International Journal of Mathematical Archive-6(2), 2015, 22-26
Available online through www.ijma.info ISSN 2229 – 5046

# APPLICATIONS OF MATRICES

**P. S. Balaram**
*Faculty of Management Studies, Aditya P.G. College, Kakinada-(A.P.), India.*

**P. S. S. R. Sujatha***
*Faculty of Mathematics, Sri Aditya Engineering College, Surampalem-(A.P.), India.*

*(Received On: 19-01-15; Revised & Accepted On: 11-02-15)*

*ABSTRACT*

*Generally Matrices have many applications in the solution of a system of equations, graph theory, operations research, statistics, etc.*

*In this paper some real life examples on matrices such as product of matrices, determinant of matrices and transition matrix of the Markov chain are discussing below.*

## BASIC RULES

1. Let $A = [a_{ik}]_{m\times n}$ and $B = [b_{kj}]_{n\times p}$.

   Then the product AB is defined as the m×p matrix $C = [c_{ij}]_{m\times p}$

   where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj} = \sum_{k=1}^{n} a_{ik}b_{kj}$.

   i.e. $AB = \left[\sum_{k=1}^{n} a_{ik}b_{kj}\right]_{m\times p}$

2. In general, AB ≠ BA.
   i.e. matrix multiplication is not commutative.

3. A square matrix A of order n is said to be non-singular or invertible if and only if there exists a square matrix B such that AB = BA = I.

   The matrix B is called the multiplicative inverse of A, denoted by $A^{-1}$ i.e. $AA^{-1} = A^{-1}A = I$.

4. Let $A = [a_{ij}]$ be a square matrix of order n. The determinant of A, det A or |A| is defined as follows:

   (a) If n=2, $\det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

   (b) If n=3, $\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

   or $\det A = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{12}a_{23} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$

5. Let A be a square matrix. If $\det A \neq 0$, then A is non-singular and $A^{-1} = \frac{1}{\det A}(adjA)$.

*Corresponding Author: P. S. S. R. Sujatha**
*Faculty of Mathematics, Sri Aditya Engineering College, Surampalem-(A.P.), India.*

**Example 1:** Let us denote $P_1$and $P_2$ as B.P. patients and Sugar patients, S and H represent Sweet and Hot and $S_1$and $S_2$ represent sugar and salt. Consider the following table, in which 1 represents that $P_i$ can eat $S_i$ and 0 represents that $P_i$ cannot eat $S_i$ .Also, table-1 represents the patients can take the ingredients and table-2 represents the ingredients can be used in the items.

| | $S_1$ | $S_2$ |
|---|---|---|
| $P_1$ | 1 | 0 |
| $P_2$ | 0 | 1 |

| | H | S |
|---|---|---|
| $S_1$ | 1 | 0 |
| $S_2$ | 0 | 1 |

Now we can represent the above 2 tables in the following two matrices in which we put $a_{ij}=1$ if $i^{th}$ person eat $j^{th}$ ingredient in the first matrix and $b_{ij}=1$ if $i^{th}$ ingredient is used in $j^{th}$ item.
$A=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$and $B=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Hence $AB=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, which concludes that the Sugar patients cannot eat sugar and B.P patients cannot eat salt.

**Example 2:** Let $A_1$, $A_2$, $A_3$ denote three people to buy Pens, Pencils, Books, and Erasers. Each of them requires these things in different amounts and will buy in three shops $S_1$, $S_2$, $S_3$. Now the question is which shop is the best for every person $A_1$, $A_2$, $A_3$ to pay the least cost?

The following 2 tables gives the individual prices and the required commodities of the product.

| Persons/Products | Pens | Pencils | Books | Erasers |
|---|---|---|---|---|
| $A_1$ | 3 | 4 | 2 | 2 |
| $A_2$ | 2 | 3 | 4 | 2 |
| $A_3$ | 4 | 3 | 2 | 1 |

| Products/Shops | $S_1$ | $S_2$ | $S_3$ |
|---|---|---|---|
| Pens | 2.00 | 2.50 | 1.50 |
| Pencils | 2.00 | 2.00 | 2.50 |
| Books | 10.00 | 12.00 | 11.00 |
| Erasers | 1.00 | 1.50 | 1.50 |

The amount paid by $A_1$ in the shop $S_1$ is 3(2) +4(2) +2(10) +2(1) =36

The amount paid by $A_1$ in the shop $S_2$ is 3(2.5) +4(2) +2(12) +2(1.5) =42.50

The amount paid by $A_1$ in the shop $S_3$ is 3(1.5) +4(2.5) +2(11) +2(1.5) =39.50

Similarly, we can calculate for the remaining also.

