONAL SECOND GRADE FLUID THROUGH INCLINED CYLINDRICAL TUBE

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ABSTRACT

Peristaltic flow of a fractional second grade fluid through inclined cylindrical tube is analyzed. Long wavelength and low Reynolds number assumptions are used to linearise the governing equations. The expression for velocity is obtained. The expressions for fractional parameter, material constant, inclined channel and amplitude on the pressure and friction force across one wavelength are discussed.

Keywords: *Peristalsis: Fractional second grade model; Pressure; Friction force; inclined channel; Caputo's fractional derivative.*

INTRODUCTION

The word peristaltic stems from the Greek word peristaltic, which means clasping and compressing. The term "peristalsis" is used for the mechanism by which a fluid can be transported through a distensible tube when contraction or expansion waves propagate progressively along its length. Peristaltic pumping has been the object of scientific and engineering researches during the recent past few decades.

In the gastrointestinal tract, the movement of spermatozoa in the ducts afferents of the male reproductive tract and the ovum in female fallopian tube, transport of lymph in the lymphatic vessels and vasomotion of small blood vessels such as arterioles, venules and capillaries are the examples of physiology and finger, roller pumps and heart lung machine are few examples of biomechanical system. In the mechanical point of view, the idea of peristaltic transport was investigated by Latham[1].Since then, other workers[2-4] studied peristaltic flow theoretically and they used perturbation techniques, long wavelength and low Reynolds approximation problem. For solving two dimensional and axi-symmetric flows. Fractional calculus has encountered much success in the description of viscoelastic characteristics. The starting point of the fractional derivative model of non-Newtonian model is usually a classical differential equation which is modified by replacing the time derivative of an integer order by the so called Riemann-Liouville fractional calculus operators. This generalization allows defining precisely non-integer order integral or derivatives. In general, fractional second grade model is derived from well known second grade model by replacing the ordinary time derivatives to fractional order time derivatives and this plays an important role to study the valuable tool of viscoelastic properties. Rathod and Asha[5] have studied the effect of magnetic field and an endoscope on peristaltic motion in uniform and non-uniform. Some authors[6-8] have investigated unsteady flow of viscoelastic fluids with fractional Maxwell model, fractional generalized Maxwell model fractional, second grade fluid, fractional Oldroyed-B model, fractional Burgers model and fractional generalized Burgers' model through channel/ annulus tube and solutions for velocity field and the associated shear stress are obtained by using Laplace transform, Fourier transform, Weber transform, Hankal transform discrete Laplace transform.

Agrawal and Anwaruddin[9] studied the effect of magnetic field on the peristaltic flow of blood using long wavelength approximation method and observed for the flow of blood in arteries with arterial stenosis or arteriosclerosis. Mekheimer[10-11] have studied Effects of heat transfer and space porosity on peristalticflow in a vertical asymmetric channel and Peristaltic flow of blood under effect of a magnetic field in a non uniform channel. Some important works[12-22] such as the flow of viscoelastic fluids, the effect of heat transfer on flow, thermal and hydrodynamic characteristics, and hydromagnetic flows, arc-shaped channels, axial coutte flow and vertical channel have been studied.

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Recently, Tripathi et.al[23-24], have studied the peristaltic flow of fractional Maxwell fluids through a channel under long wavelength and low Reynolds number approximations by using homotophy perturbation method and Adomian decomposition methods and reported the slip effects on peristaltic transport of fractional Berger's fluids through a channel and solution is obtained by homotophy analysis method. Some important works[25-29] such as Heat Transfer with hydromagnetic flow, fractional operation, fractional calculus, fractional integrals and derivatives have been studied. Rathod and Mahadev[30-34] have studied the effect of magnetic field on ureteral peristalsis in cylindrical tube. Rathod and Pallavi[35-37] have studied the peristaltic transport of dusty fluid.

Rathod and Laxmi[38-39] have studied the slip effect on peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel by adomian decomposition method and Peristaltic transport of a conducting fluid in an asymmetric vertical channel with heat and mass transfer.In this paper, we study the peristaltic transport of viscoelastic fluid with fractional second grade model through a cylindrical tube under the assumptions of long wavelength and low Reynolds number. Caputo's definition is used to find fractional differentiation and numerical results of problem for different cases are discussed graphically. The effect of fractional parameter is material constant and times on the pressure rise friction force across one wavelength are discussed. This model is applied to study the movement of chyme through small intestine and also applicable in mechanical point of view.

