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COMBINED EFFECTS OF SURFACE ROUGHNESS AND NON-NEWTONIAN COUPLE STRESSES ON SQUEEZE FILM CHARACTERSTICS BETWEEN PARALLEL STEPPED PLATES

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ABSTRACT

In this paper, an attempt has been made to investigate the theoretically the combined effects of surface roughness and couple stresses on the squeeze film characteristics between parallel stepped plates. On the basis of Christensen's stochastic theory, two types of roughness structures, namely, the longitudinal roughness and transverse roughness patterns are considered and the stochastic modified Reynolds equation for these two types of roughness patterns is derived for couple stress fluid. A closed form expressions are obtained for the mean load carrying capacity and squeeze film time. It is found that the squeeze film characteristics are improved for transverse roughness pattern. Where as, the performance of the squeeze film bearing suffers due to the presence of longitudinal roughness pattern.

***Key Words:** Parallel Stepped Plates, Couple stress, Surface roughness, Squeeze film.*

INTRODUCTION

In recent years, the surface roughness and its effects on machine design are important factors affecting the flow patterns and have been extensively investigated. In practice, it is well known that bearing surfaces becomes rough. The aspect ratio and absolute height of the asperities and valleys observed under the microscope vary greatly, depending on the material properties and on the method of surface preparation. Height of the surface roughness may range from 0.05 μm of less on polished surfaces to 10 μm on medium machined surfaces. Even the chemical degradation of the lubricants leading to the contamination of lubricants is also a plausible reason for developing roughness on the bearing surface in some cases. Several methods have been proposed to study the effects of surface roughness on the bearing performance. Due to the random structure of the surface roughness, a stochastic approach has been used to mathematically model the surface roughness Christensen [1] developed a stochastic model for the study of surface roughness on hydrodynamic lubrication of bearings. The stochastic concept of transverse and longitudinal roughness on the steady state behavior of journal bearings is analyzed by Christensen and Tonder[2,3], while Elrod [4] employed the perturbation technique to study the surface roughness effects. The effects of roughness in porous circular squeeze-plates with arbitrary wall thickness is discussed by Prakash and Tiwari [5]. Recenlty Naduvinamani *et.al.*[6] analyzed the squeeze film lubrication of rough short porous partial journal bearings with micropolar fluids.

With the evolution of modern machine elements, the extending use of small amount of additives as lubricant has given much importance. The additive of dissolved polymer in the lubricant is noticed by Oliver [7] to improve the load-carrying capacity and to reduce the friction coefficient for a short journal bearing. According to Scott and Suntiwattana[8], the pressure/anti-wear additive has the profitable effect on the frictional characteristics and the wear of the wet friction clutch material. Since the traditional continuum theory is characterized by the non-polar theory of fluids neglecting the size of fluid particles, it fails to define the features of the non -Newtonian fluids. To consider the intrinsic motion of material ingredient and better display the conducts of the lubricant oil blended with additives, several microcontinuum theories, proposed by Ariman and Sylvester[9] and Stokes[10], has been developed by polar theory of polymer fluid. Amongst these, the Stokes microcontinuum theory to couple stress fluid account for the polar effects such as couple stresses, body couples and asymmetric tensor. Owing to its relative mathematical simplicity, this theory has been widely used to investigate the effects of couple stresses on the performance of different type of fluid film bearings such as slider bearings [11,12], squeeze film[13,14], journal bearings[15,16].

In this paper, an attempt has been made to study the effects of surface roughness and non-Newtonian couple stresses on the squeeze film lubrication of parallel stepped plates.

