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NEIGHBORHOOD CONNECTED TWO OUT DEGREE EQUITABLE DOMINATION NUMBER FOR SOME STANDARD GRAPHS

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ABSTRACT

Let G= (V, E) be a connected graph. A two out degree equitable dominating set D of a graph G is called the neighborhood connected two out degree equitable dominating set (nc2oe-set) if the induced sub graph <N(D)> is connected. The minimum cardinality of a neighborhood connected two out degree equitable dominating set is called neighborhood connected two out degree equitable domination number of G and is denoted by $\gamma_{nc2oe}(G)$. In this paper we initiate a study of this parameter and we are going find $\gamma_{nc2oe}(G)$ for some standard graphs.

***Mathematics Subject Classification**: 05C69.*

***Keywords**: Two out degree equitable domination number, connected two out degree equitable number, neighborhood connected two out degree equitable number.*

INTRODUCTION

By a graph G= (V,E) we mean a finite , unordered with neither loops or multiple edges the order and size of G are denoted by p and q respectively. For graph theoretic terminology we refer to Chartand and Lesniak [5]. A subset D of V is called a dominating set if N[D]=V. The minimum (maximum) cardinality of a minimal dominating set of G is called domination number (upper domination) number of G is denoted by $\gamma(G)[\Gamma(G)]$. An excellent treatment of the fundamentals of domination is the book by Haynes etal [8]. A survey of several advanced topics in domination is given in the book edited by Haynes *et al.* [9]. Various types of domination have been defined and studied by several authors and more than 75 models of domination are listed in the appendix of Haynes *et al.* [9]. Let v ϵV the open neighborhood and the closed neighbourhood of v are donted by N(v)={u $\epsilon V, uv\epsilon E$ } and N[v]= N(v) ∪ v respectively. If D⊂ V then N(D) =$\cup_{v\epsilon D} N(v)$ and N[D]= N(D) ∪ D. If D⊂ V then vϵ D then private neighbor set of u with respect to D is defined by $P_n[u, D] = \{v: N[v] \cap S = \{u\}\}$

Sampth Kumar and Walikar [4] introduced the concept of connected domination in graphs. A dominating set D of G is called connected dominating set if the induced subgraph <D> is connected. The minimum cardinality of a connected dominating set of G is connected domination number of G and is denoted by $\gamma_c(G)$. S.Arumagam and C.Sivagnanam [3] introduced the concept of neighborhood connected domination in graphs. A dominating set D of connected graph G is called neighborhood connected dominating set (ncd-set) if the induced sub graph <N(D)> is connected. The minimum cardinality of a ncd-set D is called neighborhood connected domination number and it is denoted by $\gamma_{nc}(G)$.

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Let G= (V, E) be a graph D⊆V and v be any vertex in D. The out degree of v with respect to D is denoted by $od_D(v)$ and is defined by $od_D(v) = |N(v) \cap V - D|$. Let D be a dominating set of a graph G=(V,E) then D is called an equitable dominating set of type 1 if $|od_D(v_1) - od_D(v_2)| \leq 1$ for all $v_1, v_2 \in D$. The minimum cardinality of such a dominating set is denoted by $\gamma_{eq1}(G)$ called the 1- equitable domination number of G[2]. A.Sahal and V.Mathad [1] introduced the concept of two out degree equitable dominating set in graphs. A dominating set of D in a graph G is called a two out degree equitable dominating set if for any two vertices u,v $\in D$, $|od_D(u) - od_D(v)| \leq 2$. The minimum cardinality of a two out degree equitable domination number of G is denoted by $\gamma_{2oe}(G)$. An two out degree equitable dominating set D of connected graph G is called an connected two out degree equitable dominating set if the induced sub graph <D> is connected.[6] The minimum cardinality of connected two out degree equitable dominating set is called connected two out degree equitable domination number G and it is denoted by $\gamma_{c2oe}(G)$.