Caputo's definition

Caputo's definition [23] of the fractional –order derivative is defined as

$$D^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_b^t \frac{f^n(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \, (n-1 < \mathrm{Re}(\alpha) \le n,\ n \in N),$$

Where, α is the order of derivative and is allowed to be real or even complex, b is the initial value of function f. For the Caputo's derivative we have

$$D^{\alpha} t^{\beta} = \begin{cases} 0 & (\beta \le \alpha - 1) \\ \dfrac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} t^{\beta-\alpha} & (\beta > \alpha - 1) \end{cases}$$

MATHEMATICAL FORMULATION

The constitutive equation for viscoelastic fluid with fractional second grade model is given by

$$s = \mu \left(1 + \lambda_1^{\alpha} \frac{\partial^{\alpha}}{\partial t^{\alpha}} \right) \dot{\Upsilon} \tag{1}$$

Where $t, s, \gamma\ and\ \lambda_1,$ is the time, shear stress, rate of shear strain and material constants respectively, μ is viscosity, and α is the fractional time derivative parameters such that $0 \le \alpha \le 1$. This model reduces to second grade model with α=1, and Classical Navier Stokes model is obtained by substituting $\lambda_1 = 0$.

The governing equations of motion of viscoelastic fluid with fractional second grade model for axisymmetric flow are given by

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu \left(1 + \lambda_1^{\alpha} \frac{\partial^{\alpha}}{\partial t^{\alpha}} \right) \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial x^2} \right\} - \eta_1 \sin\theta \tag{2}$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial x} + \mu \left(1 + \lambda_1^{\alpha} \frac{\partial^{\alpha}}{\partial t^{\alpha}} \right) \left\{ \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (rv) \right) + \frac{\partial^2 v}{\partial x^2} \right\} + \eta_2 \cos\theta \tag{3}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial r} .$$

For carrying out further analysis, we introduce the following non –dimensional parameters.

$$\left. \begin{aligned} & x = \frac{x}{\lambda}, r = \frac{r}{\alpha}, t = \frac{ct}{\lambda}, \lambda_1^{\alpha} = \frac{c\lambda_1^{\alpha}}{\lambda}, u = \frac{u}{c}, v = \frac{v}{c\delta} \\ & \delta = \frac{\alpha}{\lambda}, \phi = \frac{\phi}{\alpha}, p = \frac{p\alpha}{\mu c \lambda}, Q = \frac{Q}{\pi \alpha^2 c}, \ \mathrm{Re} = \frac{\rho c \alpha \delta}{\mu} \end{aligned} \right\} \tag{4}$$

Where ρ is fluid density, δ is defined as wave number, λ,r.t,u,v,φ,p and Q stand for wavelength, radial coordinate, time, axial velocities, wave velocity, amplitude, pressure, and volume flow rate respectively in non-dimensional form.

Introducing the non-dimensional parameters and taking long wavelength and low Reynolds number approximations, Eqs. (2) reduce to

$$\frac{\partial p}{\partial x}=\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\left\{\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right\}-\eta_1\sin\theta$$
$$0=\frac{\partial p}{\partial r}+\eta_2\cos\theta \tag{5}$$
$$\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial(rv)}{\partial r}=0$$

Where $\eta_1=\frac{pg\alpha^2}{\mu c},\ \eta_2=\frac{pg\alpha^3}{\mu c\lambda}$ and g is acceleration due to gravity.

Boundary conditions are given by

$$\frac{\partial u}{\partial r}=0\ \ at\ r=0\ u=0\ at\ \ r=h \tag{6}$$

Integrating Eqn. (4) with respect to r and using first condition of Eqn. (5). The velocity gradient is obtained as

$$\frac{\partial p}{\partial x}=\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\left\{\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right\}-\eta_1\sin\theta$$
$$r\frac{\partial p}{\partial x}=\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\left\{r\frac{\partial^2 u}{\partial r^2}+\frac{\partial u}{\partial r}\right\}-r\eta_1\sin\theta$$
$$\frac{r^2}{2}\frac{\partial p}{\partial x}+c_1=\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\left(\frac{\partial u}{\partial r}r-\int\frac{\partial u}{\partial r}+\int\frac{\partial u}{\partial r}\right)-r\eta_1\sin\theta$$
$$c_1-\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)r\frac{\partial u}{\partial r}=\frac{r^2}{2}\frac{\partial p}{\partial x}-\frac{r^2}{2}\eta_1\sin\theta$$