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MATHEMATICAL FORMULATION OF THE PROBLEM

Figure 1 shows the squeeze film configuration of parallel stepped plates approaching each other with a normal velocity *V*. The lubricant in the film region is taken to be an incompressible couple stress fluid developed by Stokes[10]. It is assumed that the body forces, body couples and fluid inertia are negligible. The hydrodynamic lubrication approximation applicable to thin films[17], the governing equations in the film region are

$$\mu\frac{\partial^2 u}{\partial y^2}-\eta\frac{\partial^4 u}{\partial y^4}=\frac{\partial p}{\partial x}, \tag{1}$$

$$\frac{\partial p}{\partial y}=0, \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0. \tag{3}$$

The relevant boundary conditions are

(i) At the upper surface *y = H*

$$u=0, \qquad \frac{\partial^2 u}{\partial y^2}=0 \tag{4a}$$

$$v=-V. \tag{4b}$$

(ii) At the bearing surface *y = 0*

$$u=0, \qquad \frac{\partial^2 u}{\partial y^2}=0 \tag{5a}$$

$$v=0. \tag{5b}$$

To mathematically model the surface roughness the fluid film thickness is considered to be made up of two parts

$$H_i=h_i+h_s(x,y,\xi) \quad (i=1,2) \tag{6}$$

h_i denotes the nominal smooth part of the film geometry while h_s is the part due to the surface asperities measured from the nominal level and is regarded as a randomly varying quantity of zero mean and ξ is the index describing the definite roughness arrangement, hence for a given value of ξ, the surface component h_s of the film thickness becomes deterministic function of the space variables.

SOLUTION OF THE PROBLEM

Solution of equation (1) subject to the boundary conditions (4a) and (5a) is

$$u=\frac{1}{2\mu}\frac{dp}{dx}\left\{y^2-yH+2l^2\left[1-\frac{cosh\left(\frac{2y-H}{2l}\right)}{cosh\left(\frac{H}{2l}\right)}\right]\right\} \tag{7}$$

where $l=\sqrt{\eta/\mu}$ is the couplestress parameter.

The modified Reynolds type equation for the pressure in the film region is obtained by using Eq. (7) for u in the continuity Eq. (3) and then integrating over the film thickness and also using the boundary conditions for *v* given in Eqs. (4a), (4b) and Eqs. (5a) , (5b), as

$$\frac{d}{dx}\left[f(H,l)\frac{dp}{dx}\right]=-12\mu V \tag{8}$$

where $f(H,l)=H^3-12l^2H+24l^3\,tanh\left(\frac{H}{2l}\right)$

Stochastic Reynolds Equation

Let $f(h_S)$ be the probability density function of the stochastic film thickness h_S. Taking the stochastic average of Eq. (8) with respect to $f(h_S)$ the averaged modified Reynolds type equation is obtained in the form

$$\frac{d}{dx}\left\{E\left[f\left(H,l\right)\right]\frac{dE(p)}{dx}\right\}=-12\mu V$$

$$\frac{d}{dx}\left\{E\left[f\left(H,l\right)\right]\frac{dE(p)}{dx}\right\}=-12\mu V \tag{9}$$

where $E(.)=\int_{-\infty}^{\infty}(.)\,f(h_S)\,dh_S$ (10)

In accordance with the Christensen [1], we assume that

$$f(h_s)=\begin{cases}\dfrac{35}{32c^7}\quad (c^2-h_s^2)^3, & -c<h_s<c\\ 0, & elsewhere\end{cases} \tag{11}$$

where $\sigma = c/3$ is the standard deviation

In the context of Christensen stochastic theory for the hydrodynamic lubrication of rough surfaces the two types of one-dimensional surface roughness patterns are considered viz; the longitudinal roughness pattern and the transverse roughness pattern.

Longitudinal Roughness Pattern

For one-dimensional longitudinal roughness pattern, the roughness structure has the form of long, narrow ridges and valleys running in the *x*-direction; in this case the non-dimensional stochastic film thickness assumes the form

$$H_i = h_i + h_s\left(y,\xi\right) \qquad (i=1,2) \tag{12}$$

and the stochastic modified Reynolds equation takes the form

$$\frac{d}{dx}\left\{E\left[f\left(H,l\right)\right]\frac{dE(p)}{dx}\right\}=-12\mu V \tag{13}$$