2. NEIGHBORHOOD CONNECTED TWO OUT DEGREE EQUITABLE DOMINATION NUMBER

Definition 2.1: A two out degree equitable dominating set D of a graph G is called the neighborhood connected two out degree equitable dominating set (nc2oe-set) if the induced sub graph <N(D)> is connected. The minimum cardinality of a neighborhood connected two out degree equitable dominating set is called neighborhood connected two out degree equitable number of G and is denoted by $\gamma_{nc2oe}(G)$.

Example 2.2: Let G be a graph as in the figure 1, we can find the neighborhood connected two out degree equitable domination number

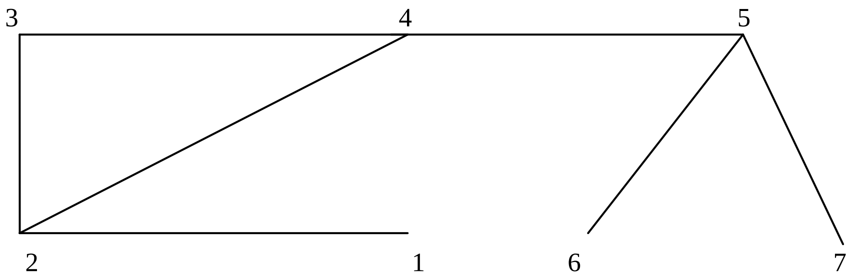

Fig 1

Let us consider a set D = {1, 3, 6, 7} and V-D= {2, 4, 5}

$od_D(1) = |N(1) \cap \{2,4,5\}| = 1$

$od_D(3) = |N(3) \cap \{2,4,5\}| = 2$

$od_D(6) = |N(6) \cap \{2,4,5\}| = 1$

$od_D(7) = |N(7) \cap \{2,4,5\}| = 1$

Then Clearly for any u,v ϵ {1,3,67} such that $|od_D(u) - od_D(v)| \leq 2$.

D= {1, 3, 6, 7} is two out degree equitable dominating set

N(D)= N(1) ∪ N(3) ∪ N(6) ∪ N(7) ={2,4,5}

<N(D)> = {2,4,5} is nc2oe-set and D={1,3,6,7} is minimum neighborhood connected two out degree equitable dominating set

$\gamma_{nc2oe}(G)$=4.

Theorem 2.3: A super set of an nc2oe – set is a nc2oe- set.

Proof: Let D be a nc2oe-set of a graph G and $D_1 = D \cup \{v\}$ where v $\in V - D$.

Since D is a dominating set v $\in N(D)$ and then D_1 is dominating set

Since every super set of two out degree equitable dominating set is two out degree equitable dominating

Then D_1 is also a two out degree equitable dominating set

Now let x, y $\in N(D_1) = N(D) \cup N(v)$

If x, y $\in N(D)$ and D is a nc2oe-set then for any $x - y$ path in $N(D)$ is a $x - y$ path in $N(D_1)$

If x $\in N(D)$ and y $\notin N(D)$, then y $\in N(v)$ and any $x - v$ path in $N(D)$ followed by the edge vy is $x - y$ path in $N(D_1)$

If x, y $\notin N(D)$ then (x, v, y) is a $x - y$ path in $N(D_1)$

Thus $N(D_1)$ is connected so that D_1 nc2oe-set of G.

Theorem 2.4: A nc2oe D of a graph G is minimal nc2oe if and only if for every $u \in D$ one of the following holds

1) $p_n[u, D] \neq \emptyset$
2) There exist two vertices x,y $\in N(D)$ such that $x - y$ path in $< N(D) >$ contains at least one vertex of $N(D) - N(D - \{u\})$.

Proof:
Let D be a minimal nc2oe-set of G.

Let u $\in D$, $D_1 = D - \{u\}$ and D_1 is not nc2oe-set

Then D_1 is not dominating set of G, or $<N(D_1)>$ is disconnected

If D_1 is not dominating set of G then $p_n[u, D] \neq \emptyset$

If $<N(D_1)>$ is disconnected then there exists two vertices x, y $\in N(D_1)$, such that there is no $x - y$ path in $<N(D_1)>$

Since $< N(D) >$ is connected it follows that every $x - y$ path in $< N(D) >$ contains at least one vertex of $N(D) - N(D - \{u\})$.