Using boundary condition

$$\frac{\partial u}{\partial r}=0\ at\ r=0$$

We get c_1=0 (7)

$$\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial u}{\partial r}=\frac{r}{2}\frac{\partial p}{\partial x}-\frac{r^2}{2}\eta_1\sin\theta \tag{8}$$

Further, integrating eqn. (7) from 0 to r we get the axial velocity as

$$\frac{\partial p}{\partial x}=\left(1+\lambda_1^{\alpha}\nearrow^{0}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\left\{\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right\}-\eta_1\sin\theta$$
$$\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial u}{\partial r}+c_2=\frac{1}{2}\frac{\partial p}{\partial x}\left[\frac{r^2}{2}\right]_0^r-\frac{r^2}{2}r\eta_1\sin\theta$$

Again using boundary condition u=0 and r=h

$$c_2=\frac{-h^2}{4}\frac{\partial p}{\partial x}\frac{-h^3}{6}\eta_1\sin\theta$$

$$\left(1+\lambda_1^{\alpha}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)u==\frac{1}{4}\frac{\partial p}{\partial x}\left(r^2-\frac{h^2}{2}\right)+\frac{h^3}{3}\eta_1\sin\theta \tag{9}$$

The volume flow rate is defined as $Q = \int_0^h 2rudr$ which by virtue of eqn (8) reduces to

$$2u = \frac{\frac{1}{2}\frac{\partial p}{\partial x}\left(r^2 - \frac{h^2}{2}\right) + \frac{h^3}{3}\eta_1 \sin\theta}{\left(1+\lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right)}$$

$$Q = \int_0^h \frac{\frac{1}{2}\frac{\partial p}{\partial x}\left(r^2 - \frac{h^2}{2}\right) + \frac{h^3}{3}\eta_1 \sin\theta}{\left(1+\lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right)} rdr$$

$$\left(1+\lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right)Q = \frac{h^4}{8}\frac{\partial p}{\partial x} + \frac{h^5}{6}\eta_1 \sin\theta \qquad (10)$$

The transformations between the wave and the laboratory frames, in the dimensionless form, are given by

$$X = x-1, R = r, U = u-1, V = v,\ q = Q - h^2 \qquad (11)$$

Where the left side parameters are in wave frame & the right side parameters are in the laboratory frame.

We further assume that the wall undergoes contraction & relaxation is mathematically formulated as

$$h = 1 - \varphi\cos^2(\pi x) \qquad (12)$$

The following are the existing relation between the averaged flow rate, the flow rate in the wave frame & that in the laboratory frame.

$$h = 1 - \varphi\cos^2(\pi x)\overline{Q} = q + 1 - \phi + \frac{3\phi^2}{8} = Q - h^2 + 1 - \phi + \frac{3\phi^2}{8} \qquad (13)$$

Eqn (9) in view of eqn (12) becomes

$$\left(1+\lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right)Q = \frac{h^4}{8}\frac{\partial p}{\partial x} + \frac{h^5}{6}\eta_1 \sin\theta \qquad (14)$$

Using Caputo's definition in eqn (13) we get

$$\frac{\partial p}{\partial x} = \frac{8\left(Q + h^2 - 1 + \phi - 3\phi^2/8\right)}{h^2}\left(1+\lambda_1^\alpha \frac{t^\alpha}{\Gamma(1-\alpha)}\right) - \frac{h^5}{6}\eta_1 \sin\theta \qquad (15)$$

The pressure difference and friction force across one wavelength are given by

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x}dx \qquad (16)$$

$$F = \int_0^1 \left(-h^2 \frac{\partial p}{\partial x}\right)dx \qquad (17)$$

The above integrals numerically evaluated using the MATHEMATICA software.

NUMERICAL RESULTS AND DISCUSSIONS

In this paper, we have studied the effect of a magnetic field on peristaltic flow of a fractional second grade fluid through a cylindrical tube under the assumption of long wavelength. The emerging parameters such as fractional effect of various parameter$\left(\alpha\right)$,inclined channel(θ),material constant$\left(\lambda_1\right)$,time$\left(t\right)$,and amplitude$\left(\phi\right)$on pressure difference across one wavelength$\left(\Delta p\right)$ and friction force across the one wavelength$\left(F\right)$ has been discussed through graphs.