Transverse Roughness Pattern

For one-dimensional longitudinal roughness pattern, the roughness structure has the form of long, narrow ridges and valleys running in the *y*-direction; in this case the non-dimensional stochastic film thickness assumes the form

$$H_i = h_i + h_s\left(x,\xi\right) \qquad (i=1,2) \tag{14}$$

and the stochastic modified Reynolds equation takes the form

$$\frac{d}{dx}\left\{\frac{1}{E\left[1/f\left(H,l\right)\right]}\frac{dE(p)}{dx}\right\}=-12\mu V \tag{15}$$

Equation (13) and (15) together can be written as

$$\frac{d}{dx}\left\{G\left[f\left(H_i,l,c\right)\right]\frac{dE(p)}{dx}\right\}=-12\mu V \tag{16}$$

where $$G\left(H_i,l,c\right)=\begin{cases}E\left[f\left(H_i,l\right)\right], & for\ longitudinal\ roughness\\ E\left[\dfrac{1}{f\left(H_i,l\right)}\right]^{-1}, & for\ transverse\ roughness\end{cases} \tag{17}$$

$$E\left[f\left(H_i,l\right)\right]=\frac{35}{32c^7}\int_{-c}^{c} f\left(H_i,l\right)\left(c^2-h_s^2\right)^3 dh_s$$

$$E\left[\frac{1}{f\left(H_i,l\right)}\right]=\frac{35}{32c^7}\int_{-c}^{c}\frac{f\left(H_i,l\right)}{\left(c^2-h_s^2\right)^3}dh_s\,.$$

On integrating Eq.(16) using the boundary condition

$$\frac{dE\left(p\right)}{dx}=0\ at\quad x=0 \tag{18}$$

Gives

$$\frac{dE\left(p_i\right)}{dx}=\frac{-12\mu Vx}{G_i\left[H_i,l,c\right]} \tag{19}$$

where
$$\begin{aligned} h_i &= h_1 \quad for \quad 0\le x\le KL \\ &= h_2 \quad for \quad KL\le x\le L \end{aligned}$$

$$G_i\left(H_i,l,c\right)=\begin{cases} E\left[f_i\left(H_i,l\right)\right], & for\, longitudinal\, roughness \\ E\left[\dfrac{1}{f_i\left(H_i,l\right)}\right]^{-1}, & for\, transverse\, roughness \end{cases}$$

$$f_i\left(H_i,l\right)=H_i^3-12l^2H_i+24l^3\,tanh\left(\frac{H_i}{2l}\right)$$

The relevant boundary conditions for the pressure are

$$E\left(p_1\right)=E\left(p_2\right)\quad a\quad tx=KL \tag{20a}$$

$$E\left(p_2\right)=0\qquad at\quad x=L \tag{20b}$$

The solution of equation (19) subject to boundary conditions (20a) and (20b) is obtained in the form

$$E\left(p_1\right)=6\mu V\left\{\frac{K^2L^2-x^2}{G_1\left(H_1,l,c\right)}+\frac{L^2\left(1-K^2\right)}{G_2\left(H_2,l,c\right)}\right\} \tag{21}$$

$$E\left(p_2\right)=\frac{6\mu V}{G_2\left(H_2,l,c\right)}\left(L^2-x^2\right) \tag{22}$$

The mean load carrying capacity $E\left(w\right)$ is given by

$$E\left(w\right)=2b\int_0^{KL}E\left(p_1\right)\,dx+2b\int_{KL}^{L}E\left(p_2\right)\,dx \tag{23}$$

The non-dimensional mean load carrying capacity W is given by

$$W=-\frac{2E\left(w\right)h_2^3}{3\mu bVL^3}=\left[\frac{K^3}{G_1\left(H^*,l^*,C\right)}+\frac{1-K^3}{G_2\left(1,l^*,C\right)}\right] \tag{24}$$

where $H^*=\dfrac{h_1}{h_2},\ l^*=\dfrac{2l}{h_2},\ C=\dfrac{c}{h_2},\ h_s^*=\dfrac{h_s}{h_2}$

$$f_1\left(H^*,l^*\right)=\left(H^*+h_s^*\right)^3-3l^{*2}\left(H^*+h_s^*\right)+3l^{*3}\,tanh\left(\frac{H^*+h_s^*}{l^*}\right)$$

$$f_1\left(H^*,l^*\right)=\left(1+h_s^*\right)^3-3l^{*2}\left(1+h_s^*\right)+3l^{*3}\,tanh\left(\frac{1+h_s^*}{l^*}\right)$$