Conversely, If D is a nc2oe-set of G satisfying the above conditions of the theorem then D is 1-minimal and hence the result by above theorem 2.3

3. NEIGHBORHOOD CONNECTED TWO OUT DEGREE EQUITABLE DOMINATION NUMBER FOR SOME STANDARD GRAPHS

Observation 3.1: For any graph G with n vertices, 2$\leq \gamma_{nc2oe}(G) \leq p$

Theorem 3.2: If G is a star then $\gamma_{nc2oe}(k_{1,p}) = p - 2$

Proof: Let V=$\{v, u_1, u_2 \dots u_p\}$ be the vertices set of $k_{1,p}$

Let D= $\{v, u_1, u_2 \dots u_{p-2}\}$ be a domination set of G and $V - D$= $\{u_{p-1}, u_p\}$

Now, $v \epsilon D$ then $od_D(v) = |N(v) \cap V - D|$
$= |\{u_1, u_2 \dots u_{p-1}, u_p\} \cap \{u_{p-1}, u_p\}|$
$= |\{u_{p-1}, u_p\}| = 2$

Now, $u_i \epsilon D$ then $od_D(u_i) = |N(u_i) \cap V - D|$
$= |\{v\} \cap \{u_{p-1}, u_p\}|$
$= \emptyset = 0$

Then $|od_D(v) - od_D(u_i)| \leq 2$. For any $u_i \epsilon D$

So D is two out degree equitable dominating set , N(v)= $\{u_1, u_2 \dots u_p\}$ N $(u_i) = v$ for all $u_i \ \epsilon \ D$

N(D)=V since <V> is connected

<N(D)> is connected

So D is a nc2oe-set and D be minimal nc2oe set

$then\ \gamma_{nc2oe}(k_{1,p}) = p - 2$

Theorem 3.3: The nc2oe number of a complete graph is 2, i.e $\gamma_{nc2oe}(k_p) = 2$

Proof: Let V=$\{u_1, u_2 \dots u_p\}$ be the vertices set of k_p

Let D= $\{u_1, u_2\}$ be a dominating set of G and V-D=$\{u_3, u_4 \dots u_p\}$

Now, $u_1 \epsilon D$ then $od_D(u_1) = |N(u_i) \cap V - D|$
$= |\{u_2, u_3, u_4 \dots u_p\} \cap \{u_3, u_4 \dots u_p\}|$
$=|\{u_3, u_4 \dots u_p\}|$=p-2

Similarly $od_D(u_2) = p - 2$

Then $|od_D(u_1) - od_D(u_2)| \leq 2$. For any $u_i, u_j \epsilon D$

So D is two out degree equitable dominating set and N $(u_i) = v$ for all $u_i \epsilon D$
N(D) =N$(u_1) \cup N(u_2)$
$= \{u_2, u_3, u_4 \dots u_p\} \cup \{u_1, u_3, u_4 \dots u_p\}$
$= \{u_1, u_2 \dots u_p\} = V$

<N(D)>=V and <V> is connected in k_p

So <N(D)> is connected
$\gamma_{nc2oe}(k_p) \leq 2$ and 2≤ $\gamma_{nc2oe}(k_p)$

Then $\gamma_{nc2oe}(k_p) = 2$

Theorem 3.4: The nc2oe number of a complete bipartite graph is
$$\gamma_{nc2oe}(k_{r,t}) = \begin{cases} 2 & if\ |r-t| \leq 2 \\ r+t & otherwise \end{cases}$$

Proof: Let V=$\{u_1, u_2, u_3 \dots u_r, v_1, v_2, v_3, \dots v_t\}$ be the vertices set of $k_{r,t}$ and $\{u_1, u_2, u_3 \dots u_r\}$ and $\{v_1, v_2, v_3, \dots v_t\}$ be the partition of V.

