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STRONG CHROMATIC NUMBER OF RESULTANT OF FUZZY GRAPHS

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ABSTRACT

Graph coloring techniques are used to solve many practical problems involving networks. Coloring of fuzzy graphs also plays a significant role in solving many network problems. The concept of strong coloring of a fuzzy graph based on strength of arcs is defined in [5] and using the strong chromatic number of union, corona and different types of products of fuzzy graphs is obtained. The types of arcs in the resultant graphs are also studied. An application of strong chromatic number in solution of transportation problem is suggested.

1. INTRODUCTION

Fuzzy graphs, introduced by Rosenfield [11] is a widely explored area of recent research in applied mathematics. It finds applications in science and technology in many streams. Fuzzy models give more precision, flexibility and compatibility to the system when compared to the classic models. Many recent research papers are available in fuzzy graph theory. The basic concepts and applications of fuzzy graphs and recent developments are detailed in [14]. Works on bipolar fuzzy graphs, interval valued fuzzy graphs etc are a few among them [12], [1], [2], [10].

The concept of chromatic number of fuzzy graphs was introduced by Munoz *et.al* [9]. Later Eslahchi and Onagh [6] defined fuzzy coloring of fuzzy graphs and defined fuzzy chromatic number $\chi^f(G)$. Anjaly and Sunitha introduced chromatic number $\chi(G)$ of fuzzy graph G incorporating the features of definitions given in Munoz *et al.* and Eslahchi and Onagh and established that $\chi(G) = \chi^f(G)$.. They also developed algorithms for the same [4].

The concept of strong coloring of a fuzzy graph based on strength of arcs is introduced in [5] and strong chromatic number $\chi_s(G)$ is defined. In this paper, the types of arcs such as $\alpha -$ strong, $\beta -$ strong and $\delta-$ arcs [13] in the union, corona and types of products of fuzzy graphs are studied. The concept of strong coloring is used in operations of fuzzy graphs and the relation between the strong chromatic number of the resultant graph and that of the individual graphs are obtained.

2. PRELIMINARIES

The following basic definitions are taken from [8]. A fuzzy graph is an ordered triple $G : (V, \sigma, \mu)$ where V is a set of vertices $\{u_1, u_2, \ldots, u_n\}$, σ is a fuzzy subset of V i.e, $\sigma : V \to [0,1]$ and is denoted by $\sigma = \{(u_1, \sigma(u_1)), (u_2, \sigma(u_2)) \ldots (u_n, \sigma(u_n))\}$ and μ is a fuzzy relation on σ, i.e $\mu(u,v) \le \sigma(u) \wedge \sigma(v) \forall u, v \in V$. We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive (i.e.,$\mu(u,u) = \sigma(u), \forall u$) and symmetric (i.e., $\mu(u,v) = \mu(v,u), \forall (u,v)$). in all the examples σ is chosen suitably. Also, we denote the underlying crisp graph of G by $G^*: (\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V : \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in V \times V : \mu(u,v) > 0\}$. Through out we assume that σ^* =V. The level set of fuzzy set σ is defined as $\lambda = \{\alpha/\sigma(u) = \alpha$ for some $u \in V\}$. For each $\alpha \in \lambda$, G_α denotes the crisp graph $G_\alpha = (\sigma_\alpha, \mu_\alpha)$ where $\sigma_\alpha = \{u \in V/\sigma(u) \ge \alpha\}$, $\mu_\alpha = \{(u,v) \in V \times V/\mu(u,v) \ge \alpha\}$. The complement of a fuzzy graph[15] $G : (V, \sigma, \mu)$ is the fuzzy graph $\bar{G} : (V, \bar{\sigma}, \bar{\mu})$ with $\bar{\sigma}(u) = \sigma(u)$ and $\bar{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v), \forall u, v \in V$.