By writing $V = -dh_2/dt$ in Eq.(23), the squeezing time for reducing the film thickness from an initial value h_0 of h_2 to a final value h_f is given by

$$t = -\frac{3\mu bL^3}{2E(w)} \int_{h_0}^{h_f} \left\{ \frac{K^3}{G_1(H_1,l,c)} + \frac{1-K^3}{G_2(H_2,l,c)} \right\} dh_2 \tag{25}$$

$$t^* = -\frac{2E(w)h_0^3 t}{3\mu bL^3} = \int_{h_f^*}^{1} \left\{ \frac{K^3}{G_1(h_3^*,h_2^*,h_s^*,l^*,C)} + \frac{1-K^3}{G_2(h_2^*,h_s^*,l^*,C)} \right\} dh_2^* \tag{26}$$

where $h_2^* = \frac{h_2}{h_0}$, $l^* = \frac{2l}{h_0}$, $C = \frac{c}{h_0}$, $h_s^* = \frac{h_s}{h_0}$, $h_3^* = \frac{h_3}{h_0}$

$$f_1(h_2^*,h_3^*,h_s^*,l^*) = h_2^{*3} + h_3^{*3} + 3h_2^{*2}h_3^* + 3h_2^*h_3^{*2} + h_s^{*3} + 3\left(h_2^{*2} + h_s^{*2} + 2h_2^*h_3^*\right) + 3\left(h_2^* + h_3^*\right)h_s^{*2}$$
$$-3l^{*2}\left(h_2^* + h_3^* + h_s^*\right) + 3l^{*3}\tanh\left(\frac{h_2^* + h_3^* + h_s^*}{l^*}\right)$$

$$f_2(h_2^*,h_s^*,l^*) = h_2^{*3} + h_s^{*3} + 3h_2^{*2}h_s^* + 3h_2^*h_s^{*2} - 3l^{*2}\left(h_2^* + h_s^*\right) + 3l^{*3}\tanh\left(\frac{h_2^* + h_s^*}{l^*}\right)$$

RESULTS AND DISCUSSIONS

In the present paper, the combined effects of surface roughness and couple stresses on the squeeze film lubrication between parallel stepped plates have been studied on the basis of Stokes microcontinuum theory for couple stress fluids and the Christensen stochastic theory for the study of rough surfaces viz; longitudinal roughness pattern and transverse roughness pattern. The parameter $l^*\left(=\frac{2l}{h_2}\right)$ is the ratio of microstructure size to the minimum film thickness. Hence, l^* gives the mechanism of the fluid with the bearing geometry. As the value of l^* approaches zero, the non-dimensional Reynolds equation reduces to the Newtonian lubricant case. When the value of l^* is large, the couple stress effects are expected to be significant. The effect of surface roughness is characterized by roughness parameter C. The limiting case of $C \to 0$ corresponds to smooth case.

Mean load carrying capacity

The variation of non-dimensional load carrying capacity W with H^* for different values of l^* with $C = 0.2, K = 0.5$ is shown in Figure 2 for both types of roughness patterns. It is observed that the load carrying capacity W increases (decreases) for increasing values of couple stress parameter l^* for transverse (longitudinal) roughness pattern. Figure 3 depicts the variation of non-dimensional load carrying capacity W with H^* for different values of C with $l^* = 0.4, K = 0.5$ for both types of roughness patterns. It is observed that W decreases for increasing values of H^* for both longitudinal and transverse roughness patterns. Further it is also observed that W increases (decreases) for increasing values of C for transverse (longitudinal) roughness pattern. The variation of non-dimensional load carrying capacity W with H^* for different values of K with $l^* = 0.4, C = 0.1$ is shown Figure 4 for both types of roughness patterns. It is observed that W decreases for increasing values of K.