Now we can represent this data in the following matrix notation:
$A=\begin{bmatrix}3 & 4 & 2 & 2\\2 & 3 & 4 & 2\\4 & 3 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2.5 & 1.5\\2 & 2 & 2.5\\10 & 12 & 11\\1 & 1.5 & 1.5\end{bmatrix}$

Hence $AB=\begin{bmatrix}36 & 42.5 & 39.5\\52 & 62 & 57.5\\35 & 41.5 & 37\end{bmatrix}$

$\therefore$ Either $A_1$ or $A_2$ or $A_3$ can buy the commodities at least cost in the Shop $S_1$.

**Example 3:** In this example, let us see how a word (small sentence) can be decoded by using a non-singular matrix.

For this, let us consider a non-singular matrix P and a number can be assigned to each letter of the alphabet according to a given table.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 7 | 5 | 13 | 9 | 16 | 18 | 22 | 4 | 23 | 11 | 3 | 21 | 1 | 6 | 15 | 12 | 19 | 2 | 14 | 17 | 20 | 25 | 24 | 10 | 26 |

Let P=$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and The text as a sequence of numbers will be organized into a square matrix A in the case that the number of letters is lower than the number of elements of the matrix A, the rest of the matrix can be filled with zero elements.

We put the text “COYOTILLO” in to the matrix A=$\begin{bmatrix} 5 & 6 & 10 \\ 6 & 14 & 4 \\ 3 & 3 & 6 \end{bmatrix}$

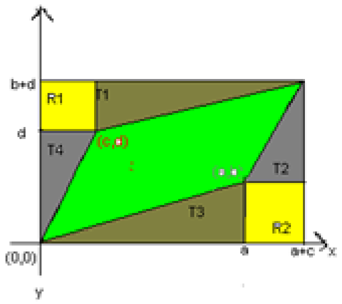
And encode the text PA=X=$\begin{bmatrix} 13 & 15 & 26 \\ 8 & 9 & 16 \\ 6 & 14 & 4 \end{bmatrix}$

To decode the message we have to multiply the matrix X by the matrix $P^{-1}$on the right:

X $P^{-1}$=$\begin{bmatrix} 13 & 15 & 26 \\ 8 & 9 & 16 \\ 6 & 14 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} -13 & 39 & 15 \\ -8 & 24 & 9 \\ -2 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 13 & 13 & 15 \\ 16 & 24 & 9 \\ 24 & 2 & 4 \end{bmatrix}$, which is”DDPFXEXSI”.

Now let us consider the examples on the determinant of a matrix.

**Example 4:** Consider a parallelogram with three vertices at (0,0), (a, b), and (c, d).




Consider the matrix A=$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Then $|A|$=ad-bc.

Let's find the area of the rectangle R with vertices at (0,0), (a+c, b+d), and (0, b+d):
AREA(R) = (b+d)(a +c)
= ab +bc + ad +cd.

Now we find the area of the small rectangles $R1$ and $R2$.
AREA($R1$)=bc= AREA($R2$)

Next we find the area of the four triangles:

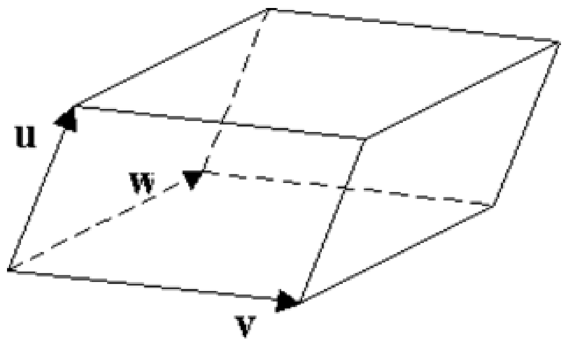
AREA ($T_1$) = $\frac{ab}{2}$ = AREA ($T_3$)

AREA ($T_2$) = $\frac{cd}{2}$ = AREA ($T_4$)

$\therefore$ Area of Parallelogram $= Area(R) - 2Area(R1) - 2Area(T_1) - 2Area(T_2)$
= ab +bc + ad +cd-2(bc)-ab-cd
= ad-bc
= $|A|$.

Hence, we see that the determinant of the matrix A whose columns contain the coordinates of two of the parallelogram vertices, gives the area of the corresponding parallelogram in the plane. Since we are working with the distance between points, we use the absolute value of the difference to represent the area of the figure.

**Example 5:** The absolute value of the determinant of an appropriate3x3matrix can give the volume of a parallelepiped (a slanted box).




If u=($a_{11}$,$a_{12}$,$a_{13}$); v=($a_{21}$,$a_{22}$,$a_{23}$) and w=($a_{31}$,$a_{32}$,$a_{33}$), then the volume of the slanted box determined by u, v, and w (see picture) is the absolute value of the determinant of the matrix

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

A three dimensional version of a two dimensional statement can be investigated further, but a discussion of finding the volume involves more mathematical tools than what we have available at this time. The key point is that determinants are closely related to area and volume formulas.