Case-(i): $|r - t| \leq 2$

Let D= $\{u_i, v_j\}$ be a dominating set of G and

V-D=$\{u_1, u_2, u_3 \dots u_{i-1}, u_{i+1} \dots u_r, v_1, v_2, v_3, \dots v_{j-1}, v_{j+1} \dots v_t\}$

Now, $u_i \epsilon D$ then $od_D(u_i) = |N(u_i) \cap V - D|$
$=|\{v_1, v_2, v_3, \dots v_{j-1}, v_{j+1} \dots v_t\} \cap \{u_1, u_2, u_3 \dots u_{i-1}, u_{i+1} \dots u_r, v_1, v_2, v_3, \dots v_{j-1}, v_{j+1} \dots v_t\}$
$=|\{v_1, v_2, v_3, \dots v_{j-1}, v_{j+1} \dots v_t\}|$=t-1

$if\ v_j \epsilon D$ then $od_D(v_j) = |N(v_j) \cap V - D|$
$=|\{u_1, u_2, u_3 \dots u_{i-1}, u_{i+1} \dots u_r\} \cap \{u_1, u_2, u_3 \dots u_{i-1}, u_{i+1} \dots u_r, v_1, v_2, v_3, \dots v_{j-1}, v_{j+1} \dots v_t\}$
$=|\{u_1, u_2, u_3 \dots u_{i-1}, u_{i+1} \dots u_r\}|$=r-1

$|od_D(u_i) - od_D(v_j)| = |t - 1 - r + 1| \leq 2$

Then $|od_D(u_i) - od_D(v_j)| \leq 2$. For any $u_i, v_j \epsilon D$

So D is two out degree equitable dominating set
N(D)=N$(u_i) \cup N(v_j)$
= V

<N(D)>=V and <V> is connected in $k_{m,n}$

So <N(D)> is connected
$\gamma_{nc2oe}(k_{r,t}) \leq 2$ and 2≤ $\gamma_{nc2oe}(k_{r,t})$
$\gamma_{nc2oe}(k_{r,t}) = 2$

Case-(ii): $|r - t| \geq 2$ and r,t ≥ 2

Let V=$\{u_1, u_2, u_3 \dots u_r, v_1, v_2, v_3, \dots v_t\}$ be the vertices set of $k_{r,t}$

Let D=$\{u_1, u_2, u_3 \dots u_r, v_1, v_2, v_3, \dots v_t\}$ be the dominating set of $k_{r,t}$

And V-D= $\emptyset$

Clearly V is nc2oe-set

Then $\gamma_{nc2oe}(k_{r,t}) \leq r + t. and\ r + t \leq \gamma_{nc2oe}(k_{m,n})$

Then $\gamma_{nc2oe}(k_{r,t}) = r + t$

Theorem 3.5: For any cycle C_P then $\gamma_{nc2oe}(C_P) = \begin{cases} \left\lfloor \frac{p}{2} \right\rfloor & p \equiv 3\ (mod\ 4) \\ \left\lceil \frac{p}{2} \right\rceil & otherwise \end{cases}$

Proof: Let V=$\{u_1, u_2 \dots u_p\}$ be the vertices set of C_P

Let D be two out degree equitable dominating set of C_P

Let $D_1 = \begin{cases} D & if\ \ p \equiv 0\ (mod\ 4) \\ D \cup \{v_p\} & if\ \ p \equiv 1\ or\ 2\ (mod\ 4) \\ D \cup \{v_{p-1}\} & if\ \ p \equiv 3\ (mod\ 4) \end{cases}$

Clearly D_1 is nc2oe-set of C_P then <D> contains atmost one isolated verteces

And <N (D)>= $\begin{cases} c_p & if\ \ p \equiv 0\ (mod\ 4) \\ p_{p-1} & otherswise \end{cases}$

Then <N (D)>is connected

Hence |D| $\geq \gamma_{nc2oe}(C_P) = \begin{cases} \left\lfloor \frac{p}{2} \right\rfloor & p \equiv 3\ (mod\ 4) \\ \left\lceil \frac{p}{2} \right\rceil & otherwise \end{cases}$