Squeeze film time

The variation of non-dimensional squeeze film time t^* with h_f^* for different values of l^* with $C = 0.2, K = 0.5, h_3^* = 0.1\ 5$ is shown in Figure 5 for both types of roughness patterns. A significant increase in t^* is observed for couple-stress fluids as compared to Newtonian case. Further, the increase (decreases) in t^* is more for the transverse (longitudinal) roughness pattern. Figure 6 depicts the variation of non-dimensional squeeze film time t^* with h_f^* for different values of C with $l^* = 0.4, K = 0.5, h_3^* = 0.1\ 5$ for both types of roughness patterns. It is observed that t^* increases (decreases) for the transverse (longitudinal) roughness patterns. This increase/decrease in t^* is pronounced for smaller values of h_f^*. The variation of non-dimensional squeeze film time t^* with h_f^* for different values of K with $C = 0.2, l^* = 0.4, h_3^* = 0.15$ is shown in Figure 7 for both types of roughness patterns. It is observed that t^* decreases for increasing values of *K*.

CONCLUSIONS

On the basis of Stokes[10] couple stress fluid theory and the stochastic theory developed by Christensen[1] for the rough surfaces on the squeeze film lubrication of rough parallel stepped plates is analyzed. On the basis of numerical computations of the results obtained, the following conclusions are drawn.

1. The load carrying capacity increases (decreases) for increasing values of l^* for transverse (longitudinal) roughness patterns.
2. The squeeze film time increases (decreases) for increasing values of l^* for transverse (longitudinal) roughness patterns.
3. The effect of couple stresses is to improve the performance of squeeze film bearing as compared with the corresponding Newtonian case.

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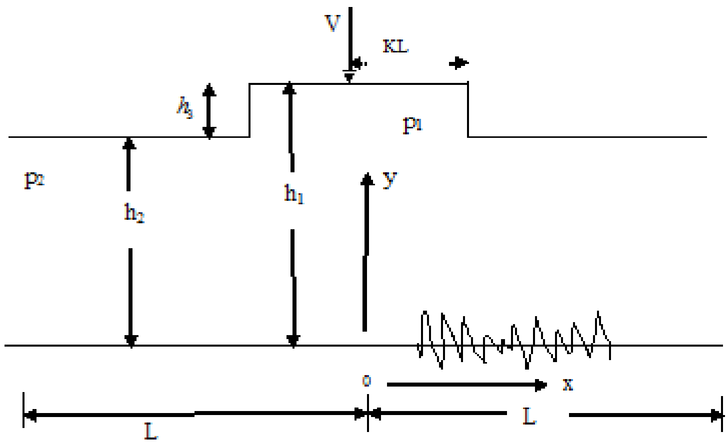