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A path P of length n is a sequence of distinct nodes $u_0, u_1, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, \ldots\ldots, n$ and the degree of membership of a weakest arc in P is defined as the strength of P. If $u_0 = u_n$ and $n \geq 3$ then P is called a cycle and P is called a fuzzy cycle, if it contains more than one weakest arc. The strength of a cycle is the strength of a weakest arc in it. A fuzzy cycle of length n is denoted by C_n. The maximum strength among all paths from u to v is denoted by $CONN_G(u, v)$. A fuzzy graph $G : (V, \sigma, \mu)$ is connected if for every u, v in V, $CONN_G(u, v) > 0$. A connected fuzzy graph $G : (V, \sigma, \mu)$ is a fuzzy tree if it has a fuzzy spanning subgraph $F : (V, \sigma, \nu)$, which is a tree where for all arcs (u, v) not in F there exists a path from u to v in F whose strength is more than $\mu(u, v)$. A maximum spanning tree of a connected fuzzy graph $G : (V, \sigma, \mu)$ is a fuzzy spanning subgraph $T : (V, \sigma, \nu)$, such that T^* is a tree, and for which $\sum_{u \neq v} \nu(u, v)$ is maximum. Also fuzzy graph is a fuzzy tree if and only if it has a unique maximum spanning tree [15]. A fuzzy graph G is said to be complete if $\mu(u, v) = \sigma(u) \wedge \sigma(v), \forall u, v \in V$. An arc (u, v) is said to be strong if $\mu(u, v) \geq CONN_{G\backslash(u,v)}(u, v)$[7]. Strong arcs are again classified as α−strong and β−strong arcs[13]. An arc (u, v) is said to be α−strong if $\mu(u, v) > CONN_{G\backslash(u,v)}(u, v)$ and if $\mu(u, v) = CONN_{G\backslash(u,v)}(u, v)$, it is said to be $\beta -$ strong . An arc (u, v) is said to be $\delta -$ arc if $\mu(u, v) < CONN_{G\backslash(u,v)}(u, v)$.

Definition 2.1: [8] If $G_1 = (V_1, \sigma_1, \mu_1)$ and $G_2 = (V_2, \sigma_2, \mu_2)$ be two fuzzy graphs with ${G_1}^* = (V_1, E_1)$ and ${G_2}^* = (V_2, E_2)$, then $G_1 \cup G_2$ is the fuzzy graph $(V_1 \cup V_2, \sigma, \mu)$

where

$$\sigma(u) = \begin{cases} \sigma_1(u), & u \in V_1 - V_2 \\ \sigma_2(u), & u \in V_2 - V_1 \end{cases} \text{ and}$$

$$\mu(u, v) = \begin{cases} \mu_1(u, v), & (u, v) \in E_1 - E_2 \\ \mu_2(u, v), & (u, v) \in E_2 - E_1 \end{cases}$$

Definition 2.2: [8] Let $G^* = {G_1}^* \times {G_2}^* = (V, E'')$ be the cartesian product of two graphs where $V = V_1 \times V_2$ and $E'' = \{(uu_2, uv_2) : u \in V_1, (u_2, v_2) \in E_2\} \cup \{(u_1v, v_1v) : v \in V_2, (u_1, v_1) \in E_1\}$, then $G_1 \times G_2$ is the fuzzy graph (V, σ, μ) where

$\sigma(u_1u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2) \forall (u_1u_2) \in V$

$\mu(uu_2, uv_2) = \sigma_1(u) \wedge \mu_2(u_2v_2) \forall u \in V_1, \forall (u_2, v_2) \in E_2$

$\mu(u_1v, v_1v) = \sigma_2(v) \wedge \mu_1(u_1v_1) \forall v \in V_2, \forall (u_1, v_1) \in E_1$.

3. STRONG CHROMATIC NUMBER

Coloring of graphs play a vital role in network problems. In any network, modeled as a fuzzy graph, the role of arcs with different strengths is significant. Note that the role of a $\delta-$ arc is negligible, as the flow is minimum along $\delta -$ arc and there is an alternate strong path (maximum flow) between the corresponding nodes. Hence strong arcs are more significant in networks. In [5], the concept of strong chromatic number of fuzzy graphs is introduced as follows.

Definition 3.1: [5] Consider a fuzzy graph $G : (V, \sigma, \mu)$. Any coloring $C : V(G) \rightarrow \mathbb{N}$ (where $\mathbb{N}$ is the set of all positive integers) such that $C(u) \neq C(v)$ if (u, v) is a strong arc($\alpha -$ strong and $\beta -$ strong) in G is called strong coloring.

A fuzzy graph G is k-strong colorable if there exists a strong coloring of G from a set of k colors.

The minimum number k for which G is k-strong colorable is called strong chromatic number of G denoted by $\chi_s(G)$.