Fig.1-5depict the variation of pressure$\left(\Delta p\right)$ with averaged flow rate $\overline{Q}$ for various values of $\alpha,\lambda_1,t,\theta, and\, \phi$. It is observed that there is a linear relation between pressure and averaged flow rate, on incresesing the averaged flow rate reduces pressure and thus, maximum averaged flow rate is achieved at zero pressure and occurs at zero averaged flow rate.

Fig.1 shows that the pressure rise Δp with averaged flow rate $\overline{Q}$ for various values of α at $x = 0.25, \phi = 0.4, \lambda = 1, \eta_1 = 0.5\, and\, \theta = \pi/4$ .It is observed that, the pressure increases with decreasing α. The fractional behavior of second grade fluid increases, as the pressure for flow diminishes. It is observed that, the time averaged flow rate $\overline{Q}$ increases with decreasing α in the pumping region$\left(\Delta p > 0\right)$, while it increases in both the free-pumping$\left(\Delta p = 0\right)$and co-pumping$\left(\Delta p < 0\right)$regions with an increase in α . The variation of Δp with $\overline{Q}$ for various values of λ at $x = 0.25, \phi = 0.4, \alpha = 0.2, \eta_1 = 0.5\, and\, \theta = \pi/6$ is presented in Fig.2. It is revealed that the pressure increases with increasing λ . This means that viscoelastic behavior (in the sense of λ) of fluids increases, the pressure for flow of fluids decreases i.e. the flow for second grade fluid is required more pressure than that for the flow of Newtonian fluids. Figs.3 depicts the variation of pressure rise Δp with averaged flow rate $\overline{Q}$ for various values of t at $x = 0.25, \phi = 0.4, \lambda = 1, \eta_1 = 0.5\, and\, \theta = \pi/6$ and this figure reveals that, the effect of time on pressure is similar to that of amplitude. Maximum flow rate are unique for various values of $\alpha,\lambda,t\, and\, \phi$.

Figs.4 depicts the variation of pressure rise Δp with averaged flow rate $\overline{Q}$ for various values of ϕ at $x = 0.25, t = 0.5, \lambda = 1, \eta_1 = 0.5\, and\, \theta = \pi/3$ .It is noted that, in the pumping region the time averaged flow rate decreases with increasing ϕ ,while it increases in both the free- pumping and co-pumping regions on increasing ϕ .

Fig.5.depicts the variation of pressure rise Δp with time averaged flow rate $\overline{Q}$ for different values of i*nclined channel*θ with $x = 0.25, \phi = 0.6, \lambda = 1, \eta_1 = 0.5\, and\, \alpha = 0.2$ It is found that, any two pumping curves intersect in the first quqdrant,to the left of this point of intersection the averaged flow rate increases on increasing θ and to the right of this point of intersection the $\overline{Q}$ decreases with increasing Θ.

Figs.6-10 shows the variations of friction force (F) with the averaged flow rate $\left(\overline{Q}\right)$under the influences of all emerging parameters such as $\alpha, \lambda_1, t, \phi, and\, \theta$. From figures, It is observed that the effects of all parameters on friction force are opposite to the effects on pressure with averaged flow rate.

Fig.10.It is noted that, the time averaged flow rate increases with decreasing θ .

5CONCLUSIONS

The assumption of long wavelength studied under the fractional models of viscoelastic fluids play important role in physics of polymers and rheology. One of the fractional models of viscoelastic fluids named as fractional second grade model has been taken to study the peristaltic flow behavior through the cylindrical tube. It is evident that less pressure is required to flow the fractional second grade fluid $\left(0 < \alpha < 1\right)$ in comparing to the flow of second grade fluid (α=1). It is revealed that the flow of Newtonian fluid $\left(\lambda_1 = 0\right)$ is taken less effort than that of the second grade fluid$\left(\lambda_1 > 0\right)$. It is also found that the pressure increases by increasing the amplitude or time. The characteristics of Δp with $\overline{Q}$, and F with $\overline{Q}$ at various parameters, are found to be opposite in nature.