Fig. 1. Squeeze film lubrication between rough parallel stepped plates

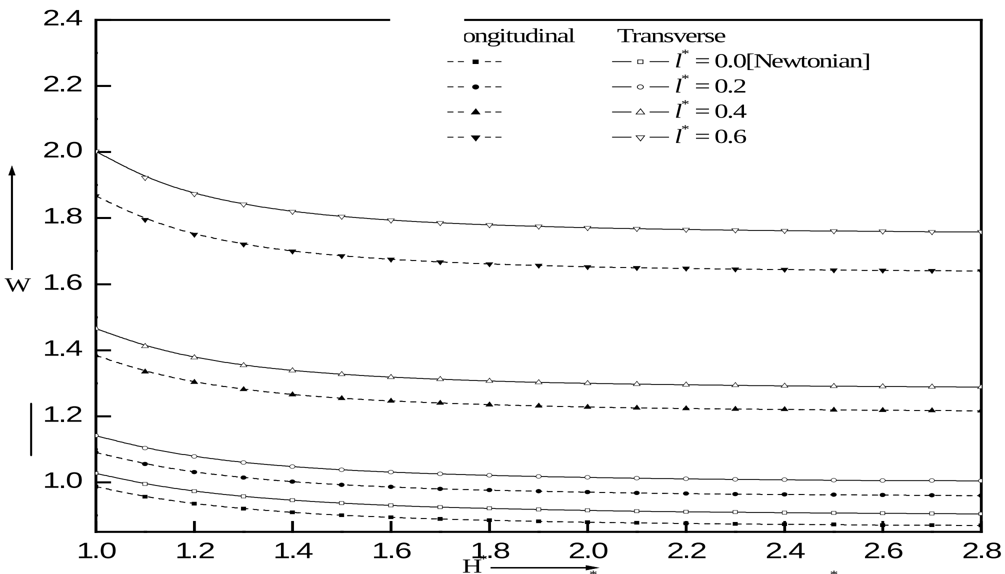

Fig 2. Variation of non-dimensional W with H^* for different values of l^* with C = 0.2, K = 0.5.

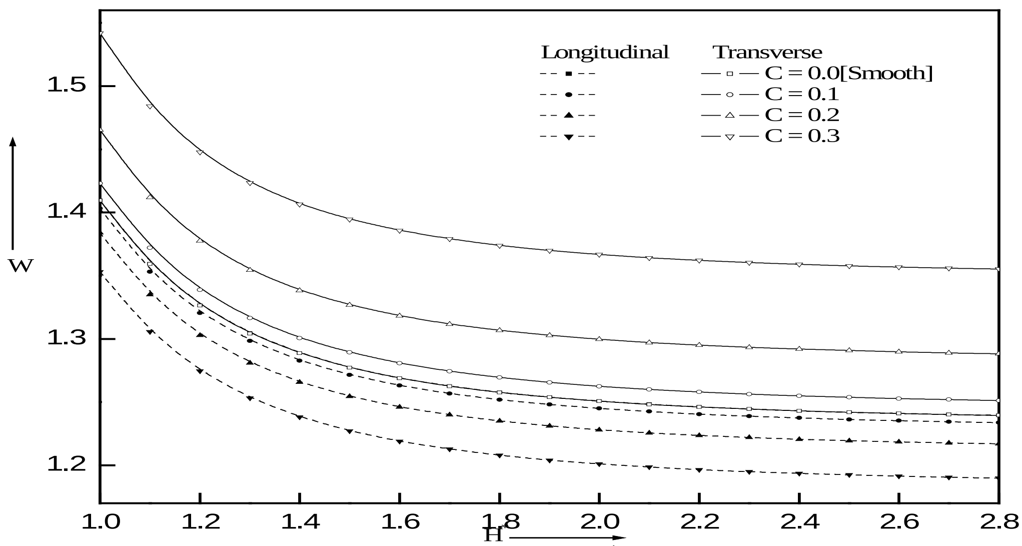

Fig 3. Variation of non-dimensional W with H^* for different values of c with K = 0.5, $l^* = 0.4$.