Note that the end nodes of a $\delta -$ arc can be assigned the same color in strong coloring.

Illustration 1. Consider the following fuzzy graph in Fig. I.

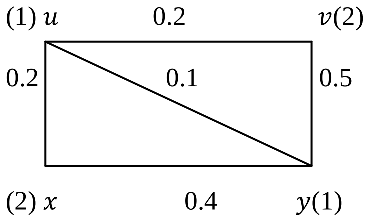

Fig. I

The arcs (u, v) and (u, x) are $\beta-$ strong arcs, since $\mu(u, v) = 0.2 = CONN_{G\backslash(u,v)}(u, v) = 0.2$ and $\mu(u, x) = 0.2 = CONN_{G\backslash(u,x)}(u, x) = 0.2$. The arcs (x, y) and (v, y) are $\alpha-$ strong as $\mu(x, y) = 0.4 > CONN_{G(x,y)}(x, y) = 0.2$ and $\mu(v, y) = 0.5 > CONN_{G\backslash(v,y)}(v, y) = 0.2$. But arc (u, y) is a $\delta-$ arc, since $\mu(u, y) = 0.1 < CONN_{G\backslash(u,y)}(u, y) = 0.2$. Hence the nodes u and y are assigned the same color (say) color 1. The nodes v and x are assigned color 2. Hence $\chi_s(G)$ = 2. Note that the chromatic number $\chi(G)$= 3 since u and y are adjacent and hence colored differently.

4. STRONG COLORING OF RESULTANT GRAPHS

In this section, strong chromatic number of the resulting graphs obtained by different operations on fuzzy graphs are studied. First we discuss the types of arcs in operations of fuzzy graphs.

Theorem 4.1: *The $\alpha-$ strong, $\beta-$ strong and $\delta-$ arcs are preserved in union of fuzzy graphs.*

Proof: The definition of union of fuzzy graphs indicates the connectivity and adjacencies of the nodes in fuzzy graphs are unaffected. Hence the result.

Theorem 4.2: $\chi_s(G_1 \cup G_2) = max\ \{\chi_s(G_1), \chi_s(G_2)\}$

Proof: The arc strengths are preserved in union. Hence the result follows.

Theorem 4.3: *There are no $\delta -$ arcs in $G_1 \times G_2$ if and only if both G_1 and G_2 are complete fuzzy graphs, fuzzy cycles or fuzzy trees.*

Proof: Since there are no $\delta -$ arcs in complete fuzzy graphs, fuzzy cycles and fuzzy trees [13], the proof follows from the definition of cartesian product of fuzzy graphs.
For other operations, depending upon the arc strength and node strength, $\alpha-$ strong, $\beta-$ strong and $\delta-$ arcs may vary in their behaviour in the resultant graphs.

We now have the results on strong coloring as follows.

Theorem 4.4: $\chi_s(K_n \times K_m) = max\ \{m, n\}$.

Proof: Without loss of generality, let us assume that $n > m$. Let $u_1, u_2, \ldots, u_n$ be the nodes of K_n and $v_1, v_2, \ldots, v_m$ be the nodes of K_m. Take a node say u_1v_1 of $K_n \times K_m$. The node u_1v_1 is adjacent to all nodes u_1v_j, $j = 1, 2, 3, \ldots, m$ and u_iv_1 $i = 2, 3, \ldots, n$, by the definition of cartesian product. Also by theorem 4.3, there are no $\delta-$ arcs in $K_n \times K_m$ and all adjacencies are by strong arcs. Hence all nodes u_1v_j must be colored differently. Hence m colors are required. Also all nodes u_iv_1 are colored with n colors. Since the nodes u_iv_j and u_rv_s are adjacent if and only if either $i = r$ or $j = s$, all nodes u_iv_1 and u_1v_j are not adjacent. Hence the colors of the nodes u_iv_1 can be used for coloring the nodes u_1v_j. Hence for strong coloring of $K_n \times K_m$, a minimum of n colors are required. Hence the proof.

Definition 4.5: The corona $G_1 \circ G_2$ is the fuzzy graph obtained by taking one copy of G_1 having n_1 nodes and n_1 copies of G_2. Then join the node i of G_1 to every node in the i^{th} copy of G_2.