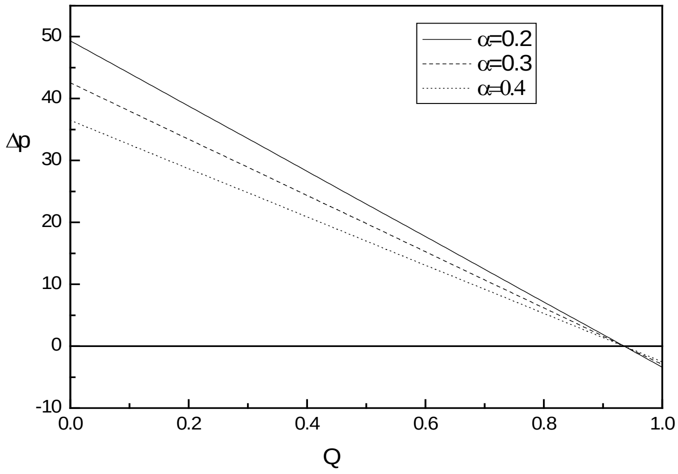

Figure 1: Pressure verses averaged flow rate for various values of α at
$$x = 0.25, \phi = 0.4, \lambda = 1, \eta_1 = 0.5, \theta = \pi / 4$$

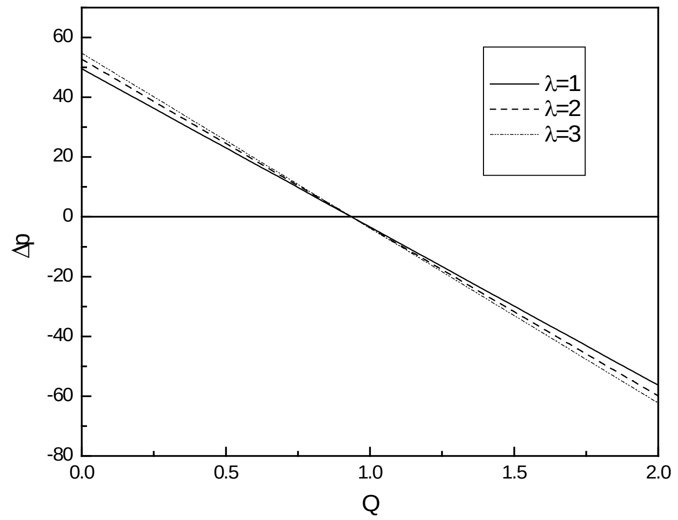

Figure 2: Pressure verses averaged flow rate for various values of λ at
$$x = 0.25, \phi = 0.4, \alpha = 0.2, \eta_1 = 0.5, \theta = \pi / 6$$

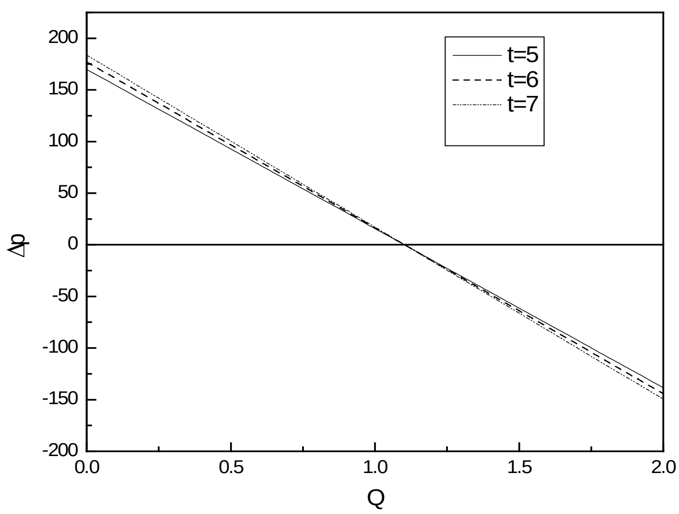

Figure 3: Pressure verses averaged flow rate for various values of t
at $x = 0.25, \phi = 0.4, \lambda = 1, \eta_1 = 0.5, \theta = \pi / 6$

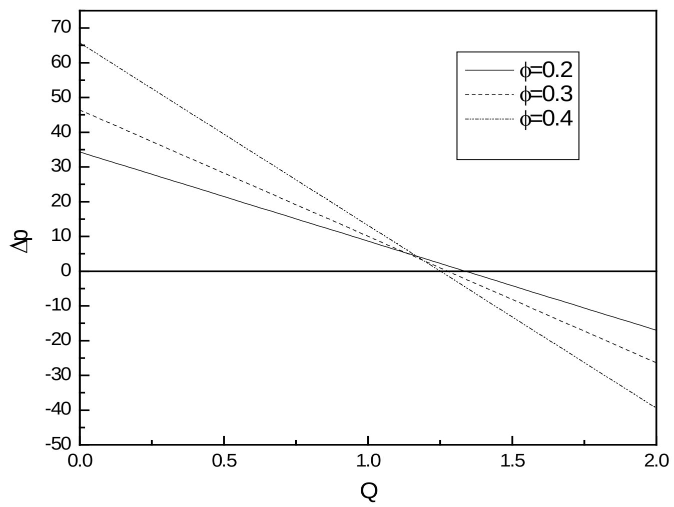

Figure 4: Pressure verses averaged flow rate for various values of ϕ at $x = 0.25, t = 0.5, \lambda = 1, \eta_1 = 0.5, \theta = \pi/3$