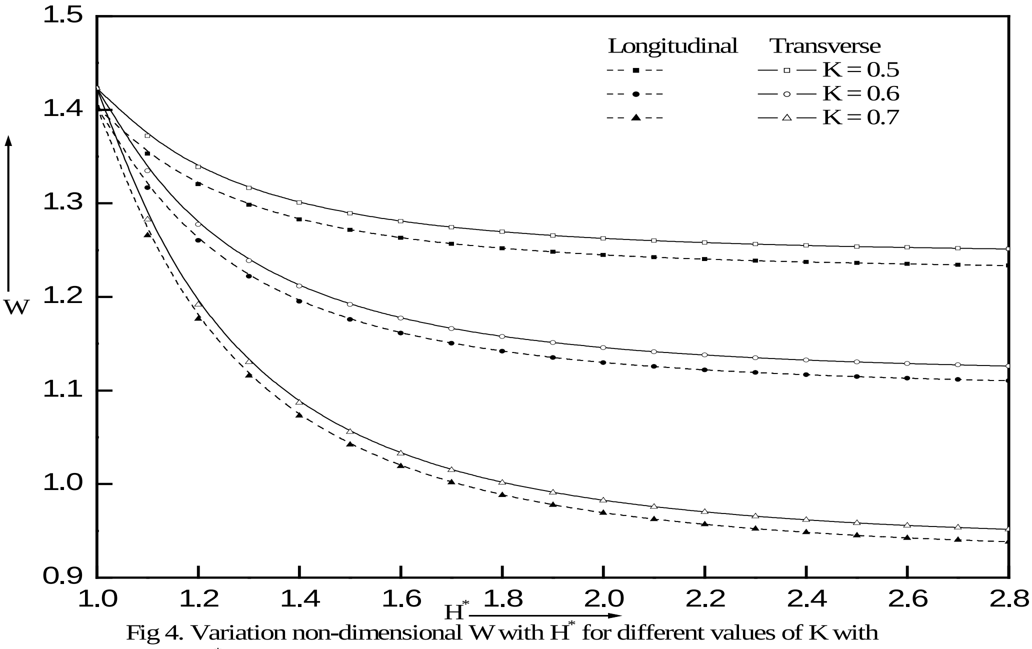

Fig 4. Variation non-dimensional W with H^* for different values of K with $l^* = 0.4$, C = 0.1.

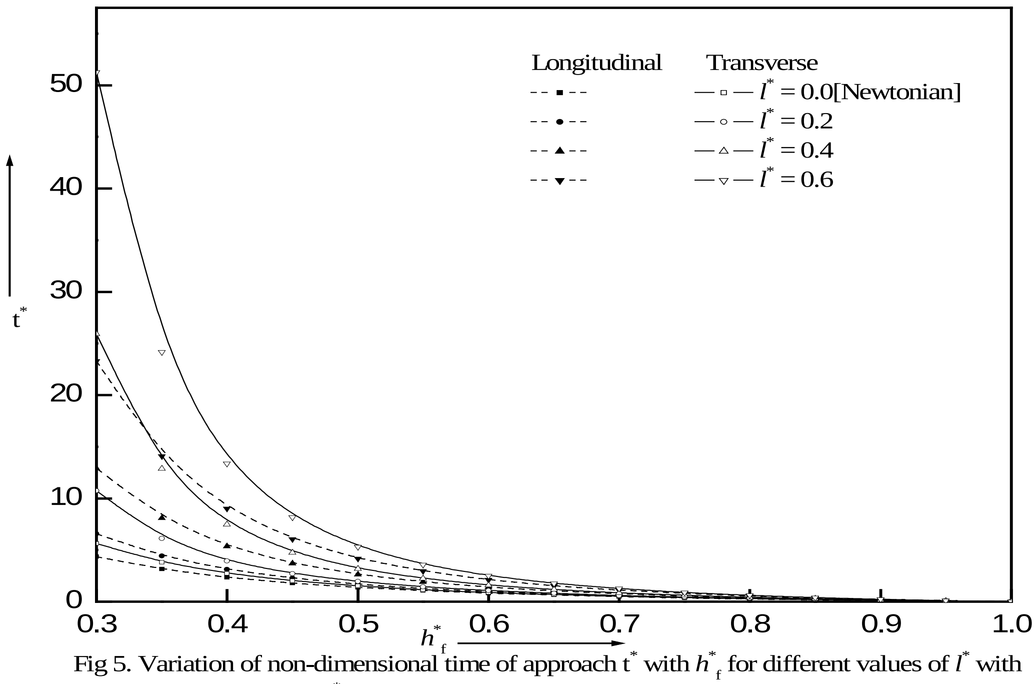

Fig 5. Variation of non-dimensional time of approach t^* with h^*_f for different values of l^* with C = 0.2, K = 0.5, $h^*_3 = 0.15$.