The fuzzy graph $G = G_1 \circ G_2 = (V, \sigma, \mu)$ where $\sigma(u_i) = \sigma_1(u_i)$, $\sigma(v_i) = \sigma_2(v_i)$ for all $u_i \in V_1$, $v_i \in V_2$ and $\mu(u_i, u_j) = \mu_1(u_i, u_j)$, $\mu(v_i, v_j) = \mu_2(v_i, v_j)$ and $\mu(u_i, v_j) = \sigma_1(u_i) \wedge \sigma_2(v_j)$ for all $u_i \in V_1$, $v_j \in V_2$.

Theorem 4.6: *If $\sigma_1(u_i) > \sigma_2(v_j)$ for all $u_i \in V_1$ and $v_j \in V_2$, then (u, v) is a $\delta-$ arc of $G_1 \circ G_2$ if and only if (u, v) is either a $\delta -$ arc of G_1 or G_2.*

Proof: Let (u, v) be a $\delta -$ arc of $G_1 \circ G_2$. If both u and v are nodes of G_1 or G_2, then by definition of corona, (u, v) is a $\delta -$ arc of G_1 or G_2 respectively. Consider arc (u, v) is such that $u \in V_1$ and $v \in V_2$. Then by our assumption, $\mu(u, v) = \sigma_2(v)$. Now there are two cases:

Case-I: $\sigma_2(v_j) \geq \sigma_2(v)\ \forall v_j \in V_2$. By definition of corona, there is atleast one arc (v, v_k) in G_2 of strength equal to $\sigma_2(v)$ in the path joining u and v in $G_1 \circ G_2$. But this contradicts the assumption that (u, v) is a $\delta -$ arc of $G_1 \circ G_2$. Hence an arc (u, v) such that $u \in V_1$ and $v \in V_2$ cannot be a $\delta -$ arc of $G_1 \circ G_2$.

Case-II: $\sigma_2(v_j) \leq \sigma_2(v)\ \forall v_j \in V_2$. By definition of corona, there is atleast one arc $((v, v_k)$ in G_2 of strength less than $\sigma_2(v)$ in the path joining u and v in $G_1 \circ G_2$. Again this contradicts the assumption that (u, v) is a $\delta -$ arc of $G_1 \circ G_2$. Hence an arc (u, v) such that $u \in V_1$ and $v \in V_2$ cannot be a $\delta -$ arc of $G_1 \circ G_2$. Similarly in any case we can prove that an arc (u, v) such that $u \in V_1$ and $v \in V_2$ is not a $\delta -$ arc of $G_1 \circ G_2$.

Conversely, assume that (u, v) is a $\delta -$ arc of G_1 or G_2. To prove that (u, v) is a $\delta -$ arc of $G_1 \circ G_2$. By definition of corona, the $\delta -$ arc in G_1 remains $\delta -$ arc in $G_1 \circ G_2$ since the path joining any two nodes of G_1 is the same in $G_1 \circ G_2$. Now consider a $\delta -$ arc of G_2 say (v_k, v_l). Even if there are arcs (u_i, v_j) in the path between v_k and v_l in $G_1 \circ G_2$, all arcs will have strength greater than $\mu_2(v_k, v_l)$ since $\mu(u_i, v_j) = \sigma_1(u_i) \wedge \sigma_2(v_j) = \sigma_2(v_j)$ and $\mu_2(v_k, v_l) < \sigma_2(v_k) \wedge \sigma_2(v_l)$. Hence $CONN_{G_1 \circ G_2 \setminus (v_k, v_l)}(v_k, v_l) \geq CONN_{G_2 \setminus (v_k, v_l)}(v_k, v_l)$ and (v_k, v_l) remain as a $\delta -$ arc in $G_1 \circ G_2$. Hence the proof.

Theorem 4.7: If $\mu_1(u_i,u_j) \geq \mu_2(v_i,v_j)$ for all arcs (u_i,u_j) in G_1 and (v_i,v_j) in G_2, then $\chi_s(G_1 \circ G_2) \leq \{\chi_s(G_1) + \chi_s(G_2)\}$.

Proof: In $G_1 \circ G_2$, a node $u_i \in V_1$ is adjacent to each node $v_j \in V_2$. All adjacencies of G_1 and G_2 are also preserved with same arc strengths. For strong coloring of $G_1 \circ G_2$, the nodes of G_1 are assigned the same colors and the colors can be repeated for coloring the nodes of G_2 such that end nodes of strong arcs are colored differently. Hence the total number of colors required will be always less than or equal to the sum of strong chromatic numbers of G_1 and G_2. Hence the proof.