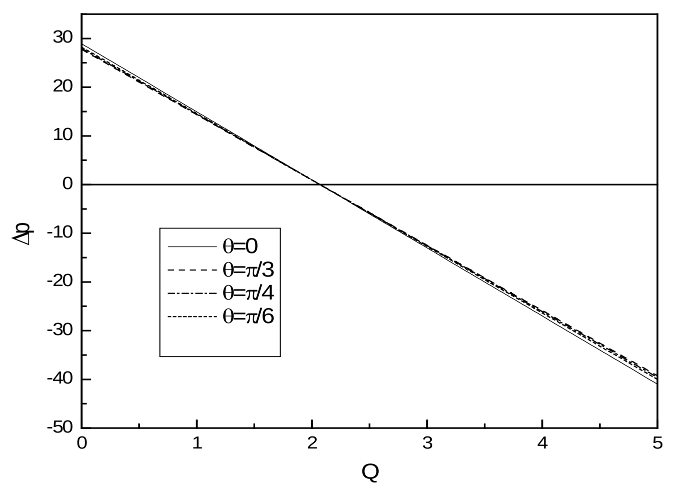

Figure 5: Pressure verses averaged flow rate for various values of θ at $x = 0.25, \phi = 0.6, \lambda = 1, \eta_1 = 0.5, \alpha = 0.2$

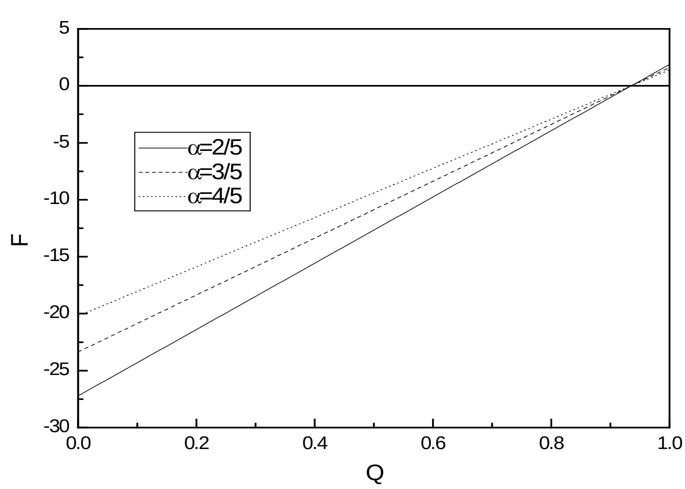

Figure 6: Friction force verses averaged flow rate for various values of α at $x = 0.25, \phi = 0.4, \lambda = 1, \eta_1 = 0.5, \theta = \pi/4$