Hence theorem

Theorem 3.6: The neighborhood connected two out degree equitable domination number of Peterson graph is 5

Proof:

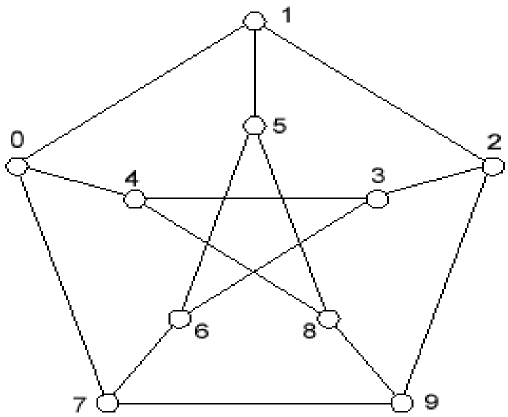

Figure-2

Let us consider D={ 0,1,2,7,9} or D= {3,4,5,6,8}

If D= {0, 1, 2, 7, 9} and V-D = {3, 4, 5, 6, 8}

Now $od_D(0) = 1, od_D(1) = 1, od_D(2) = 1, od_D(7) = 1, od_D(9) = 1$

So D is minimal two out degree equitable dominating set

And <N(D)> = < V> is connected
γ_{nc2oe}(G)=5

Theorem 3.7: For any Path P_p, $\gamma_{nc2oe}(P_p) = \left\lceil\frac{p}{2}\right\rceil$

Proof: Let P_p={$v_1, v_2, v_3 \ldots v_p$}

If p not ≡ 1 (mod 4)

Then D= $\{v_j, j = 2k, 2k + 1 and\ k\ is\ odd\}$

Since G is path, then deg (v) ≤ 2, clearly D is neighborhood connected two out degree equitable dominating set. set N<D> is connected

So D is a nc2oe-set of P_p

If p ≡ 1 (mod 4)

Then $D_1 = D \cup \{v_{p-1}\}$ is a a nc2oe-set of P_p

Hence $\gamma_{nc2oe}(P_p) \leq \left\lceil\frac{p}{2}\right\rceil$

Since $\gamma_{nc}(G) = \left\lceil\frac{p}{2}\right\rceil$ and $\gamma_{nc}(G) \leq \gamma_{nc2oe}(G)$

We have $\left\lceil\frac{p}{2}\right\rceil \leq \gamma_{nc2oe}(P_p)$ and $\left\lceil\frac{p}{2}\right\rceil \leq \gamma_{nc2oe}(P_p)$
$\gamma_{nc2oe}(P_p) = \left\lceil\frac{p}{2}\right\rceil$

Theorem 3.8: For the Wheel W_p, the neighborhood connected two –out degree equitable domination number is:
$\gamma_{nc2oe}(W_p)\begin{cases} 2 & if\ p = 4,5 \\ p-4 & if\ \ p \geq 6 \end{cases}$

Proof: Let W_p be a when with m-1 vertices on the cycle and a single vertex at the center.

Let V (W_p) = {u,v_1, v_2, $v_3 \ldots v_{p-1}$}, where u is the center and v_i (1≤ $i \leq p-1$) is on the cycle. Clearly deg (v_i) = 3 for all 1≤ $i \leq p-1$ and deg (u) = $p-1$.

Clearly p≥ 4. We have the following cases

Case -1: p=4 and 5
If p=4 then W_4 forms a complete graph then by theorem 3.1 $\gamma_{c2oe}(W_4) = 2$

If p=5. Let us take D= {u, v_i} and V-D = { $v_i, v_i, \ldots v_{i-1}, v_{i+1} \ldots v_{p-1}$} since u is adjacent with v_i for all i 1≤ $i \leq 4$, V-D ⊂ N (u) so N(u)∩ V-D ⊂ V-D
$od_D(u)$= |N(u)∩ V-D| = |V-D|=3

Now for v_i
Since deg (v_i) =3 and v_i is adjacent to u∈D then N (v_i) = {u, v_j, v_k} and $N(v_i)$∩ V-D= {v_j, v_k}