Now we find strong chromatic number of the resultant graph of three types of products of fuzzy graphs introduced in [3].

Definition 4.8: Direct Product [3]: If $G_1 = (V_1,\sigma_1,\mu_1)$ and $G_2 = (V_2,\sigma_2,\mu_2)$, then the direct product $G_1 \sqcap G_2$ is the fuzzy graph $G = (V_1 \times V_2,\sigma,\mu)$ where E = $\{(u_1v_1,u_2v_2): (u_1,u_2) \in E_1, (v_1,v_2) \in E_2\}$,
$\sigma(uv) = min\{\sigma_1(u),\sigma_2(v)\}\ \forall uv \in V_1 \times V_2$,
$\mu(u_1v_1,u_2v_2) = \mu_1(u_1,u_2) \wedge \mu_2(v_1,v_2)$.

Theorem 4.9: *If $G_1 = (V_1,\sigma_1,\mu_1)$ and $G_2 = (V_2,\sigma_2,\mu_2)$ are connected, then there are no $\delta-$ arcs in $G_1 \sqcap G_2$.*

Proof: If there are no δ arcs in G_1 as well as in G_2, the definition of direct product indicates that all nodes are connected with strong arcs. By definition of direct product, any node (u,v) has strength $min\{\sigma_1(u),\sigma_2(v)\}$.

Case-I: All nodes and all arcs of G_1 have strength greater than $\sigma_2(v)\forall\ v \in V_2$. Then each node uv of $G_1 \sqcap G_2$ has strength $\sigma_2(v)$ and each arc has strength $\mu(u_1v_1,u_2v_2) = \mu_2(v_1,v_2)$. Let (v_i,v_j) be a δ arc in G_2,. Then by assumption, $\mu(u_iv_i,u_jv_j) = \mu_2(v_i,v_j)$. Since both G_1 and G_2 are connected, there exists a path joining the nodes u_iv_i and u_jv_j with arcs (u_kv_i,u_mv_j) having strength $\mu_2(v_i,v_j)$. Hence there is at least one arc of the same minimum strength as that of (v_i,v_j) in $G_1 \sqcap G_2$. Hence any $\delta-$ arc in G_2 is not a $\delta-$ arc in $G_1 \sqcap G_2$.

Case-II: The nodes and arcs of G_1 and G_2 are of arbitrary strength. Let (u_r,u_s) be a $\delta-$ arc in G_1. Then two cases arise.

Subcase-I: $\mu_1(u_r,u_s) \leq \mu_2(v_i,v_j)\forall\ (v_i,v_j) \in E_2$. Then by definition of $G_1 \sqcap G_2$, $\mu(u_rv_i,u_sv_j) = \mu_1(u_r,u_s)$. Hence for any path joining the nodes u_rv_i and u_sv_j, there exist at least two intermediate nodes u_rv_k and u_sv_l with arc (u_rv_k,u_sv_l) of strength $\mu_1(u_r,u_s)$. Thus for any path joining u_rv_i and u_sv_j, the strength will be same as $\mu_1(u_r,u_s)$. Hence (u_rv_i,u_sv_j) is not a $\delta-$ arc.

Subcase-II: There exists at least one arc (v_p,v_m) in G_2 with strength $\mu_2(v_p,v_m) \leq \mu_1(u_r,u_s)$. Then since (u_r,u_s) being a $\delta-$ arc in G_1., there are other arcs say (u_k,u_l) in G_1 of strength > $\mu_1(u_r,u_s)$. Hence for any path joining the nodes u_rv_p and u_sv_m, there exists atleast two intermediate nodes u_kv_p and u_lv_m with arc (u_kv_p , u_lv_m) of strength $\mu_2(v_p,v_m)$ Hence (u_rv_i,u_sv_j) is not a δ arc . Hence there are no $\delta-$ arcs in direct product of two connected fuzzy graphs. Now interchanging the roles of G_1 and G_2, a similar argument holds.

Theorem 4.10: $\chi_s(G_1 \sqcap G_2) = m_1m_2$ where $m_1 =|E_1|$ and $m_2 =|E_2|$.