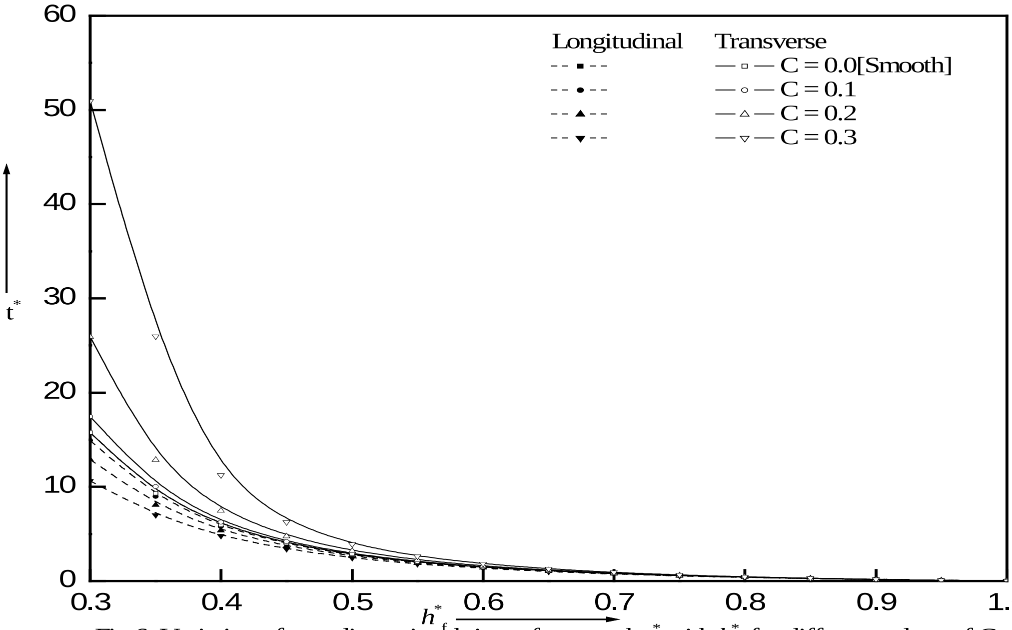

Fig 6. Variation of non-dimensional time of approach t^* with h^*_f for different values of C with K = 0.5, $l^* = 0.4$, $h^*_3 = 0.15$.

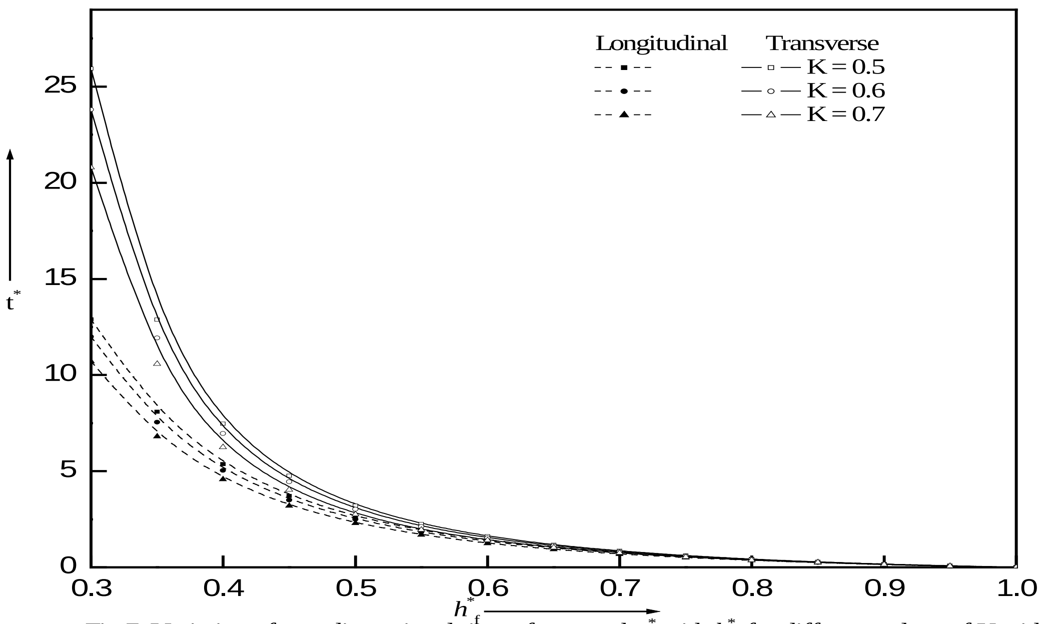

Fig 7. Variation of non-dimensional time of approach t^* with h^*_f for different values of K with C = 0.2, $l^* = 0.4$, $h^*_3 = 0.15$.

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