**Example 6:** Determinants can also be used to produce equations of lines and of planes.
(a). Consider two distinct points ($x_1$, $y_1$) and ($x_2$,$y_2$). We know that we can determine the equation of the line through those two points by finding the slope: m $=\frac{y_2-y_1}{x_2-x_1}$ *and the equation is*
($y$-$y_1$)=m($x$-$x_1$).

which can also generate the equation of the line by setting the determinant of a 3X3 matrix *A* equal to zero. where A is given by $\begin{bmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix}$.

i.e., $|A| = x(y_1-y_2) - y(x_1-x_2) + 1(x_1y_2 - x_2y_1) = 0$

$\Rightarrow xy_1 - xy_2 - yx_1 + yx_2 + x_1y_2 - x_2y_1 = 0$

$\Rightarrow (y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$

$\Rightarrow (y-y_1) = \frac{(y_2-y_1)}{(x_2-x_1)}(x-x_1).$

(b).The three points ($x_1$,$y_1$), ($x_2$,$y_2$) and ($x_3$,$y_3$) are said to be collinear i. e, the three points will lie on the same line if

$$\begin{vmatrix} x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

(c). The equation of a plane is: *ax* + *by* + *cz* = *d*

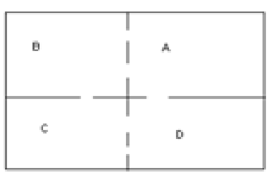
If we know the coordinates of three points ($x_1$,$y_1$, $z_1$), ($x_2$,$y_2$, $z_2$), ($x_3$, $y_3$, $z_3$) that lie on the plane, then we have three equations :

$a_1x+b_1y+c_1z=d_1$
$a_2x+b_2y+c_2z=d_2$
$a_3x+b_3y+c_3z=d_3$

Let A be the coefficient matrix of the system above,

So the equation of a plane through three distinct points is $|A|$=0.

**Example 7:** The maze below was built as a test container for rats.



The figure shows four compartments with door leading from one to another. A rat in any compartment is equally likely to pass through each of the doors of the compartment. Then the transition matrix of the Markov chain is given by $\begin{bmatrix} 0 & \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}$

**Example 8: (**Adjacency matrices and Airlines): You have been hired by the website travelocity.com to help people plan trips between various cities. Often your customers are business travelers so that they want to travel between cities in the morning to conduct a day's business. Large cities often provide flights to many cities, but small cities often are quite limited in the number of cities that they service. Your customers are particularly interested in travel between the following cities. **A**lbany, **B**oston, **N**ew York, **P**hilly, Wash, Richmond, Detroit, and LasVegas. For simplicity, we will only use the first letter to refer to the city. Here is the flight information that you are given.

From Boston there are flights to N, P, W, D
From Albany there are flights to N, W
From New York there are flights to B, P, W, R, D, L
From Philly there are flights to N, B, W, R
From Wash there are flights to B, A, N, R, P, L
From Richmond there are flights to N, P, W
From Detroit there are flights to B, N
From Las Vegas there are flights to N, W

An adjacency matrix for the data is listed below

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Like this the concepts of matrices have so many applications in different areas.

## BIBLIOGRAPHY


1. Several Simple Real-world Applications of Linear Algebra Tools-E. Ulrychov´a1
2. Friedberg, S.H., Insel, A.J., Spence, L.E.: Linear Algebra, 4th edition, Prentice Hall, 2003.
3. Poole, D.: Linear Algebra: A Modern Introduction. Brooks/Cole, 2003.
4. Coufal, J., Ulrychov´a, E.: ˇ Sifrov´an´ı a deˇsifrov´an´ı uˇzit´ım regul´arn´ıch matic, Mundus Symbolicus,
5. Leon, S.J.(1994). *Linear algebra with applications*, New Jersey: Prentice Hall.
6. Poole, D. (2006). *Linear algebra a modern introduction*, second edition, Belmont: Thompson Higher Education.
7. Strang, G. (1988). *Linear algebra and its applications*, third edition, New York: Harcourt Brace Jovanovich Inc.
8. Williams, G. (1984). *Linear algebra with applications*, Boston: Allan and Bacon, Inc.
9. http://www.intmath.com/Matrices-determinants/1_Determinants.php.
10. http://www.maths.surrey.ac.uk/explore/emmaspages/option1.html#Det
11. http://www.college-cram.com/study/algebra/matrix-algebra/determinant-of-a-2x2-matrix/
12. http://www.mathwarehouse.com/algebra/matrix/multiply- matrix.php
13. http://www.answers.com/topic/determinant.
14. Amazing Applications of Determinants- Keri Witherell
15. www. courses.ncssm.edu/math.


**Source of support: Nil, Conflict of interest: None Declared**