Proof: By theorem 4.9, all arcs in $G_1 \sqcap G_2$ are strong. Thus strong coloring gives different colors for each adjacent node. Hence the number of colors required is equal to the number of pairs of adjacent nodes or number of arcs. By definition, the number of arcs in $G_1 \sqcap G_2$ is m_1m_2 and the result follows.

Definition 4.11: Semi product[3]:If If $G_1 = (V_1,\sigma_1,\mu_1)$ and $G_2 = (V_2,\sigma_2,\mu_2)$, then the semi product $G_1 \diamond G_2$ is the fuzzy graph $G = (V_1 \times V_2,\sigma,\mu)$ where E = $\{(uv_1,uv_2): u \in V_1, (v_1,v_2) \in E_2\} \cup \{(u_1v_1,u_2v_2): (u_1,u_2) \in E_1, (v_1,v_2) \in E_2\}$,
$\sigma(uv) = min\{\sigma_1(u),\sigma_2(v)\}\ \forall uv \in V_1 \times V_2$,
$\mu(uv_1,uv_2) = \sigma_1(u) \wedge \mu_2(v_1,v_2)$.
$\mu(u_1v_1,u_2v_2) = \mu_1(u_1,u_2) \wedge \mu_2(v_1,v_2)$.

Similar to that of direct product we get the following results for semi product and strong product.

Theorem 4.12: *If $G_1 = (V_1,\sigma_1,\mu_1)$ and $G_2 = (V_2,\sigma_2,\mu_2)$ are connected, then there are no $\delta-$ arcs in $G_1 \diamond G_2$.* All arcs in $G_1 \diamond G_2$ are $\beta-$ strong.

Proof: By definition, the arc set of semi product is $\{(u_1v_1, u_2v_2): (u_1,u_2) \in E_1,\ (v_1,v_2) \in E_2 \cup \{(uv_1, uv_2): u \in V_1,\ (v_1,v_2) \in E_2\}$ .i.e the arc set of semi product consists of arcs of direct product along with arcs of the form $\{(uv_1, uv_2)/\, u \in V_1,\ (v_1,v_2) \in E_2\}$. Then from the proof of theorem 4.9. there exists a path between u_iv_i and u_jv_j . In $G_1 \diamond G_2$, the path becomes a cycle since there are arcs (u_jv_k, u_iv_m) such that the cycle joining u_iv_i and u_jv_j is $u_iv_i - u_iv_m - u_jv_k - u_jv_j$ together with the arc (u_iv_i, u_jv_j) . Argument similar to the direct product shows that all arcs (u_iv_i, u_jv_j) in $G_1 \diamond G_2$ have their strength same as that of some intermediate arc in the cycle joining the end nodes u_iv_i and u_jv_j of the cycle $\forall i, j$. Hence the proof.

Theorem 4.13: $\chi_s(G_1 \diamond G_2) \le n_1m_2$ where $n_1 = |V_1|$ and $m_2 = |E_2|$.

Proof: By theorem 4.12, all arcs in $G_1 \diamond G_2$ are $\beta-$ strong. Thus strong coloring gives different colors for each adjacent node. Hence the number of colors required is less than or equal to the degree of each vertex. By definition, the degree is less than n_1m_2 and the result follows.

Theorem 4.14: $\chi_s(G_1 \diamond G_2) \le \{\chi_s(G_1) + \chi_s(G_2)\}$

Proof: The same color can be used for nodes of the form $u_1v\ and u_2v$, where $u_i \in V_1$ and $v_j \in V_2$. All other nodes of the form $(uv_1, uv_2)/\, u \in V_1,\ (v_1,v_2) \in E_2$are to be colored differently. For strong coloring of these nodes, $|V_1|$ colors are required. These colors can be repeated to color nodes of the form u_1v_1 and u_2v_2, since the fuzzy graph $G_1 \diamond G_2$ is not a complete fuzzy graph. Also sum of strong chromatic number of G_1 and G_2 is greater than the number of pairs of adjacent nodes in $G_1 \diamond G_2$. Hence the strong chromatic number of the semi product will always be less than or equal to the sum of strong chromatic numbers of G_1 and G_2.