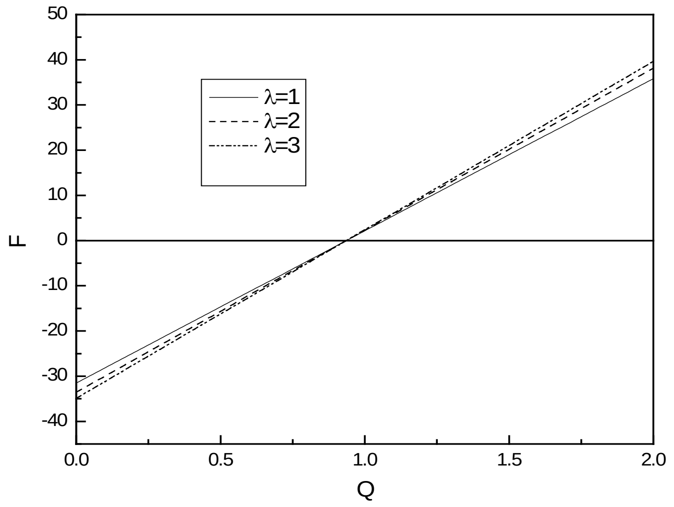

Figure 7: Friction force verses averaged flow rate for various values of λ_1 at

$$x = 0.25, \phi = 0.4, \alpha = 0.2, \eta_1 = 0.5, \theta = \pi / 3$$

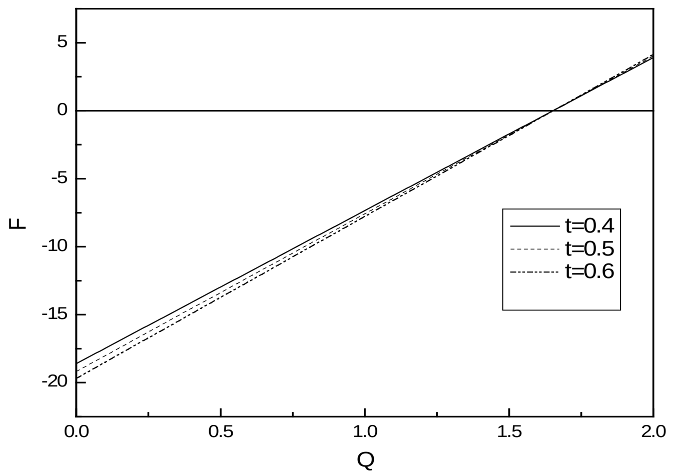

Figure 8: Friction force verses averaged flow rate for various values of t at

$$x = 0.25, \phi = 0.6, \alpha = 0.2, \eta_1 = 0.5, \theta = \pi / 3$$

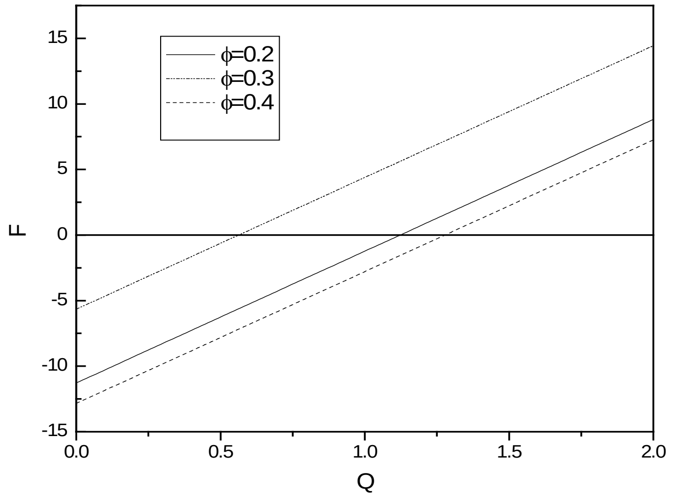

Figure 9:.Friction force verses averaged flow rate for various values of ϕ at

$$x = 0.25, \lambda = 1, \alpha = 0.2, \eta_1 = 0.5, \theta = \pi / 4$$

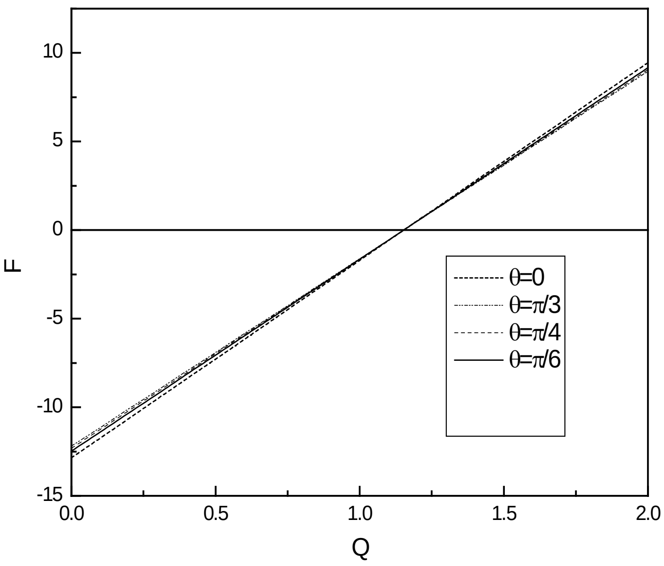

Figure 10: Friction force vs. averaged flow rate for various values of θ at $x = 0.25, \lambda = 1, \alpha = 0.2, \eta_1 = 0.5, \phi = 0.6$

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