$od_D(v_i)$= |N(u)∩ V-D| = 2

$|od_D(u) - od_D(v_i)| = 1 \leq 2$ and clearly <N(D)>= V is connected

So D is neighborhood connected two degree equitable dominating set

Hence $\gamma_{nc2oe}(W_p)$ ≤2 and 2≤ $\gamma_{nc2oe}(W_p)$

Hence $\gamma_{nc2oe}(W_p)$ =2

Case-2: p≥6
In this case deg (u) =p, while deg (v_i) =3 for all i, 1≤ $i \leq 5$,

Let us take D={u, $v_1, v_2, v_3 \ldots v_{p-4},$ } be a dominating set and $V - D = \{v_{p-3}, v_{p-2,} v_{p-1}\}$ since u is adjacent with v_i for all i, V-D ⊂ N(u)
So N(u)∩ V-D ⊂ V-D
$od_D(u)$= |N(u)∩ V-D| = 4

Now for v_i and v_j

If v_i and v_j is adjacent N(v_i)= {u, v_j , v_k} and N(u)∩ $V - D$= {v_k}
$od_D(v_i$)= |N(u)∩ $V - D| = 1$

If v_i and v_j are not adjacent but v_i and v_j are adjacent with u so N(u)∩ V-D contains two elements so
$od_D(v_i)$= |N(u)∩ $V - D| = 2$

So for any elements u,v ∈ D
$|od_D(u) - od_D(v)| \leq 2$ and clearly <N(D)>= V is connected

So D is neighborhood connected two degree equitable dominating set

Hence $\gamma_{nc2oe}(W_p)$=$p - 4$

Theorem: 3.9: For the double star$S_{r,t}$, the neighborhood connected two –out degree equitable domination number is: $\gamma_{nc2oe}(S_{r,t})$= $r + t$

Proof: Let $\{\boldsymbol{u}, u_1, u_2, u_3 \ldots u_r, v, v_1, v_2 v_3 \ldots v_t\}$ are the vertices of $S_{r,t}$ and all u_i is adjacent to u and v_i is adjacent to v.

Here { $u_1, u_2, u_3 \ldots u_r, v_1, v_2 v_3 \ldots v_t\}$ be the isolated vertices and D= {u, v} is support vertices and is connected

So let us take D={ $u_1, u_2, u_3 \ldots u_r, v_1, v_2 v_3 \ldots v_t\}$ and $V - D$ ={u,v}

Since every vertices has one neighborhood so clearly D is two out degree equitable dominating set N(u_i)=u and N(v_i)=v for all i. <N(D)>= <u, v> is connected $\gamma_{nc2oe}(S_{r,t})$= $r + t$

Theorem 3.10: Let T be a tree with two pendent vertices and two support vertices then $\gamma_{nc2oe}(T) = p - 2$

Proof: Let T be any tree of order at least two.

Let V(T)= $\{v_1, v_2 \ldots v_p\}$

If there exists two pendent vertices $\{v_1, v_p\}$ which is adjacent to two support vertices $\{v_2, v_{p-1}\}$ respectively

Let D= $\{v_2, v_3 \ldots v_{p-1}\}$ and V-D= $\{v_1, v_p\}$

Clam D is two out degree equitable dominating set

Since G is a tree and V-D= $\{v_1, v_p\}$ are pendent vertices

Since D is connected and $\{v_2, v_{p-1}\}$ is support vertex to $\{v_1, v_p\}$ then D is dominating set

Let v_i ∈ D such that v_i is support vertex and each support vertex adjust vertex then $od_D(v_i) =$ |N(v_i) ∩ $V - D$ |= 1

Let v_i ∈ D such that v_i is not support vertex so
$od_D(v_i) =$ |N(v_i) ∩ $V - D$ |= 0
$|od_D(u) - od_D(v)| \leq 2$

So <N(D)> is connected two out degree equitable dominating set and D is minimal neighborhood connected two out degree equitable dominating set

Hence $\gamma_{nc2oe}(T) = p - 2$.

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