Definition 4.15: Strong Product [3]: If $G_1 = (V_1, \sigma_1, \mu_1)$ and $G_2 = (V_2, \sigma_2, \mu_2)$, then the strong product $G_1 \otimes G_2$ is the fuzzy graph $G = (V_1 \times V_2, \sigma, \mu)$
where E = $\{(uv_1, uv_2): u \in V_1,\ v_1, v_2) \in E_2\} \cup \{(u_1w, u_2w): (u_1,u_2) \in E_1, w \in V_2\} \cup \{(u_1v_1, u_2v_2): (u_1,u_2) \in E_1, (v_1,v_2) \in E_2\}$,
$\sigma(uv) = min\{\sigma_1(u), \sigma_2(v)\}\ \forall uv \in V_1 \times V_2$,
$\mu(uv_1, uv_2) = \sigma_1(u) \wedge \mu_2(v_1,v_2)$.
$\mu(u_1w, u_2w) = \mu_1(u_1,u_2) \wedge \sigma_2(w)$.
$\mu(u_1v_1, u_2v_2) = \mu_1(u_1,u_2) \wedge \mu_2(v_1,v_2)$.

Remark 4.16: If $\mu_1(u_1,u_2) > 0$ and $\mu_2(v_1,v_2). > 0\ \forall u_1,u_2 \in V_1$ and $v_1,v_2 \in V_2$, then the support of $G_1 \otimes G_2$ is a complete graph and the number of nodes in G1 ⊗ G2 = $|V_1|.|V_2|$

Theorem 4.17: *If $G_1 = (V_1, \sigma_1, \mu_1)$ and $G_2 = (V_2, \sigma_2, \mu_2)$ are connected, then there are no δ- arcs in $G_1 \otimes G_2$.* All arcs in $G_1 \otimes G_2$ are $\beta-$ strong.

Proof: By definition, arc set of strong product is $\{(uv_1, uv_2): u \in V_1,\ v_1, v_2) \in E_2\} \cup \{(u_1w, u_2w): (u_1,u_2) \in E_1, w \in V_2\} \cup \{(u_1v_1, u_2v_2): (u_1,u_2) \in E_1,\ (v_1,v_2) \in E_2\}$. Hence an argument similar to that in the proof of theorem 4.12 gives the proof.

Theorem 4.18: If $\mu_1(u_1,u_2) > 0$ and $\mu_2(v_1,v_2). > 0\ \forall u_1,u_2 \in V_1$ and $v_1,v_2 \in V_2$, then $\chi_s(G_1 \otimes G_2) = |V_1|.|V_2|$.

Proof: By remark 4.16, strong product gives a support which is a complete graph. All arcs in $G_1 \otimes G_2$ are strong. Hence the proof.

Remark 4.19: If δ is the minimum degree and Δ is the maximum degree of the underlying crisp graph G^*, then $\delta \le \chi_s(G) \le \chi(G) \le \Delta + 1$.

Theorem 4.20: $\chi_s(G_1 \otimes G_2) \ge \chi_s(G_1) + \chi_s(G_2)$. Equality holds if $\chi_s(G_1) = \chi(G_1)$ and $\chi_s(G_2) = \chi(G_2)$, where there are no δ− arcs in G_1 or G_2.

Proof: The arcs of strong product are all strong. All nodes that are adjacent are to be colored differently. The only non adjacent nodes are u_iv_j and u_kv_l where $(u_i,u_k) \notin E_1,\ (v_j,v_l) \notin E_2$. Since the nodes u_i, u_k and v_j , v_l are non adjacent in G_1 and G_2, they are assigned the same color in strong coloring of G_1 and G_2. Also $\deg(u_iv_j) = |V_1| + |V_2| + min\{deg(u_i), deg(v_j)\} - 1$. Also $\delta(G1 \otimes G2) = |V_1| + |V_2| + min\ \{\delta(G_1), \delta(G_2)\} - 1 \ge \Delta(G_1) + \Delta(G_2)$. Hence $\chi_s(G_1 \otimes G_2) \ge \Delta(G1) + \Delta(G2) \ge \chi_s(G_1) + \chi_s(G_2)$.

5. APPLICATION

Consider a transportation channel covering n places. Let the fuzzy graph model is as follows. The nodes represents the places and arcs represent accessibility with varying strengths indicating the quality of transportation. The more number of strong arcs indicates that the channel is more efficient. i.e., more number of places are interconnected with good transportation or are easily accessible. Correspondingly the cost of transportation can be reduced.

Illustration 2: Consider a transportation problem with four places denoted as u_1, u_2, u_3, u_4. Consider the following fuzzy graphs G_1 in Fig II and G_2 in Fig.III.

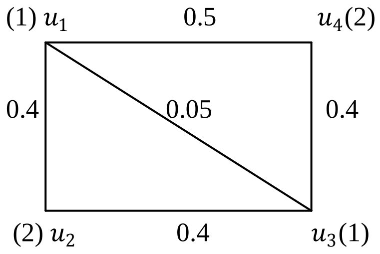

Fig. II

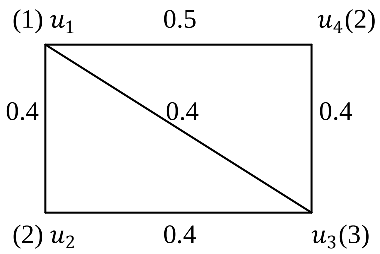

Fig. III

In Fig. II, (u_1, u_3) is a $\delta -$ arc since $\mu(u_1, u_3) = 0.05 < CONN_{G\setminus(u_1,u_3)}(u_1, u_3) = 0.4$, where as all other arcs are strong. Here $\chi_s(G) = 2$. If some goods are to be transported from u_1 to u_3 the path with maximum strength $u_1 - u_2 - u_3$ is chosen. But in Fig. III, (u_1, u_3) is a strong arc and hence $\chi_s(G) = 3$. and the goods are directly transported from u_1 to u_3. Hence larger the strong chromatic number,lesser the transportation cost.

6. CONCLUSION

In this paper, we have studied the types of arcs in union, corona and products of fuzzy graphs and analyzed whether the presence of δ- arcs in G_1 and G_2. guarantee the presence of $\delta -$ arc in the resultant fuzzy graph. The strong chromaticity of resultant graphs of various operations are also obtained. An application of strong chromatic number in solving transportation problem is suggested.

REFERENCES

1. Akram.M, Davvaz.B, Bipolar fuzzy Graphs, Information Sciences, 181(2011), 5548-5564.
2. Akram.M, Davvaz.B, Strong Intuitionistic fuzzy Graphs, Filomat, 26(1) (2012), 177-195.
3. Al. Hawary. Talal., Complete Fuzzy Graphs, International Journal of Mathematical Combinatorics, Dec (2011), Vol.4,p. 26.
4. Anjaly Kishore, M.S.Sunitha, Chromatic Number of Fuzzy Graphs, Annals of Fuzzy Mathematics and InformaticsVol.7, No.4,(2014) pp 543-551.
5. Anjaly Kishore,M.S.Sunitha, Strong Chromatic Number of Fuzzy Graphs, Annals of Pure and Applied mathematics, 7(2), (2014),52-60.
6. Eslahchi.C , Onagh.B.N., Vertex Strength of fuzzy graphs, International Journal of Mathematics and Mathematical Sciences(2006).
7. K.R.Bhutani, Rosenfeld, A., Strong arcs in fuzzy graphs, Information Sciences, 152 (2003) 319-322.
8. Mordeson, J. N., Nair, P.S., Fuzzy Graphs and Fuzzy Hypergraphs, Physica- Verlag (2000).
9. Munoz.S,Ortuno.T,Ramirez.J,Yanez.J., Coloring fuzzy graphs, Omega, The International Journal of Management Science 33 (2005) 211-221.
10. M.Pal, H.Rashmanlou., Irregular Interval valued fuzzy graphs, Annals of Pure and Applied mathematics, 3(1),(2013),425-438.

11. A. Rosenfeld, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu, M. Shimura (Eds.), Fuzzy Sets and their Applications to Cognitive and Decision Processes, Academic Press, New York, 1975, 77-95.
12. Samantha.S, M.Pal Irregular Bipolar fuzzy graphs, International Journal of Applications of Fuzzy Systems, Vol.2, (2012) pp91-102.
13. Sunil Mathew., M.S.Sunitha., Types of arcs in a fuzzy graph, Information Sciences, (2009) 1760-1768.
14. Sunil Mathew., Fuzzy Graphs - Basics, Concepts and Applications, Lambert Academic Publishing, (2012).
15. M.S.Sunitha., A.Vijayakumar., Complement of a fuzzy graph, Indian Journal of pure and applied Mathematics (2002) 1451-1464.

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